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STRUCTURAL RESPONSE TO TRAVELING SEISMIC WAVES[a]

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INTRODUCTION

In most analyses of the response of structures to earthquake ground motions, the seismic excitation is assumed to be identical at all points along the base of the structure. However, this assumption only approximates the excitation actually applied to the structure, since it does not account for the spatial variations of the incident seismic waves. These spatial variations cause different locations along the structure foundations to be subjected to excitations that differ in both amplitude and phase. Such excitations can have an important effect on structural response.

The influence of traveling seismic waves has been studied for several different types of structures, such as rigid footings (12,15), conventional buildings (14,17), bridges (1,5,9), buried pipelines and tunnels (8,11), earth dams (6,10), and nuclear power plants (13,19). These investigations have provided insights into traveling wave effects and, in addition, have pointed out where deficiencies in the understanding of such effects still exist. The main insights are that: (1) Traveling wave effects become pronounced when the wavelengths of the incident waves are comparable to or less than a characteristic length of the structure or foundation; (2) the net translational excitation of shallow foundation elements caused by nonvertically incident P- and S-waves can be reduced relative to the excitations caused by vertically incident waves; (3) nonvertically incident SH-waves lead to torsional excitation of the structure and nonvertically incident P- and SV-waves lead to rocking excitation; and (4) traveling Rayleigh waves may excite all six

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components of response, depending on their direction of incidence relative to the structure. The primary deficiencies that still exist are related to: (1) The lack of suitable recorded strong-motion data necessary to guide the specification of spatially varying input motions for seismic response analyses; and (2) the lack of available engineering guidelines for assessing the behavior of structures subjected to traveling seismic waves.

This paper presents results from the first phase of an ongoing research program directed toward the second deficiency noted in the foregoing, i.e., the development of engineering guidelines for assessing the effects of traveling waves on the

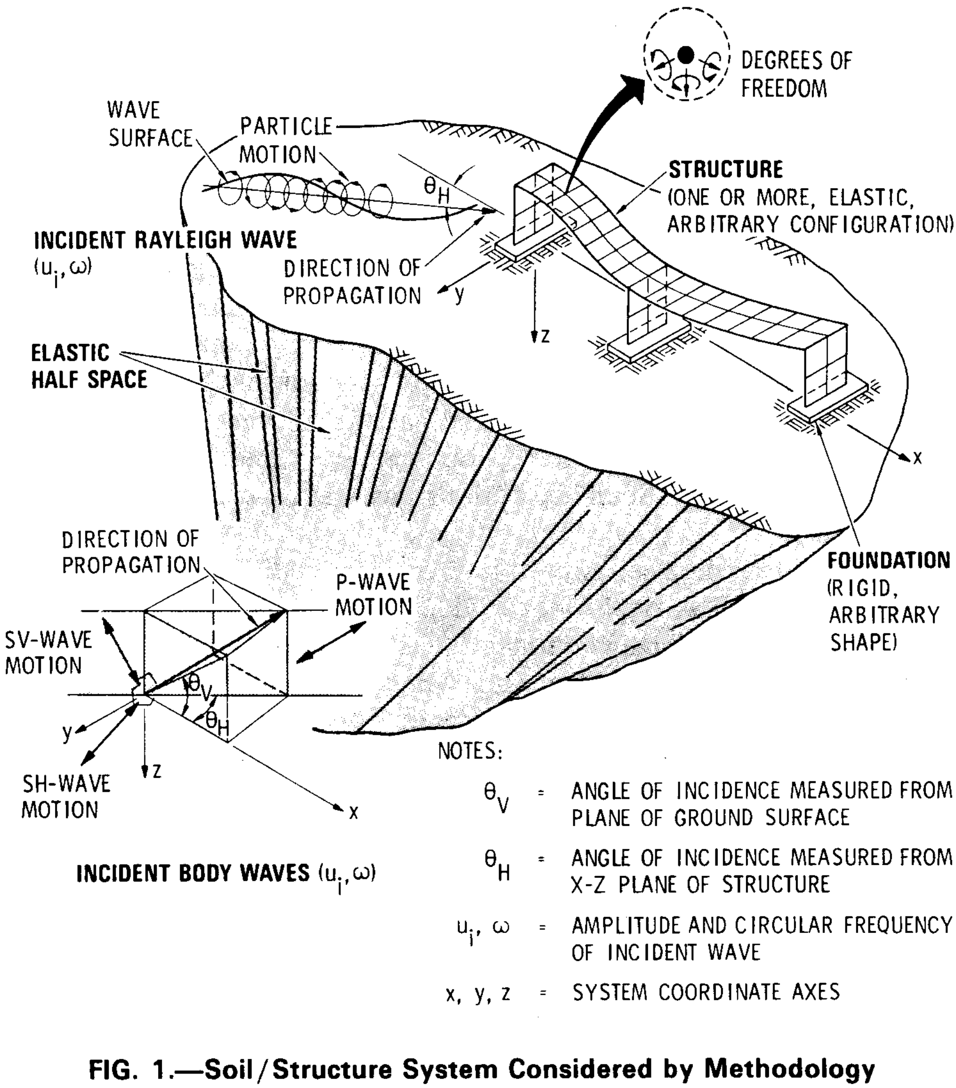


FIG. 1.—Soil/Structure System Considered by Methodology

response of structures. The paper consists of two main parts. The first part briefly summarizes a new methodology for analyzing the three-dimensional dynamic response of soil/structure systems subjected to traveling seismic waves; this summary is extracted from a detailed description contained in a prior report (18). The second and principal part of the paper describes an example application of the methodology to a single-span bridge subjected to incident plane SH-waves. The purpose of this application is to demonstrate basic phenomena associated with the three-dimensional vibrations induced in bridge-type structures by traveling seismic waves. To this end, a simple model of the bridge is studied, and the basic effects of the excitation frequency and the direction of incidence of the incoming wave on the three-dimensional bridge displacements and deformed shapes are presented.

Methodology

The methodology presented here has the following features (see Fig. 1):

1. It computes the three-dimensional dynamic response of an arbitrarily configured, elastic, aboveground structure. It can also consider two or more closely spaced structures.
2. It assumes each structure to be supported on any number of rigid foundations of arbitrary shape that are bonded to the surface of an elastic half-space.
3. It represents input motions as any desired combination of plane, harmonic body, or surface waves with arbitrary excitation frequencies, amplitudes, and angles of incidence.

This methodology represents a first step in the development of a more general analysis procedure that, in the future, will involve viscoelastic and horizontally

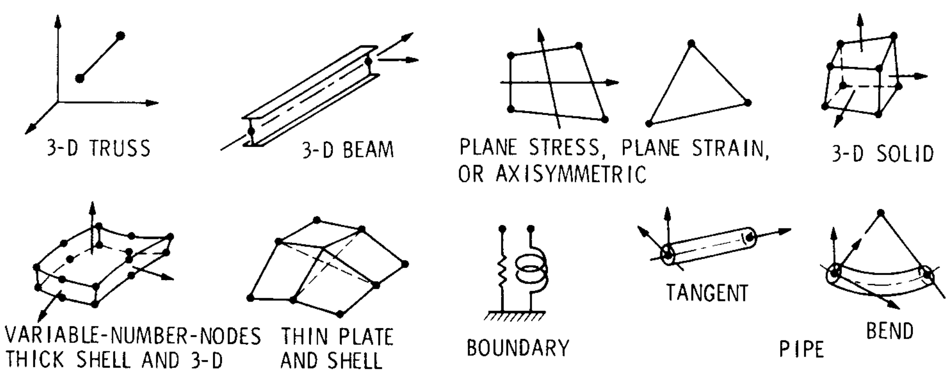


FIG. 2.—Superstructure Element Types

layered soil media, embedded foundations, and arbitrary transient input motions. The manner in which the methodology represents the superstructure and the foundation/soil system and then performs the overall system response analysis is summarized in the following.

Representation of Superstructure.—Typically, a three-dimensional finite element model is employed to represent the superstructure. The model can comprise any combination of the available finite element types, with some examples shown in Fig. 2 (4); it is used to define the stiffness matrix, mass matrix, and fixed-base mode shapes and frequencies of the superstructure. Either a determinant-search method or a subspace-iteration approach can be used to carry out the mode shape and frequency calculations (2,3). Damping in the superstructure is represented by defining a damping ratio for each fixed-base normal mode considered in the analysis.

Representation of Foundation/Soil System.—Foundation/soil interaction effects under the action of the incident-wave motions are represented by using a continuum solution based on the work of Wong (20). This solution characterizes the foundation/soil system in terms of complex, frequency-dependent driving force vectors and impedance matrices. The driving forces correspond to the reaction forces that result when each foundation is fixed and subjected to the incident waves transmitted through the soil medium [Fig. 3(*a*)]. The elements of the *j*th column of the impedance matrix are computed as foundation reaction forces caused by a unit harmonic displacement of the *j*th foundation degree-of-freedom when all other foundation degrees-of-freedom are fixed [Fig. 3(*b*)]. These quantities are derived for one or more foundations of arbitrary shape

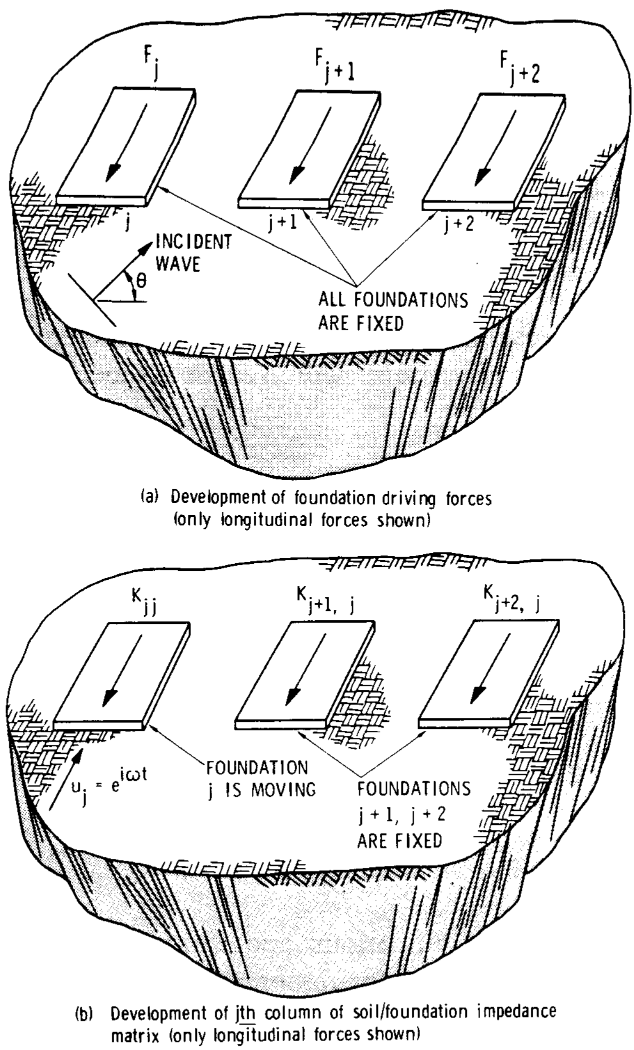


FIG. 3.—Development of Foundation/Soil Driving Forces and Impedance Matrix

by first using Green's functions for an elastic half-space to define stress/displacement relationships for various subregions of each foundation, and then by imposing rigid-body displacement boundary conditions and equilibrium requirements (18).

System Response Analysis.—Once the superstructure and foundation/soil systems are characterized as described in the foregoing, compatibility and equilibrium requirements at the superstructure/foundation interface are used to couple these two sets of results and to thereby represent the complete soil/foundation/superstructure system (16). The steady-state response of this system is then computed using an extension of a procedure described by Clough and Penzien (7). A formulation of this analysis procedure is provided in Ref. 18.

Response of Single-Span Bridge to Incident SH-Waves

The foregoing methodology is used to compute the three-dimensional response of a single-span bridge resting on the surface of an elastic half-space and subjected

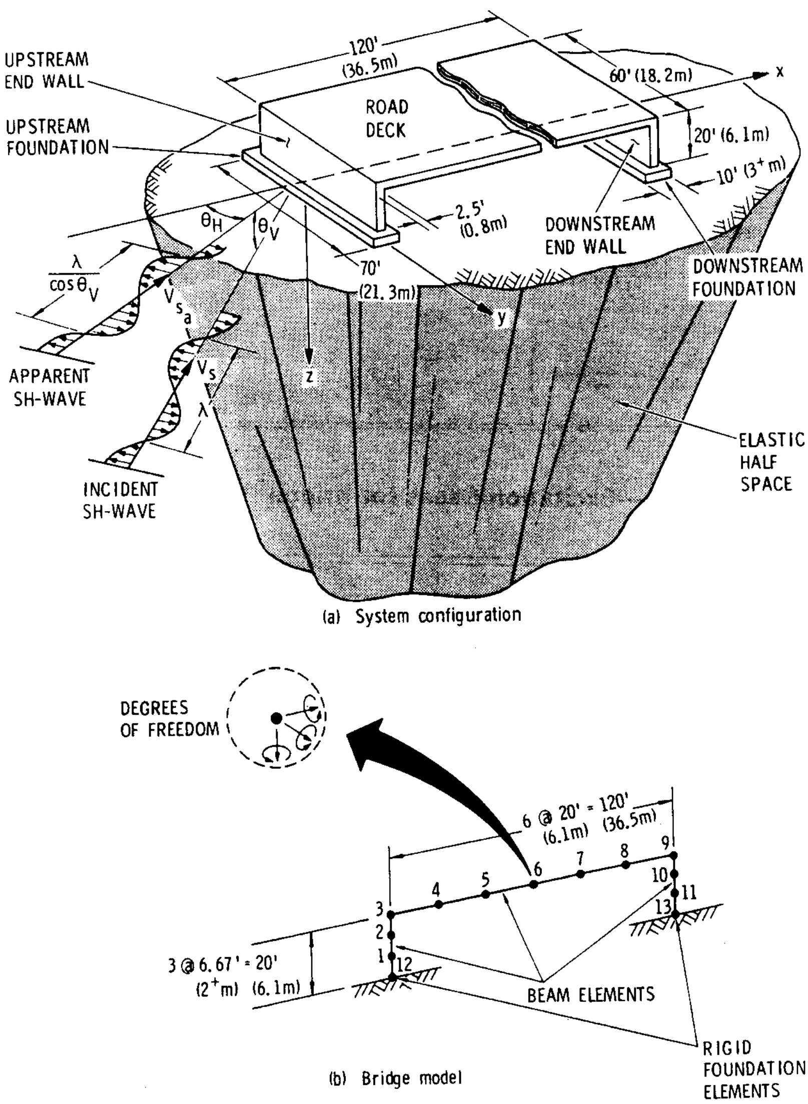


FIG. 4.—Bridge Configuration and Model Used in Example Analysis

to incident plane SH-waves. Parametric analyses are carried out to show how the structure response is influenced by the excitation frequency and the angles of incidence of the incoming wave.

Bridge Model and Excitation.—The bridge chosen for this example is shown in Fig. 4(*a*) to be 120 ft (36.5 m) long, 70 ft (21.5 m) wide, and 20 ft (6.1 m) high. The bridge is modeled using the system of undamped beam elements shown in Fig. 4(*b*). Table 1 furnishes section and material properties for the bridge and material properties for the soil medium.

The free-field excitations from the incident SH-waves have a surface amplitude of 2.0, an arbitrary excitation frequency (up to 25 Hz maximum), and a zero phase angle at the upstream foundation, which is the origin of the coordinate

TABLE 1.—Section and Material Properties Considered in Bridge Response Analysis

	SUPERSTRUCTURE SECTION PROPERTIES				Material Properties		
	Cross-sectional area, in square feet (square meters)	Moment of Inertia, in feet^4 (meters^4)			Shear wave velocity, in feet per second (meters per second)	Unit weight, in pounds per cubic foot (kilograms per cubic meter)	Poisson's ratio
Element (1)	(2)	About strong axis (3)	About weak axis (4)	Torsion (5)	(6)	(7)	(8)
Road deck	0.98×10^2 (9.1)	3.56×10^4 (306.9)	3.29×10^2 (2.8)	1.01×10^3 (8.7)	—	—	—
End walls	1.48×10^2 (13.7)	4.28×10^4 (369.4)	0.77×10^2 (0.7)	0.31×10^3 (2.7)	—	—	—
Elastic half-space	—	—	—	—	500 (150)	110 (1,760)	0.33
Superstructure	—	—	—	—	6,900 (2,100)	150 (2,400)	0.15

TABLE 2.—Excitation Cases for Bridge Response Analysis

Case number (1)	θ_H, in degrees (radians) (2)	θ_V, in degrees (radians) (3)	Description of incident SH-wave motions (4)
1	90 (1.57)	90 (1.57)	Vertically incident waves with particle motions directed along span of bridge.
2	90 (1.57)	0 (0)	Horizontally incident waves propagating in y direction, with particle motions directed along span of bridge.
3	0 (0)	90 (1.57)	Vertically incident waves with particle motions perpendicular to span of bridge.
4	0 (0)	0 (0)	Horizontally incident waves propagating in x direction, with particle motions perpendicular to span of bridge.
5	45 (0.79)	45 (0.79)	Waves traveling at angle of 45° to ground surface and in plane oriented at 45° to span of bridge. Particle motion directed at angle of 45° relative to x- and y-axes.

Note: For each case, excitations have a surface amplitude of 2.0, a variable frequency (up to 25 Hz maximum), and a zero phase angle at the upstream foundation.

system for these analyses. The orientation of these excitations and the direction of wave propagation are represented by the two angles of incidence, θ_H and θ_V, as shown in Fig. 4(*a*). Five different combinations of these angles are used to define five different excitation cases for which the bridge response is analyzed.

These cases, listed in Table 2, were selected from a more extensive set described in Ref. 18 because they illustrate some of the more interesting features of

TABLE 3.—Dimensionless Frequency Definitions

Angle of incidence, θ_H, in degrees (radians) (1)	Orientation of incident wave propagation path (2)	Definition of l (Eq. 1) Description (3)	Definition of l (Eq. 1) Numerical value [Fig. 4(a)] (4)	Dimensionless Frequency, R_L (Eq. 1) (5)
0 (0)	Within vertical plane parallel to x-z plane of bridge	Distance between the two bridge foundations	120 ft (36.6 m)	$R_{L_x} = 120 \text{ ft} \times \omega/(2\pi V_s)$
90 (1.57)	Within vertical plane normal to x-z plane of bridge (or parallel to y-z plane)	Length of foundations along y-axis	70 ft (21.3 m)	$R_{L_y} = 70 \text{ ft} \times \omega/(2\pi V_S)$
45 (0.79)	Within vertical plane oriented at 45° (0.79 rad) to x-z plane	—[a]	—[a]	R_{L_x} and R_{L_y}

[a]Same as for θ_H = 0° and 90° (0 rad and 1.57 rad).

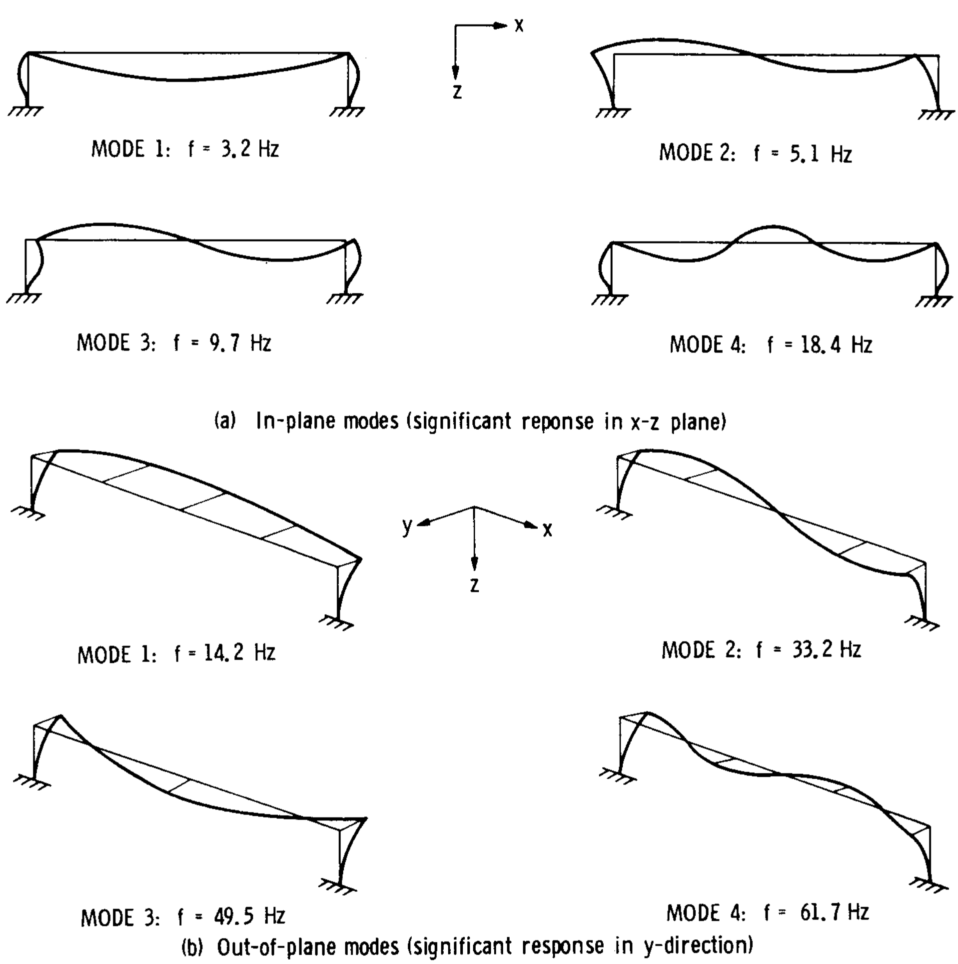


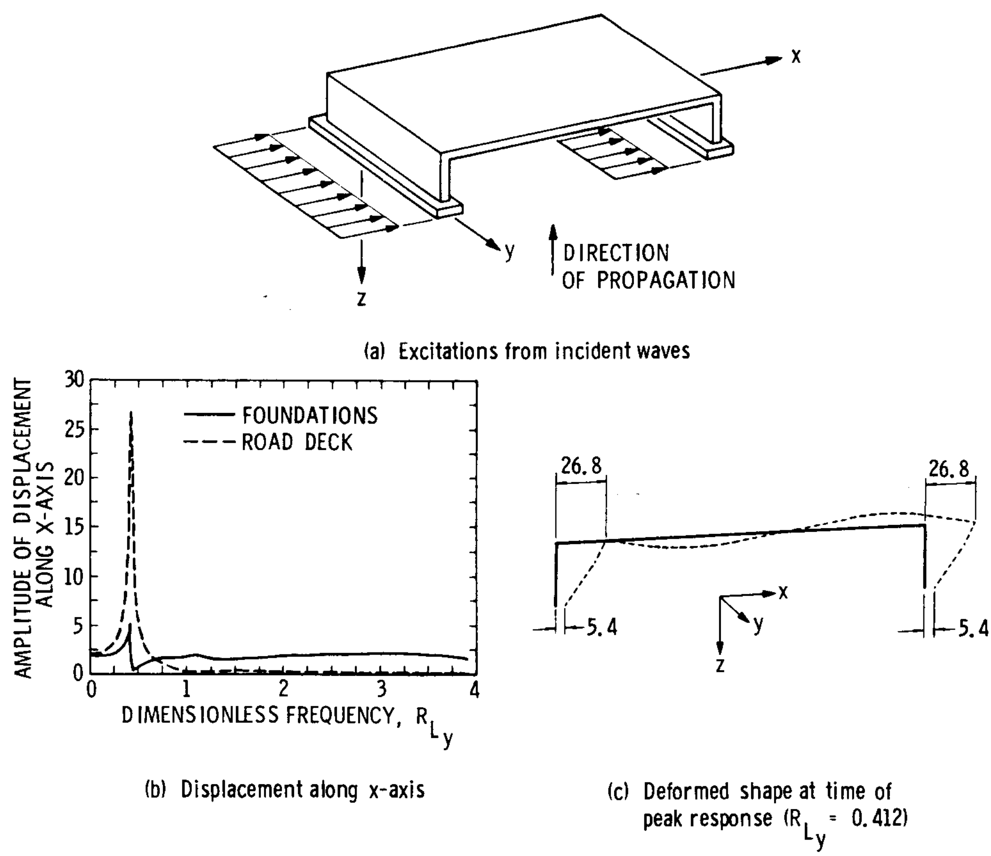
FIG. 5.—Fixed-Base Mode Shapes and Frequencies of Bridge

the three-dimensional bridge response.

For each excitation case, bridge motions are presented in the form of response amplitude versus dimensionless frequency curves and as three-dimensional representations of the deformed shape of the bridge at times of peak cyclic response. The dimensionless frequencies used in presenting these results are defined as

$$R_L = \frac{l}{\lambda} = \frac{l\omega}{2\pi V_s} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (1)$$

in which λ = the wavelength of the incident wave along its propagation path; V_s = the shear wave velocity of elastic half-space; ω = the circular frequency



(a) Excitations from incident waves

(b) Displacement along x-axis

(c) Deformed shape at time of peak response (R_{L_y} = 0.412)

FIG. 6.—Case 1: Bridge Response to Incident SH-Waves with $\theta_H = \theta_V = 90°$ (1.57 rad)

of the excitation; and l = a characteristic structural dimension that is selected according to the orientation of the propagation path of the incident wave, as defined by θ_H. The selection of l for the three values of θ_H considered in these example analyses is shown in Table 3.

Fixed-Base Mode Shapes and Frequencies.—The fixed-base mode shapes and frequencies for the bridge are shown in Fig. 5. This figure shows that the out-of-plane modes have significantly higher frequencies than do the corresponding in-plane modes, a direct result of the greater stiffness of the bridge in the y direction. A total of 29 modes was used to characterize the superstructure to provide adequate convergence of the response computations within the range of excitation frequencies considered in this analysis (up to 25 Hz).

Case 1: $\theta_H = \theta_V =$ **90° (1.57 rad).**—The first set of bridge response results ensues from the application of vertically incident SH-waves with particle motions directed along the x-axis of the bridge (i.e., along its span). Such excitations are uniformly distributed along the length of the foundations [Fig. 6(a)].

The only significant bridge response for this case is in the x direction. It corresponds to sidesway motions that occur over a single, narrow frequency range centered about the resonant frequency of the bridge/soil system for this mode of response [Figs. 6(b) and 6(c)]. This resonant frequency ($R_{L_y} = 0.412$,

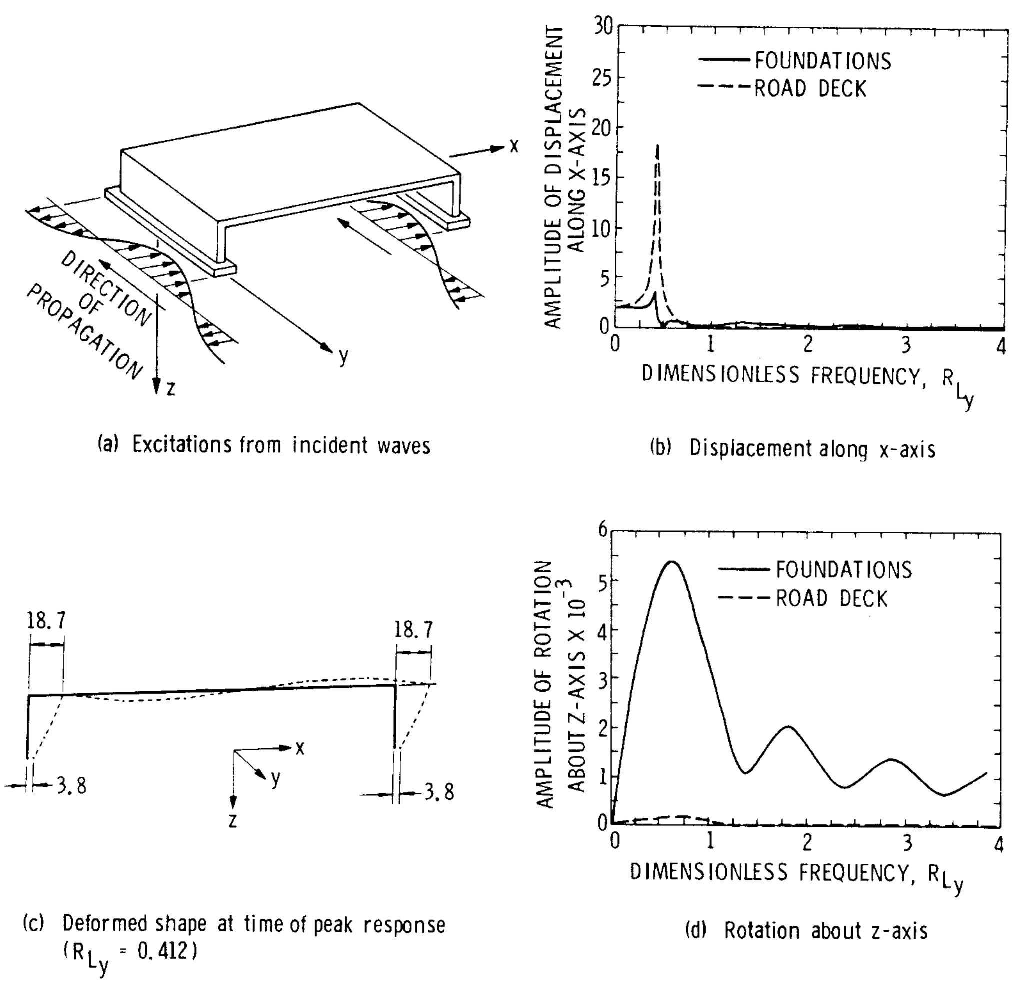


FIG. 7.—Case 2: Bridge Response to Incident SH-Waves with θ_H = 90° (1.57 rad), θ_V = 0° (0 rad)

or 2.94 Hz) is reduced from that of the corresponding fixed-base mode [5.1 Hz for Mode 2 of Fig. 5(a)] because of the presence of the underlying soil medium. At excitation frequencies beyond this resonance range, the bridge response is characterized by very small displacements of the road deck in the x direction and by displacements of the foundations that are similar in amplitude and phase to those of the free field (18). No foundation rotations about the z-axis are generated by the excitation because of the complete symmetry of the bridge and the excitation with respect to the x-axis. This response, which is the simplest of the various cases considered, is analogous to that of a simple single-degree-of-freedom system whose natural frequency corresponds to this resonant frequency of the bridge/soil system.

Case 2: $\theta_H = 90°$ (1.57 rad), $\theta_V = 0°$ (0 rad).—The second case considers SH-waves with particle motions oriented in the same direction as for Case 1 (in the x direction, or along the bridge span); however, the waves are now horizontally incident, rather than vertically incident, with a propagation path in the y direction (i.e., normal to the span of the bridge). As a result, the excitations are no longer uniform over the entire length of the two foundations;

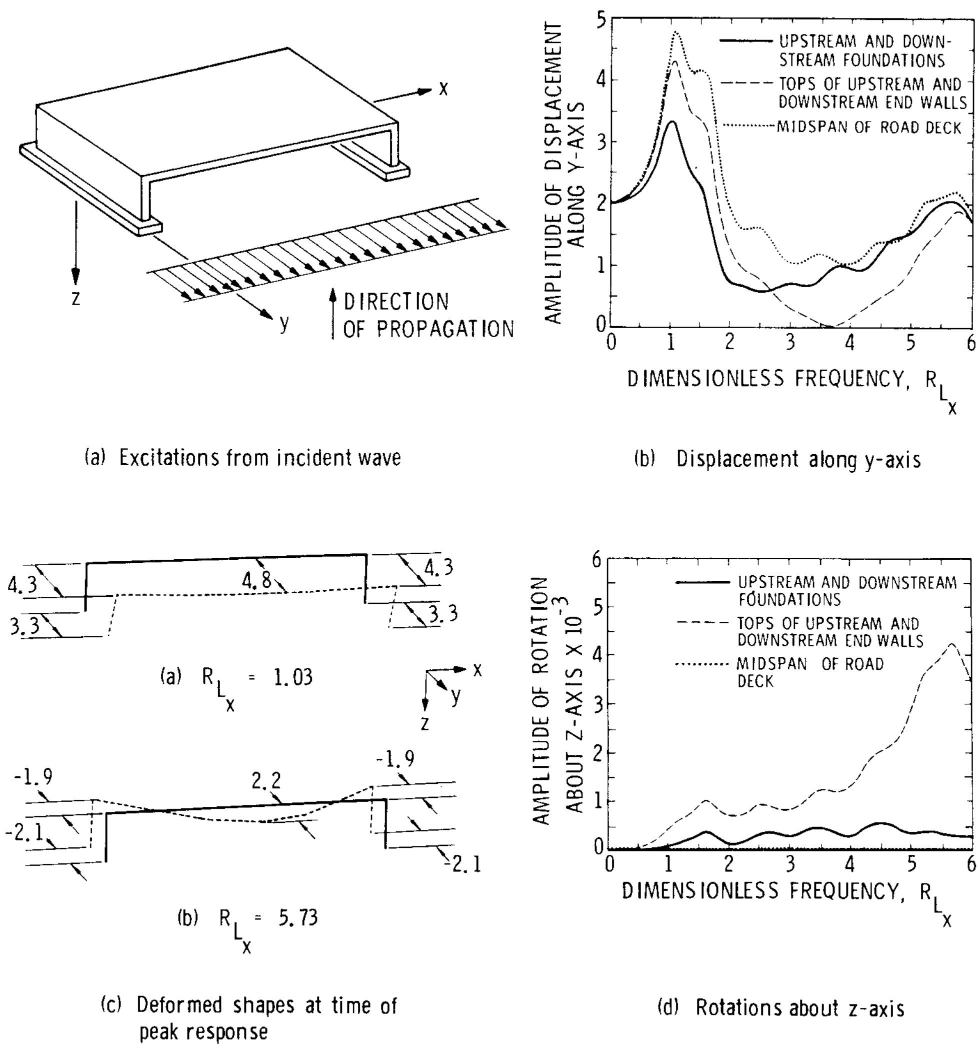


FIG. 8.—Case 3: Bridge Response to Incident SH-Waves with $\theta_H = 0°$ (0 rad), $\theta_V = 90°$ (1.57 rad)

instead, they exhibit spatial variations as the wave propagates in the y direction [Fig. 7(*a*)].

The bridge response for this case exhibits significant sidesway displacements, which occur over the same narrow resonant frequency range as that for the vertically incident waves of Case 1 [Figs. 7(*b*) and 7(*c*)]. However, the amplitudes of these displacements are reduced from those of Case 1 because, as shown in Ref. 18, only part of the energy from the spatially varying excitation—i.e., that corresponding to excitation components symmetric about the x-axis—is now available to drive the structure in the x-direction. The remaining energy—which corresponds to antisymmetric excitation components—causes rotations of the foundation about the z-axis [Fig. 7(d)]. Since these foundation rotations are large relative to those of the road deck, they correspond to significant torsional deformations in the end walls. They are seen to be largest in the frequency range of about $0.25 \leq R_{L_y} \leq 1$, but are still prominent at higher

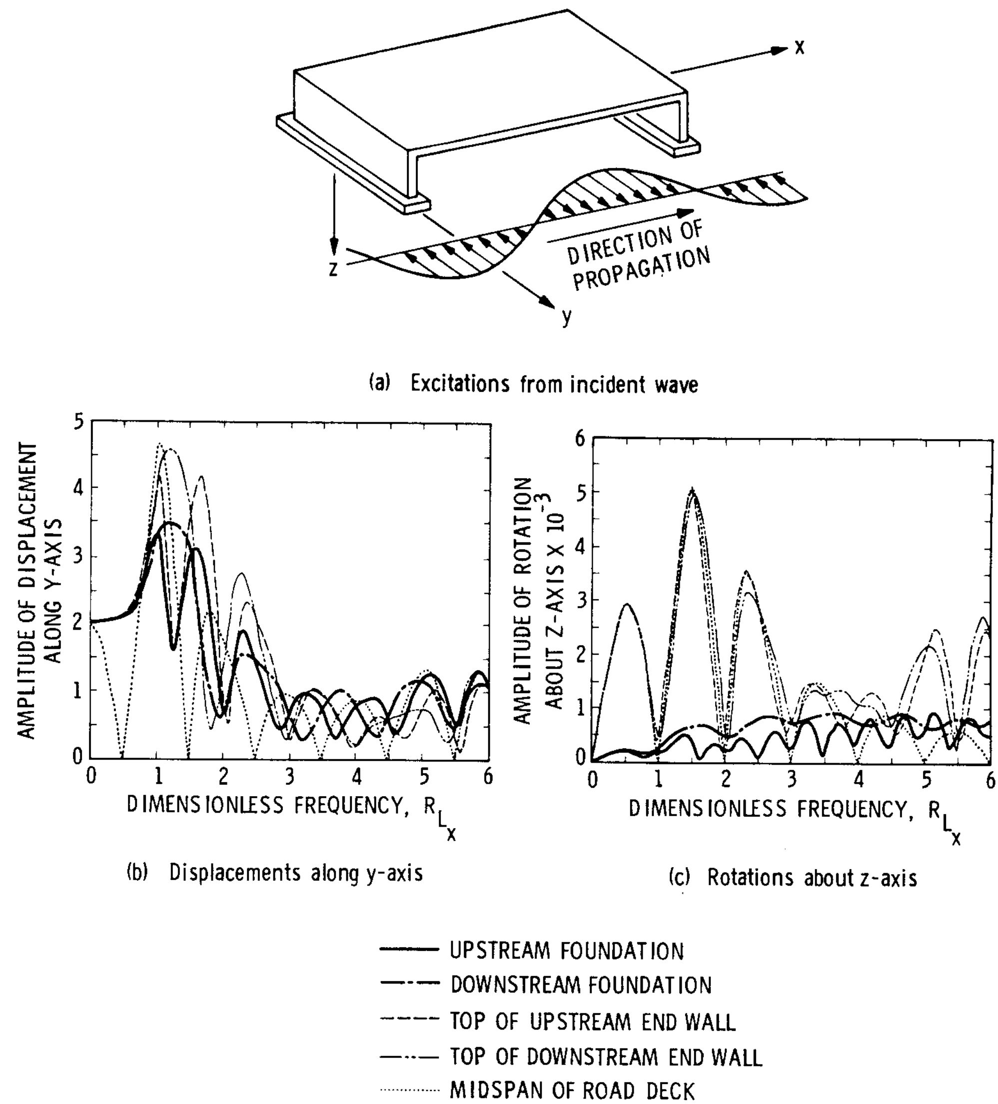


FIG. 9.—Case 4: Bridge Response to Incident SH-Waves with $\theta_H = \theta_V = 0°$ (0 rad)

excitation frequencies. Such rotations are not induced by the vertically incident waves of Case 1.

Still another important feature of the bridge response for this case is the nature of the foundation displacements in the x direction that result from incident waves whose wavelengths are short relative to the foundation length (i.e., for high excitation frequencies that are represented by large values of R_{L_y}). In contrast to Case 1, these foundation displacements are now substantially smaller in amplitude than the corresponding displacements of the free field [Fig. 7(b)]. This is because, for incident waves with short wavelengths, the net loadings applied along the length of the rigid foundation by the symmetric components of the spatially varying excitations are reduced; therefore, they are less effective in driving the foundations in the x direction (18).

Case 3: $\theta_H = 0°$ (0 rad) and $\theta_V = 90°$ (1.57 rad).—The third case considers vertically incident SH-waves that differ from Case 1 in that they propagate in a plane parallel to (rather than normal to) the bridge span. For this case, the corresponding input motions applied to each foundation are identical in amplitude and phase and are directed along the y-axis [Fig. 8(a)].

The resulting bridge response consists of displacements in the y direction and rotations about the z-axis that are symmetric about the midspan of the bridge [Figs. 8(b)–8(d)]. Peaks in these response amplitudes occur at frequencies

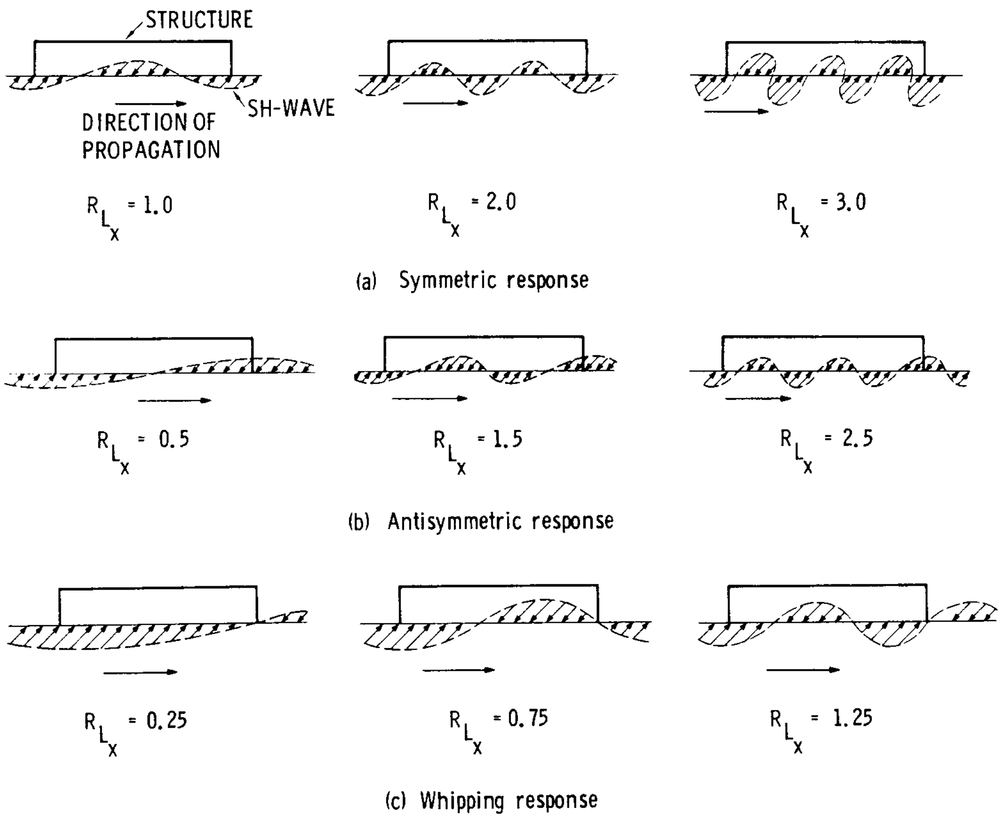


FIG. 10.—Case 4: Relationship between Wavelength of Incident SH-Waves and Bridge Response Characteristics with $\theta_H = \theta_V = 0°$ (0 rad)

of $R_{L_x} = 1.03$ (4.3 Hz), where the response primarily comprises rigid-body motions, and at $R_{L_x} = 5.73$ (23.9 Hz), where the response features prominent bending deformations in the road deck and torsional deformations in the end walls [Figs. 8(c) and 8(d)]. Because of the underlying soil medium, these frequencies are much lower than those of the corresponding out-of-plane fixed-base modes shown in Fig. 5(b) (i.e., Modes 1 and 3, which have frequencies of 14.2 Hz and 49.5 Hz, respectively).

Case 4: $\theta_H = \theta_V = 0°$ (0 rad).—The fourth case considers horizontally incident SH-waves that, like the waves considered in Case 3, propagate in a plane parallel to the bridge span and apply excitations to the bridge that are directed along the y-axis [Fig. 9(a)]. However, since the waves are not vertically incident, there is now a phase difference between the excitations applied to each foundation.

As for Case 3, the principal bridge response consists of displacements along the y-axis and rotations about the z-axis. Response amplitudes are given in Figs. 9(b) and 9(c) and show two important trends. First, the bridge response is clearly more complex than that resulting from the vertically incident SH-waves of Case 3. Second, close examination of response amplitudes given in Figs.

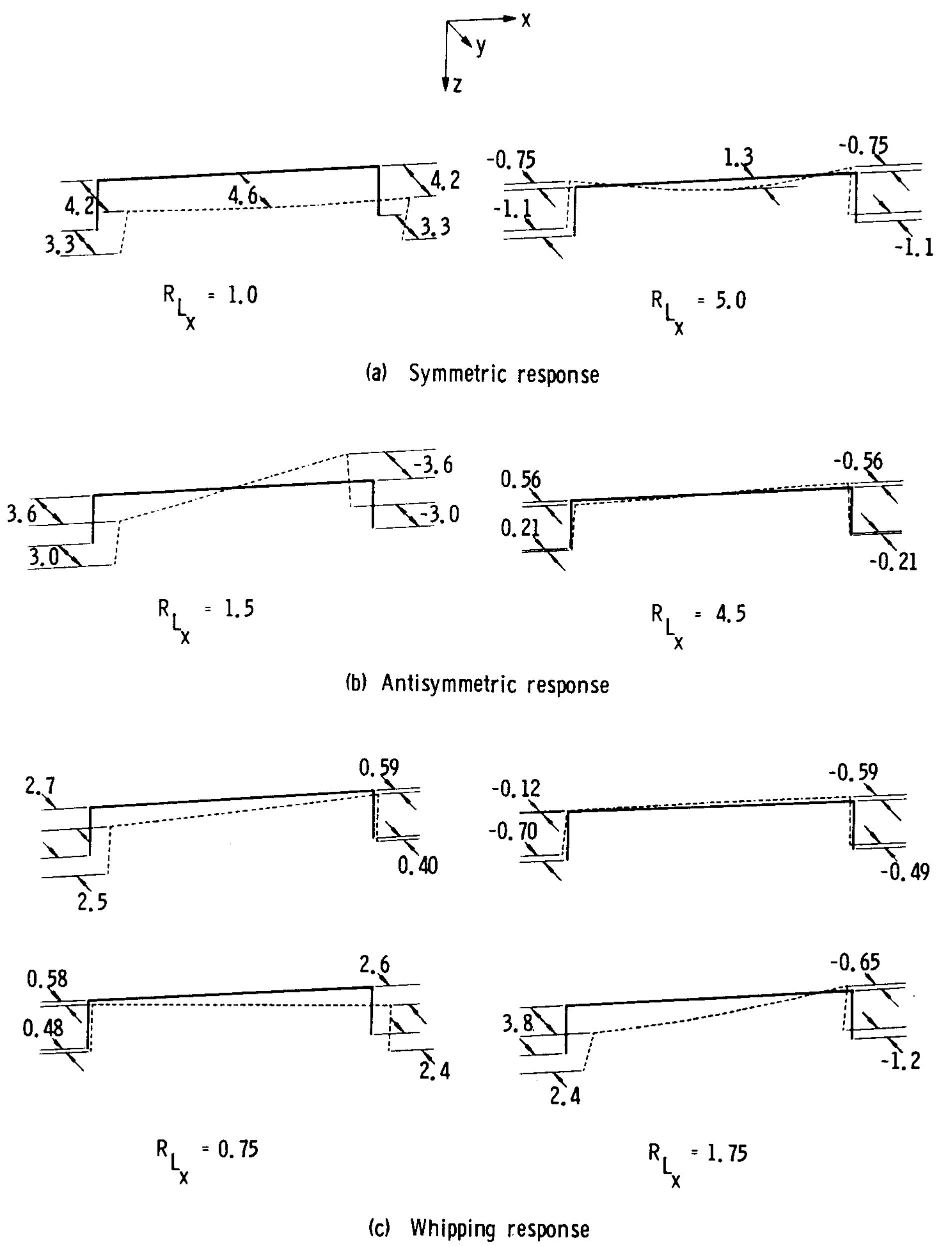


FIG. 11.—Case 4: Deformed Shapes of Bridge at Times of Peak Response to Incident SH-Waves with $\theta_H = \theta_V = 0°$ (0 rad)

9(b) and 9(c) and phase angles given in Ref. 18 shows the existence of distinct patterns of bridge response (symmetric, antisymmetric, and whipping) that occur at particular sets of wavelengths of the incident wave.

Bridge responses that are symmetric about the midspan occur when the wavelength of the incident wave is such that the excitations applied to each foundation are identical in amplitude and phase; this occurs at dimensionless frequencies of R_{L_x} = 1.0, 2.0, 3.0, etc. [Fig. 10(*a*)]. Deformed shapes of the bridge for this case are shown in Fig. 11(*a*) and, from comparisons with Fig. 8(*c*), are seen to be similar to those resulting from vertically incident waves.

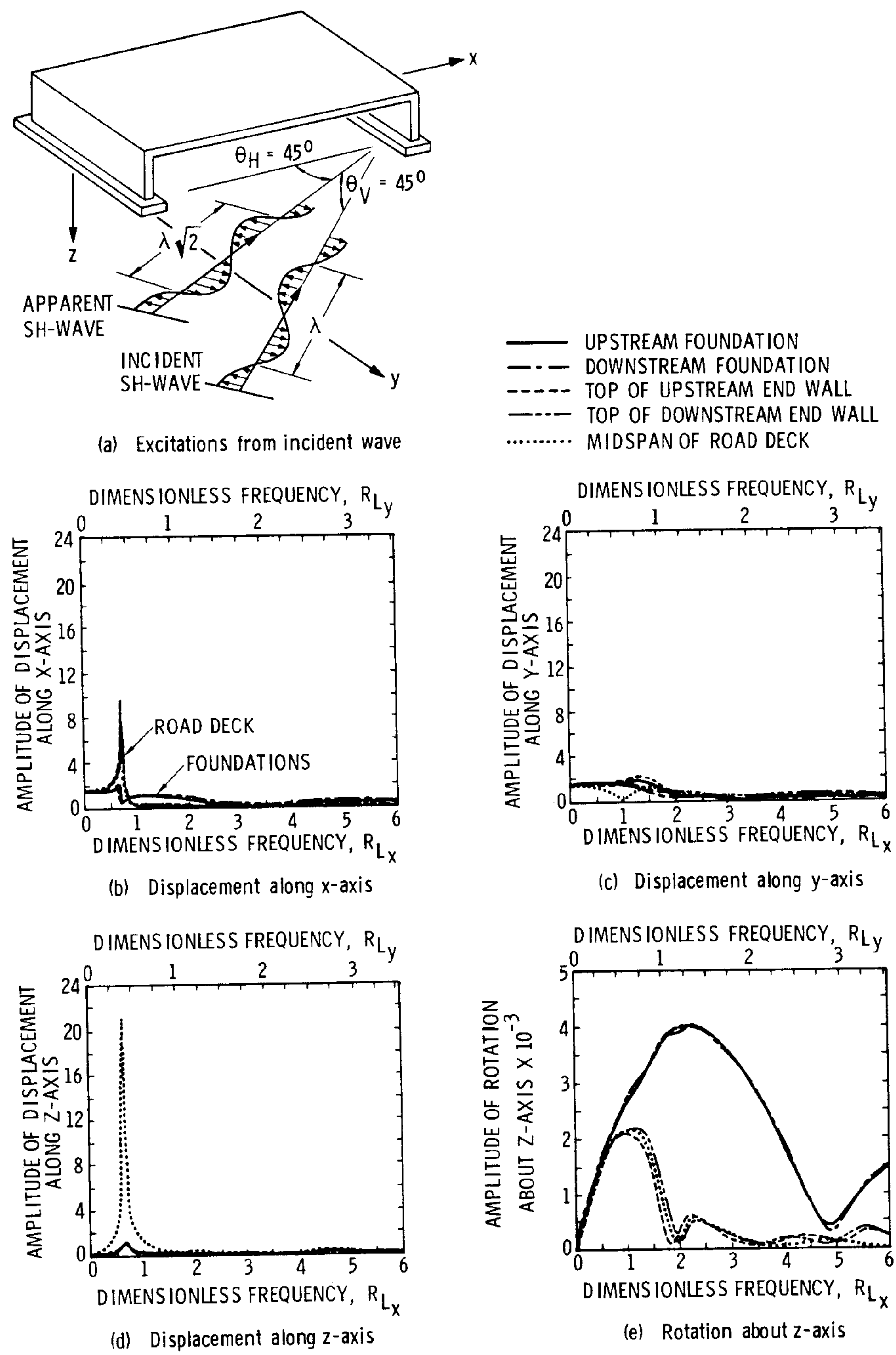


FIG. 12.—Case 5: Bridge Response to Incident SH-Waves with $\theta_H = \theta_V$ = 45° (0.79 rad)

Responses of the bridge that are antisymmetric about its midspan occur when the wavelength of the incident wave is such that the excitations applied to each foundation are of equal amplitude and opposite phase, as shown in Fig. 10(*b*). This occurs at dimensionless frequencies of R_{L_x} = 0.5, 1.5, 2.5, etc. The antisymmetric responses at lower frequencies in this type of excitation (e.g., R_{L_x} = 0.5 and 1.5) feature rigid-body rotations of the road deck about the z-axis that are much larger than the corresponding rotations of the foundations [Fig. 9(*c*)]. Therefore, the end walls are undergoing torsional deformations. At higher frequencies (e.g., $R_{L_x} > 3.5$) these torsional deformations are reduced. Antisymmetric deformed shapes of the bridge are shown in Fig. 11(*b*).

A third type of response (whipping) occurs when the wavelengths of the incident waves are such that the excitations applied to each foundation are 90° (1.57 rad) out of phase; this occurs at dimensionless frequencies of R_{L_x} = 0.25, 0.75, 1.25, 1.75, etc. [Fig. 10(*c*)]. The resulting bridge response features large displacements in the y direction at one end while, at the same time, the other end is experiencing relatively small displacements; i.e., the bridge is "whipping" about a center of rotation near the end with the small displacements. At low excitation frequencies (R_{L_x} = 0.75) this response comprises displacements at the two ends of the bridge that are of nearly equal amplitude and are nearly

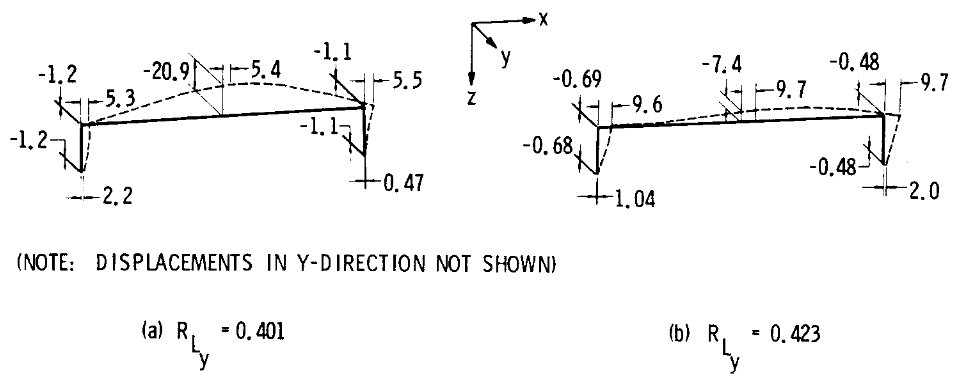


FIG. 13.—Case 5: Deformed Shapes at Times of Peak Displacement in X and Z Direction-Incident SH-Waves with $\theta_H = \theta_V$ = 45° (0.79 rad)

in phase with the free-field excitations (i.e., they are about 90° (1.57 rad) out of phase with each other). This low-frequency response can therefore be envisioned as whipping about first one end of the bridge and then the other, and is seen to consist primarily of rigid-body displacements [Fig. 11(*c*)]. At higher excitation frequencies (R_{L_x} = 1.75) the whipping response is more complex because of the increased effects of wave diffraction about the upstream foundation and wave scattering from the downstream foundation. For this case, the displacements of the two end walls now exhibit sizable differences in amplitude, and the bridge is whipping about its downstream end only [Fig. 11(*c*)]. In addition, the bridge response now features prominent bending deformations; and as shown in Ref. 18, the two ends of the bridge are no longer responding in phase with the free-field excitations.

Case 5: $\theta_H = \theta_V$ = 45° (0.79 rad).—The final case considers waves that propagate at an angle of 45° relative to the ground surface and in a plane at 45° (0.79 rad) relative to the x-z plane of the bridge [Fig. 12(*a*)]. The resulting bridge response is fully three dimensional, as shown by the response amplitudes for all three displacement components [Figs. 12(*b*)–12(*d*)] and for the rotations about the z-axis [Fig. 12(e)]. These figures show the following trends:

1. All three displacement components are coupled at excitation frequencies below $R_{L_y} = 1$, with the most significant coupling occurring over a narrow frequency range ($R_{L_y} = 0.4$ to about 0.5) and involving large amplitudes of displacement along the x- and z-axes [Figs. 12(b)–12(d) and Fig. 13]. This can be contrasted with Cases 1 to 4, where no such coupling effects were induced. At higher excitation frequencies the displacement amplitudes are small.
2. The largest displacements are those of the road deck in the x direction and the z direction. The peak values of these displacement components occur at nearly the same frequency ($R_{L_y} = 0.423$ and 0.401, respectively) and result from two resonance phenomena that are coupled for these particular angles of incidence (Fig. 13). The displacements of the road deck in the x direction are analogous to the sidesway vibrations already considered for Cases 1 and 2, and occur at nearly the same frequency (i.e., $R_{L_y} = 0.412$ for Cases 1 and 2, and occur at nearly the same frequency (i.e., $R_{L_y} = 0.412$ for Cases 1 and 2 versus 0.423 for this case). The displacements of the road deck in the z direction correspond (18) to a resonance with the fundamental in-plane mode of the bridge/soil system [the associated fixed-base mode is shown as Mode 1 in Fig. 5(a)]. These large *vertical* displacements result from the *horizontal* incident wave motions because of the off-diagonal coupling terms in the foundation/soil impedance matrix and from the different phases of the wave motion at the two foundations for this particular excitation frequency and these angles of incidence (18).
3. The displacements along the y-axis do not exhibit any prominent amplifications of the incident wave motions [Fig. 12(c)]. The amplitudes of these displacement components fall below those of Case 4 [Fig. 9(b)].
4. At lower excitation frequencies ($R_{L_y} < 0.5$), significant rotations about the z-axis are induced in both the foundations and the road deck [Fig. 12(e)]. This differs from Case 2, where such rotations were only generated in the foundations, and from Case 4, where much larger rotations occurred in the road deck [Figs. 7(d) and 9(c)]. At higher excitation frequencies, there is a marked increase in the rotations of the foundations relative to those of the road deck. This is similar to the foundation rotations observed in Case 2 [Fig. 7(d)] except for a reduction in the peak amplitudes of these rotations and an expansion of their frequency scale because of apparent wavelength effects (18).

Conclusions

A new methodology has been developed for analyzing the three-dimensional response of soil/structure systems excited by traveling seismic waves. To provide insights into the response of such systems, the methodology has been used to analyze a simple single-span bridge supported on an elastic half-space and subjected to incident SH-waves. The results of this analysis lead to two main conclusions. First, phase differences in the input ground motions applied to the bridge foundations can have significant effects on the bridge response. Therefore, it is important to consider such traveling wave effects when designing earthquake-resistant structures of this type. Second, the nature of the structure response to these traveling seismic waves is strongly dependent on the direction of incidence as well as on the excitation frequency of the seismic waves. Therefore, it is not sufficient to consider only a single direction of propagation when evaluating the effects of traveling waves on the three-dimensional response of a bridge structure.

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

l = structure dimension used in defining dimensionless frequencies;
R_L, R_{L_x}, R_{L_y} = dimensionless frequencies;
u_i = amplitude of incident wave motion;
V_s = shear wave velocity of soil medium;
θ_H, θ_V = angles of incidence;
λ = wavelength of incident wave along its propagation path; and
ω = circular frequency of free-field excitation.