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ATTENUATION OF MODIFIED MERCALLI INTENSITY FOR SMALL EPICENTRAL DISTANCE IN CALIFORNIA

by

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ABSTRACT

Four regression equations describing the attenuation of Modified Mercalli Intensity (MMI) in terms of local earthquake magnitude, M, epicentral distance, R, focus depth, H, fault size, S, and the local geological conditions at the recording site (s = 0 for alluvium, s = 2 for basement rocks and s = 1 for intermediate sites) have been investigated. The equation (2.29):

$$I_{MM} = 1.5\ M - A - B\ell n\Delta - C\Delta/100 - Ds$$

where I_{MM} is the level on MMI scale and $\Delta = (R^2 + H^2 + S^2)^{1/2}$, leads to the best fit. For R = 0, H = 10 to 15 km and s = 0 the results of this equation become equivalent to $I_{MM} = 1.5\ M - 1.5$ proposed by Gutenberg and Richter (1956). The predicted I_{MM} is about 1/2 intensity level higher for alluvium sites relative to the basement rock sites.

INTRODUCTION

Many intensity scales have been proposed and used to describe destructive effects of strong earthquake shaking on man made structures and on the geological environment. Since the sixteenth century more than 50 different intensity scales have been proposed and used all over the world. A summary on some of these scales can be found in the papers by Gorshkov and Shenkareva (1958) and by Barosh (1969). With the development of instrumental seismology and earthquake engineering and with more recent rapid growth of the number of recorded strong motion accelerograms, it has become possible to associate various intensity levels with the quantitative measures of strong ground motion (e.g., Medvedev 1953, Kawasumi, 1951).

Rapid development of the statistical methods for characterization of the expected levels of strong ground motion for use in design (e.g., Anderson and Trifunac 1978) motivated detailed probabilistic analyses of the amplitudes of strong shaking in terms of the Modified Mercalli Intensity Scale (Wood and Neumann, 1931), because many areas of the central and eastern United States as well as many other parts of the world still do not have adequate instrumental data base to characterize seismicity (e.g., Trifunac and Brady 1975, Trifunac and Anderson 1977, 1978a,b, Trifunac 1976). At present the strong motion data recorded in California and in Japan is begining to become adequate for some preliminary empirical scaling of strong ground motion in terms of magnitude and intensity scales. When this data is employed in engineering design considerations elsewhere, special care must be exercised to account for systematic differences that exist between different magnitude and intensity scales in different regions as well as in different ways the same scales are employed by different organizations.

With a more systematic instrumental coverage of the central and eastern United States and the studies of the appropriate magnitude scales (e.g., Nuttli and Herrmann, 1982), there is at present an increasing tendency to characterize all earthquakes by a magnitude scale for the purpose of developing the seismic risk estimates. Usually this necessitates conversion of older earthquake catalogues with data in terms of intensity scale and assignment of the new equivalent magnitude scales. Furthermore, conversions from locally applicable magnitude scales to the magnitude scales used in California where most of the strong motion data has been recorded are required. This conversion process is further complicated by different anelastic absorption coefficients in western versus central and eastern United States, (Nuttli and Herrmann, 1981). Though there is some evidence to suggest that it might be better to develop seismic risk estimates on the basis of models which consider intensity scales only (Anderson, 1979) it is useful in general to better understand the relationship between the magnitude and intensity scales.

Many empirical equations have been proposed to relate the observed intensities with computed magnitude scales (e.g., Gutenberg and Richter, 1956; Nuttli and Zollweg,1974; Howell and Schultz, 1975). However, the effects of fault size, source depth and the geological setting (alluvium versus basement rock) of the recording station are usually not analyzed in detail. Thus, to better understand the nature of strong motion accelerograms recorded in California, and to facilitate development of rational and well calibrated empirical equations for correct transfer of the characteristics of strong motion accelerograms into other regions outside California, in this paper we analyze the relationship between the reported earthquake magnitude and the observed Modified Mercalli Intensity at a site, which is less than 50 km away from the epicenter. We choose distances less than 50 km to focus on the near source effects and to de-emphasize the effects of attenuation.

I. DATABASE

The data on reported M.M.I. corresponds to the data collected from earthquakes in the Western United States from the year 1933 to 1971 that were used previously by the authors in the statistical studies of the dependence of Fourier Amplitude Spectra, Response Envelope Spectra, peak amplitudes and other strong-motion parameters on magnitudes, epicentral distances, intensity levels (MMI) and local site classification and depth of alluvium beneath recording stations (e.g., Trifunac and Brady, 1975). More than 50% of this data set corresponds to the San Fernando earthquake of 1971 with magnitude 6.5.

II. EMPIRICAL EQUATIONS

The empirical equations used in this analysis relate the MM intensity, I_{MM}, with the epicentral distance, R, focal depth, H, source dimension, S, magnitude, M, and local geology, characterized by site condition parameter, s, or depth of sediments, h. The source dimension, $S = \hat{S}(M,R)$, is an empirically determined function of magnitude, M, and epicentral distance R (Trifunac and Lee, 1985) given by

$$S = \hat{S}(M,R) = S(M)(1 - \exp(\ell n(.1)R/S(M)) \tag{2.1}$$

where S(M) is a linear function of magnitude, M, such that

$$\begin{aligned} \text{for} \quad M &= 3 \qquad S(M) = 0.2 \text{ km} , \\ M &= 6.5 \qquad S(M) = 17.5 \text{ km} \end{aligned} \tag{2.2}$$

The parameters, R, H and S, all measured in km, are lumped together in this analysis as

$$\Delta = (R^2 + H^2 + S^2)^{1/2} , \tag{2.3}$$

and Δ is used as a measure of the "representative" distance from the earthquake source.

Previous analyses related the MM intensity, I_{MM}, with only the epicentral distance, R, focal depth, H, and maximum intensity, I_o, or magnitude, M (e.g., Guttenberg and Richter, 1942; Neumann, 1954; Cornell, 1968, Ergin, 1969; Milne and Davenport, 1969; Brazee, 1972; Howell and Schultz, 1975). The present analysis is an attempt to generalize such empirical relations to include the effects of source dimensions and the local site geology.

The first group of regression equations could be chosen by assuming that the MM intensity, I_{MM}, is proportional to a certain power of the earthquake energy density, E (Howell and Schultz, 1975):

$$I_{MM} = \beta E^{\alpha} \tag{2.4}$$

Assuming the attenuation of energy with distance to be described by

$$E = K\Delta^{b} \, e^{c\Delta} \tag{2.5}$$

where K is a constant, b is associated with the grometric spreading and c is a constant describing the rate of absorption. Substituting (2.5) into (2.4) gives

$$I_{MM} = \beta K^{\alpha} \Delta^{b\alpha} \, e^{c\alpha\Delta} \tag{2.6}$$

At the epicenter R = 0, Δ = H, I_{MM} is the maximum intensity I_{MAX}, so that

$$I_{MAX} = \beta K^{\alpha} \, H^{b\alpha} \, e^{c\alpha H} \qquad (2.7$$

Combining (2.6) and (2.7) gives an equation for I_{MM} of the form:

$$I_{MM} = I_{MAX} \, \Delta^{B} \, e^{A+c\Delta/100} \tag{2.8}$$

where A, B, and C are all constants to be determined. This last term in the exponent for Δ is divided by 100 to control the numerical value of C only. To include the effects of local geology of the site on the intensity, (2.8) is modified to

$$I_{MM} = I_{MAX} \, \Delta^{B} \, e^{A+c\Delta/100} + a\,s + b \tag{2.9}$$

when using the site parameter, s, to describe the site (s = 0 for alluvium, s = 2 for basement rock, s = 1 for intermediate sites). Instead the depth of sediments, h, can be used:

$$I_{MM} = I_{MAX}\, \Delta^{B}\, e^{A+C\Delta/100} + a\,h + b \qquad (2.10)$$

Both (2.9) and (2.10) are variations of (2.8), and show that the attenuation of intensity I_{MM} with distance is of the form

$$\partial I_{MM}/\partial\Delta = I_{MM}(B/\Delta + C/100) \qquad (2.11)$$

and this results in the attenuation of intensity with distance also being a linear function of intensity. For attenuation to be only a function of distance Δ and be uniform for earthquakes of all magnitudes and intensities, (2.8) can be changed to become

$$I_{MM} = I_{MAX} - \Delta^{B}\, e^{A+C\Delta/100} \qquad (2.12)$$

so that it is the difference $I_{MAX} - I_{MM}$ and not I_{MM}/I_{MAX} in (2.8) that enters into the attenuation law. (2.9) and (2.10) can be modified to

$$I_{MM} = I_{MAX} - \Delta^{B}\, e^{A+C\Delta/100} + a\,s + b, \qquad (2.13)$$

and $$I_{MM} = I_{MAX} - \Delta^{B}\, e^{A+C\Delta/100} + a\,h + b\ . \qquad (2.14)$$

In both cases then, $\partial I_{MM}/\partial\Delta$ is only a function of Δ, and not I_{MM} itself. The regression analysis for any of the four equations (2.9), (2.10), (2.13) or (2.14) can be performed in two steps:

(1) Either starting from (2.8) or (2.12), linear regression can be performed to estimate the constants A, B and C.

(2) Calculate the residuals, $\varepsilon = I_{MM} - \hat{I}_{MM}$, the difference of the recorded and the estimated intensities from the first step. The residuals are then fitted with the site parameters of local geology to estimate the constants a and b.

The second group of equations which we consider here can be formulated by assuming that the MM intensity, I_{MM}, is proportional to the logarithm of the seismic energy density, E, at the site (Howell and Schultz, 1975):

$$I_{MM} = \alpha \ell n E + \beta \tag{2.15}$$

which together with (2.5) gives

$$I_{MM} = \alpha K + \alpha b \ell n \Delta + \alpha c \Delta + \beta. \tag{2.16}$$

Again at the epicenter R = 0, $\Delta = H$, I_{MM} is the maximum intensity I_{MAX}, so that

$$I_{MAX} = \alpha K + \alpha b \ell n H + \alpha c H + \beta \tag{2.17}$$

Combining (2.16) and (2.17) gives an equation for I_{MM} of the form:

$$I_{MM} = I_{MAX} - A - B \ell n \Delta - C\Delta/100 , \tag{2.18}$$

where A, B, and C are all constants to be determined. The last term with Δ is divided by 100 again only to control the numerical value of C. To include the effect of local geology of the site on the intensity, (2.18) is modified to

$$I_{MM} = I_{MAX} - A - B \ell n \Delta - C\Delta/100 - Ds \tag{2.19}$$

for using the site classification s = 0, 1 or 2, and

$$I_{MM} = I_{MAX} - A - B \ell n \Delta - C\Delta/100 - Dh \tag{2.20}$$

for using the depth of sediments h. The constants A, B, C and D can be estimated by linear regression in one step.

In all of the above equations, the MM intensity, I_{MM}, is related to the maximum intensity I_{MAX} by terms describing its attenuation with distance and local geology at the site. If the maximum intensity for the earthquake is not available, it can be approximated by (1.5M - 1.5), based on Gutenberg and Richter's (1956) empirical relation between magnitude and maximum intensity:

$$M = 1 + \frac{2}{3} I_{MM} \quad (2.21)$$

A new set of equations can thus be obtained by substituting 1.5M - 1.5 for I_{MAX} and since a constant term appears in all of these equations, the constant term 1.5 can be omitted. The new set of equations then becomes:

(2.13) becomes $$I_{MM} = 1.5M - \Delta^{B} e^{A+C\Delta/100} + a\,s + b \quad (2.22)$$

(2.14) becomes $$I_{MM} = 1.5M - \Delta^{B} e^{A+C\Delta/100} + a\,h + b \quad (2.23)$$

(2.19) becomes $$I_{MM} = 1.5M - A - B\ell n\Delta - C\Delta/100 - Ds \quad (2.24)$$

(2.20) becomes $$I_{MM} = 1.5M - A - B\ell n\Delta - C\Delta/100 - Dh \quad (2.25)$$

Equations (2.22) through (2.25) have been considered in the regression analysis of the available database.

III. RESULTS

Regression analyses were performed on all four equations: (2.22), (2.23), (2.24) and (2.25). However, only the results for (2.24) will be presented since (2.24) gives the best fit and all of its coefficients are statistically significant. The other equations either resulted in one of the two attenuation coefficients for Δ, having wrong sign or in insignificant coefficients.

Figure 3.1 is a plot of the estimated MMI levels versus epicentral distance R for a focal depth of H = 5 km and for magnitudes ranging from 3 to 8 in steps of .5. The figure shows that all MMI levels have the same attenuation rate with distance at all magnitude levels. At the epicenter, R = 0, the MMI levels reach the maximum intensity level, I_{MAX} and their values agree favorably with Gutenberg and Richter's (1956) empirical relation between magnitude and maximum intensity (equation (2.21)). Figure 3.2 is the same plot but with distance shown in logarithmic scale.

To test the quality of fit, the coefficient of correlation has been calculated from the formula

$$\rho = \frac{\sum(I_{MM} - \bar{I}_{MM})\ (\hat{I}_{MM} - \bar{\hat{I}}_{MM})}{\sqrt{\sum(I_{MM} - \bar{I}_{MM})^2 \sum(\hat{I}_{MM} - \bar{\hat{I}}_{MM})}} \qquad (3.1)$$

where $\bar{I}_{MM}$ is the mean of the observed MMI levels and $\bar{\hat{I}}_{MM}$ the mean of the estimated MMI levels. The summation is over all the MMI levels used in the regression analysis. A plot of the estimated versus observed MMI levels is given in Figure 3.3. It gives a correlation coefficient of $\rho = .707$.

Table 3.1 gives the regression coefficients A, B, C and D in (2.24) together with their 90% confidence intervals. The coefficients in

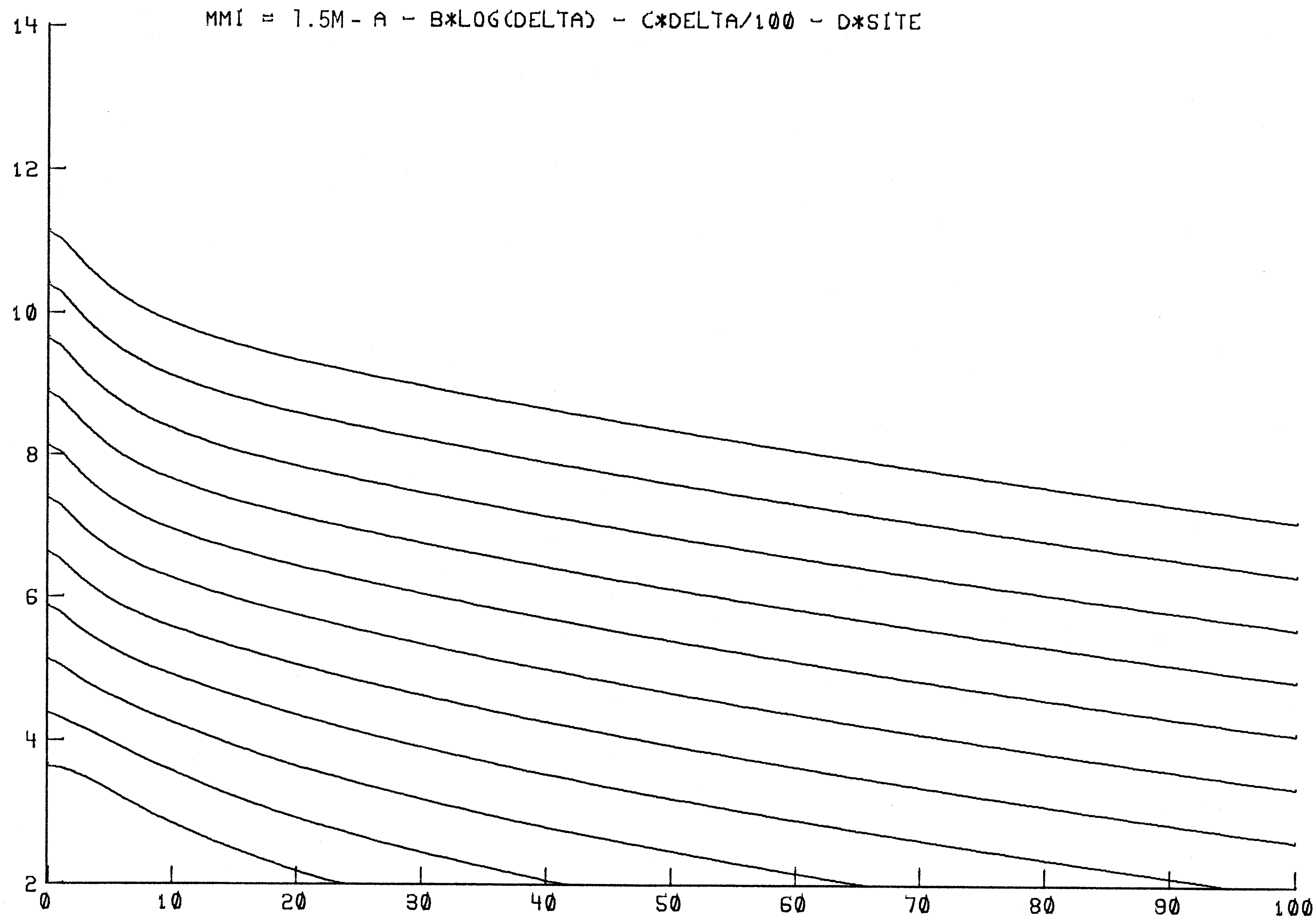
MMI = 1.5M - A - B*LOG(DELTA) - C*DELTA/100 - D*SITE
14
12
10
8
6
4
2
0
10
20
30
40
50
60
70
80
90
100

Figure 3.1

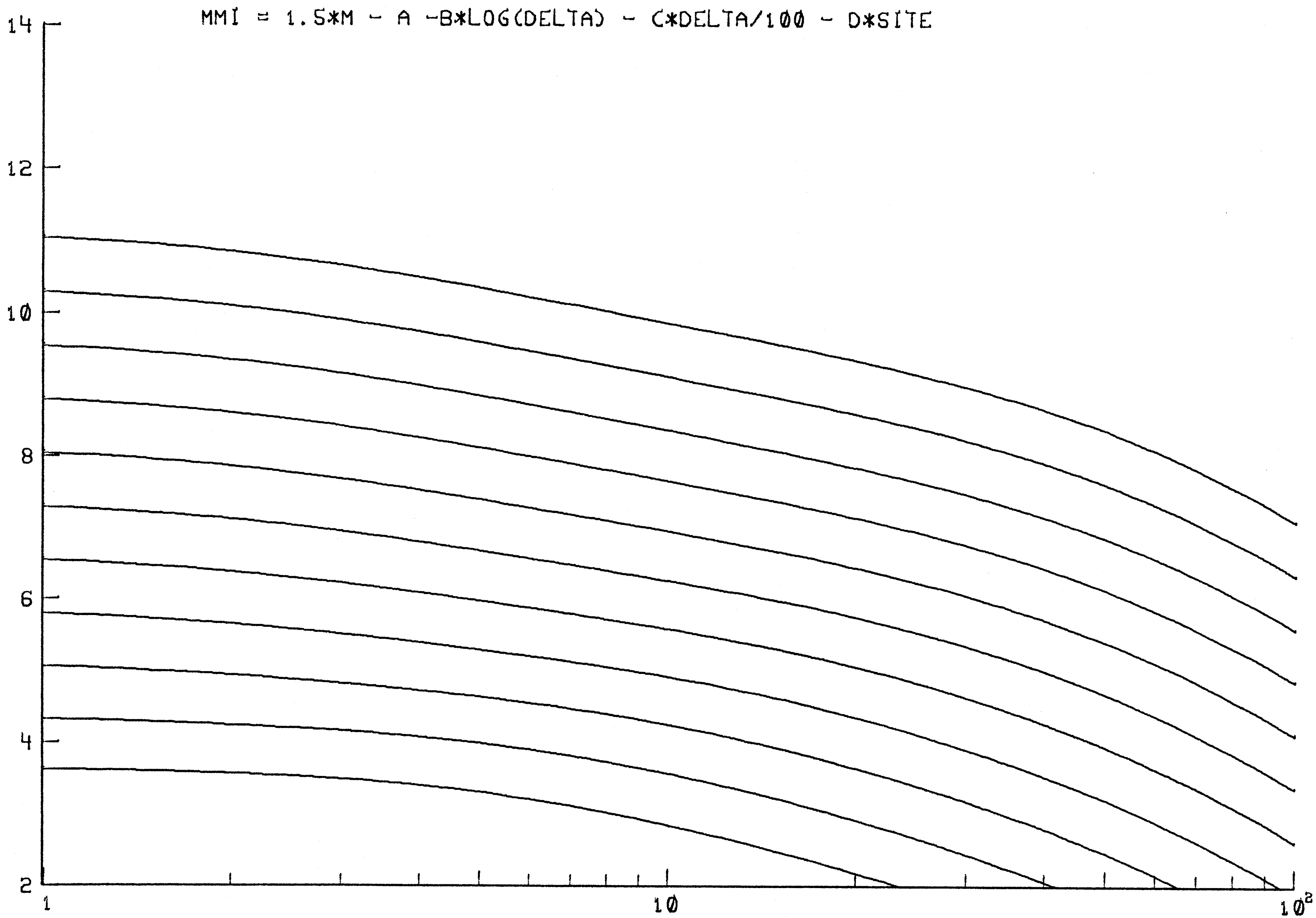

Figure 3.2

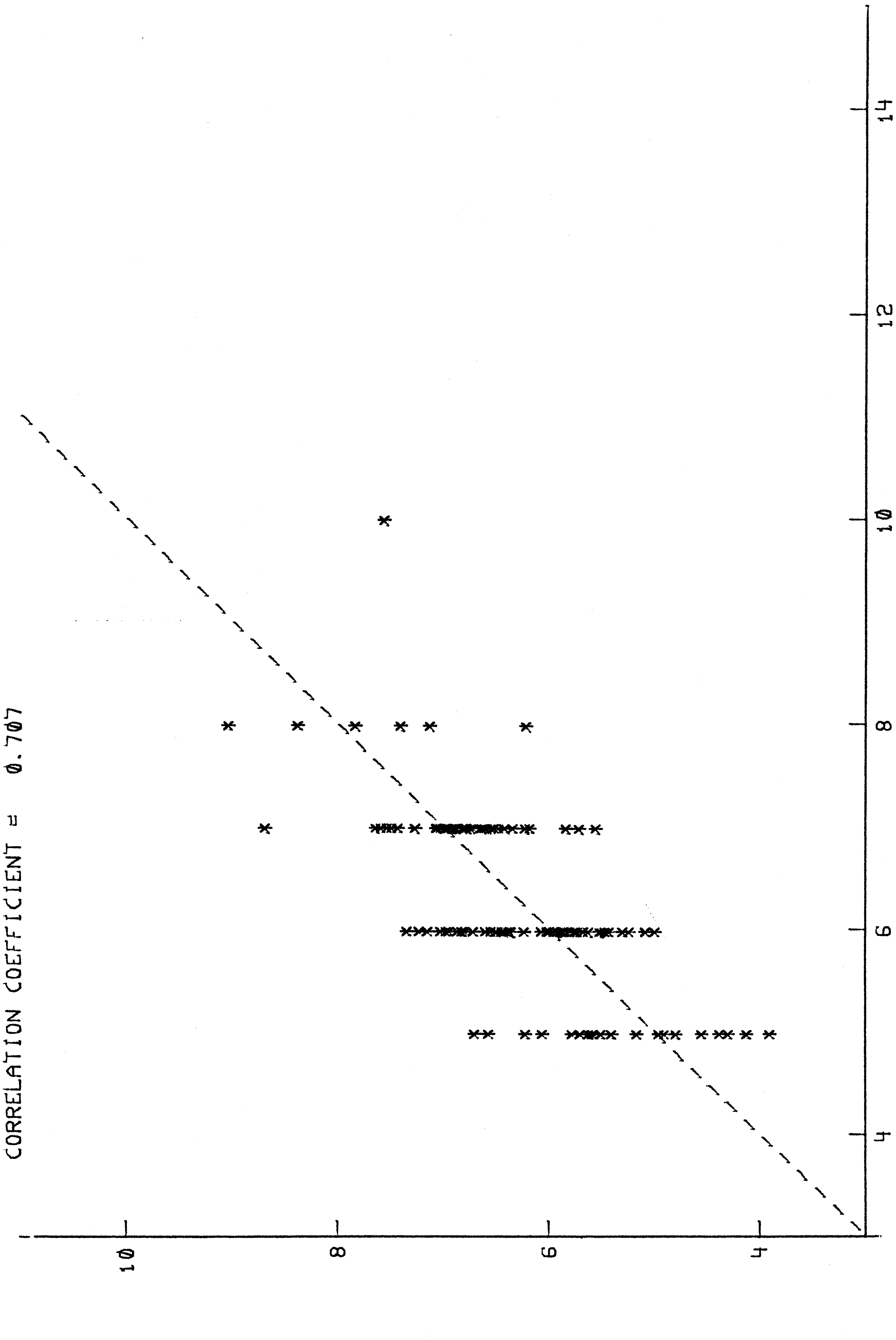

Figure 3.3

TABLE 3.1

A	=	- 1.12	±	1.49
B	=	0.856	±	0.555
C	=	1.50	±	0.14
D	=	0.26	±	0.13

Table 3.1 show a number of trends which can be compared with other related results. The coefficient B = 0.856, for example, multiplying $-\ell n\Delta$ in (2.24) has a value which can be compared with the coefficient C_0 in $C_0 \log_{10}\Delta$ in describing the frequency dependent attenuation of Fourier spectrum amplitudes (Trifunac and Lee, 1985). After correcting for the difference between the natural and common logarithms ($\ell nx = 2.305 \log_{10}x$) it is seen that B would "correspond" to $C_0 = 1.97$ and this would be associated with C_0 describing attenuation of the high frequency spectral amplitudes (> 10 Hz). Since the 90% confidence interval for B is from 0.30 to 1.41 it is seen that it overlays the entire range of C_0 (from 0.54 at 0.2 Hz to 1.69 at 15 Hz, see Trifunac and Lee, 1985) and it can be concluded that both attenuation rates are in good agreement, but that the attenuation of intensity levels for distances much less than 50 km might be better described by the attenuation rate for high frequency waves. With Δ increasing the term $C\Delta/100$ begins to contribute to even faster intensity attenuation rate.

The estimate for D = 0.26 suggests that the intensities recorded on alluvium (s = 0) tend to be about 1/2 intensity level higher than the intensities recorded on basement rock (s = 2). This is in qualitative agreement with reports that strong shaking on alluvium may lead to larger reported intensities (e.g., Richter, 1958) relative to the intensities reported for sites on the basement rock. Letting R = 0, equation (2.24) reduces to

$$I_{MM} = 1.5\ M - f(H,S)$$

where

$$f(H,S) = A + B\ell nH + CH/100 + DS$$

for s = 0 and H = 5, 10, 15 and 20 km. f(H,S) yields 0.34, 1.00, 1.45 and 1.74. For s = 2 and H = 5, 10, 15 and 20 km, f(H,S) is equal to 0.86, 1.52, 1.95 and 2.26, respectively. Remembering the Gutenberg and Richter (1956) relationship $I_{MM} = 1.5\ M - 1.5$ it is seen that (2.24) reduces to their equation for s = 0 and H ~ 15 km or s = 2 and H = ~10 km. Since most earthquakes in California occur at depths between 0 and 15 km the estimates in this paper based on (2.24) appear to be consistent with previous experience.

IV. CONCLUSIONS

In this paper an empirical scaling equation expressing the expected level of shaking on the Modified Mercalli Intensity scale for an earthquake with local magnitude M, focus at depth H, at epicentral distance R and in terms of simplified classification of local site conditions has been investigated. The quantitative properties of this equation are in agreement with Gutenberg and Richter's (1956) equation and the common expectation that the reported intensity levels on alluvium tend to be higher than those reported on the basement rock.

Other related functional relationships have been studies but did not lead to regression coefficients which are significantly different from zero, or are consistent with other independent physical constraints.

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