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Distribution of Peaks in Linear Earthquake Response

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Introduction

Through generalizations and extensions of Rice's work on the distribution of amplitudes of random noise (3), Longuet-Higgins (2), and Cartwright and Longuet-Higgins (1) presented results which find useful applications in earthquake engineering and strong motion seismology. In particular, they show that the distribution of amplitudes of a random function, $f(t)$, which can be represented by the series

$$f(t) = \sum_{n} c_n \cos(\omega_n t + \phi_n) \quad \ldots\ldots (1)$$

may be characterized by only two parameters, $m_0^{1/2}$, and ϵ, in which

$$\epsilon = 1 - \frac{m_2^2}{m_0 m_4} \quad \ldots\ldots (2)$$

represents a measure of the width of the energy spectrum, $E(\omega)$, of $f(t)$. In Eq. 1, c_n are related to $E(\omega)$ through

$$\sum_{\omega_n = \omega}^{\omega + d\omega} \frac{1}{2} c_n^2 = E(\omega)\, d\omega \quad \ldots\ldots (3)$$

in which ω_n represents circular frequency; t = time; and ϕ_n = assumed to be randomly and uniformly distributed between 0 and 2π. In Eq. 2

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$$m_n = \int_0^\infty E(\omega)\,\omega^n\,d\omega \quad \ldots \quad (4)$$

represents the nth moments of the energy spectrum $E(\omega)$.

Cartwright and Longuet-Higgins (1) also present the distribution function, the expected values, and the most probable values of the peaks of $f(t)$ and the most probable values of the largest among the considered N peaks. These results are particularly useful in earthquake engineering applications when one of the most frequently used functionals for characterization of strong motion amplitudes is the response spectrum. This spectrum represents the amplitudes of the maximum relative (or absolute) response of a viscously-damped, single-degree-of-freedom oscillator excited by strong earthquake shaking. In the linear response range, structural vibration can be represented by a function of the form equivalent to $f(t)$ in Eq. 1 and therefore, results on the distribution of

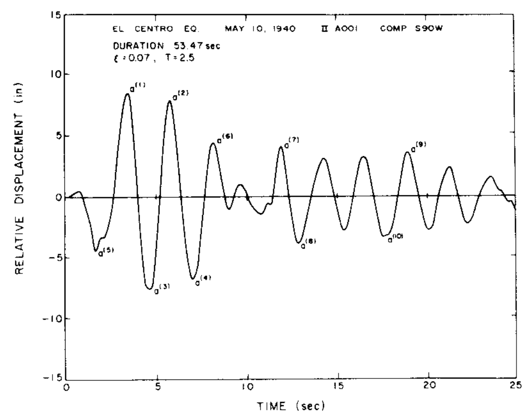

FIG. 1.—Relative Displacement of Single-Degree-of-Freedom System for Natural Period T_n = 2.5 sec and Damping ζ = 0.07; Peak Labeled a^1 is Largest Relative Displacement; Peaks a^2, a^3, ..., Represent "Second Largest," "Third Largest," etc., Peaks of $f(t)$

amplitudes of $f(t)$ can be applied directly to the analyses which deal with response spectrum estimation and its use in design (4).

Fig. 1 shows an example of $f(t)$ for small ϵ. It represents relative displacement of a single-degree-of-freedom system for natural period $T_n = 2.5$ sec, and fraction of critical damping $\xi = 0.07$, excited by the El Centro earthquake acceleration. The peak labeled a^1 is the largest relative displacement during this excitation and corresponds to the relative displacement spectrum amplitude. For the purposes of this paper, a^1 is referred to as the largest peak of $f(t)$. The peaks a^2, a^3, ..., then represent the "second largest," the "third largest," etc., peaks of $f(t)$.

The use of a^1 only in the current definition of the response spectrum technique for design of earthquake-resistant structures clearly ignores much valuable information on overall structural response. By ignoring a^2, a^3, ..., and the total number of peaks, N, the response spectrum functionals disregard explicit data on the distribution of response maxima and on the overall duration of response. By strict adherence to the formalism of the response spectrum superposition approach, the design of structures in the linear response range can, of course, be carried out in terms of a^1 only. Unfortunately, the common practice in earthquake-resistant design which often extrapolates arbitrarily the response spectrum superposition approach to design structures for nonlinear response, not only violates the rational principles of mechanics, but also requires a considerable degree of engineering judgment which may be difficult to justify.

The aim of this paper is not to reconcile the use of the response spectrum approach in the design of nonlinear systems, nor is it to advocate the use of this method when means are available to do better. Its objective is to present some new results on the distribution of a^n amplitudes and thus help a designer to consider the relationship between all response maxima (their number and amplitudes) to the physical characteristics of the structural system which is designed. The estimates of the amplitudes of the second a^2, third a^3, etc., largest peaks of the equivalent linear system should be helpful in understanding the number of times certain response levels may be exceeded as the structural system is progressing into nonlinear response. These results may further be useful for qualitative interpretation of the observed damage of structures in terms of the number of the equivalent linear excursions beyond the assumed design strength.

It is noted that the results of the expected and most probable amplitudes of a^n as presented here are applicable to numerous other problems outside the field of earthquake engineering, in which the function $f(t)$, as used in this paper, adequately describes the process which is studied. Thus, while the motivation for this work comes from the need to understand in greater detail the response of structures to earthquake excitation, the theoretical derivations presented here are general.

Expected Value of nth Local Maximum, $E(a^n)$

It is convenient to normalize the subsequent results for the amplitudes a^n in terms of $\bar{a}$ which represents the root-mean-square (rms) amplitude of the peaks of $f(t)$; thus

$$\bar{a} = \left[\frac{1}{N}(a^{1,2} + a^{2,2} + \ldots + a^{N,2})\right]^{1/2} \quad (5)$$

For narrow-band function $f(t)$ (when $\epsilon \approx 0$), $\bar{a}$ is closely approximated in terms of

$$a_{\text{rms}} = \left[\frac{1}{T}\int_0^T f^2(t)\,dt\right]^{1/2} \quad (6)$$

the rms of $f(t)$, as follows (4):

$$\bar{a} = \sqrt{2}\,a_{\text{rms}} = \sqrt{2}\,m_0^{1/2} \quad (7)$$

In Eq. 6, T represents duration of $f(t)$.

It can be shown that the expected value of a^n, when $\epsilon = 0$, is given by (see Appendix I):

$$\frac{E(a^n)}{\bar{a}} = \frac{n\sqrt{\pi}}{2}\left[N - \frac{N(N-1)}{2!\sqrt{2}} + \frac{N(N-1)(N-2)}{3!\sqrt{3}} + \ldots (-1)^{N+1}\frac{1}{\sqrt{N}}\right]$$

$$+ \frac{-n(n-1)}{3!}\cdot\frac{\sqrt{\pi}}{2}\left[2N - \frac{2N(2N-1)}{2!\sqrt{2}} + \frac{2N(2N-1)(2N-2)}{3!\sqrt{2}}\right.$$

$$\left. + \ldots (-1)^{2N+1}\frac{1}{\sqrt{3N}}\right] + \frac{n(n-1)(n-2)}{3!}\cdot\frac{\sqrt{\pi}}{2}\left[3N - \frac{3N(3N-1)}{3!\sqrt{2}}\right.$$

TABLE 1.—$E(a^n)/\bar{a}$ from Eq. 8 as a Function of N and n ($\epsilon = 0$)

	N						
n (1)	2 (2)	4 (3)	6 (4)	8 (5)	10 (6)	100 (7)	1,000 (8)
1	1.19	1.42	1.56	1.64	1.71	2.28	2.74
2	0.94	1.20	1.35	1.45	1.52	2.13	2.61
3		1.10	1.26	1.36	1.43	2.07	2.56
4		1.03	1.21	1.30	1.37	2.03	2.53
5			1.16	1.26	1.33	2.00	2.51
6			1.12	1.24	1.29	1.98	2.49
7				1.22	1.26	1.96	2.48
8				1.21	1.22	1.95	2.48

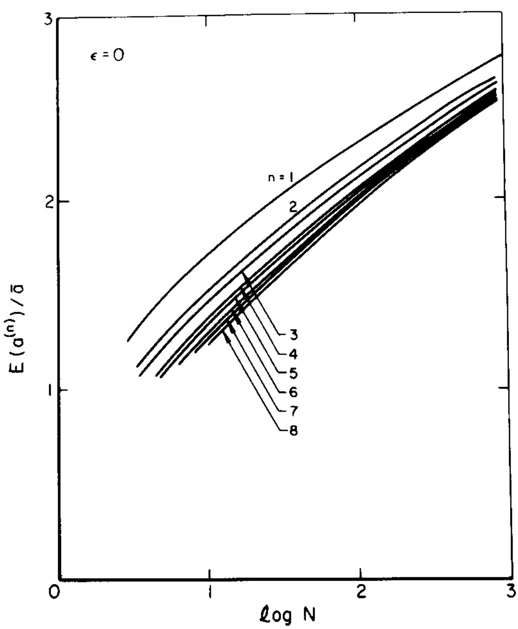

FIG. 2.—$E(a^n)/\bar{a}$ Versus $\log_{10} N$ for $\epsilon = 0$

$$+ \frac{3N(3N-1)(3N-1)}{3!\sqrt{3}} + \dots (-1)^{3N+1} \frac{1}{\sqrt{3N}} \Bigg] - (-1)^n \frac{\sqrt{\pi}}{2} \Bigg[nN$$

$$\vdots$$

$$- \frac{nN(nN-1)}{2!\sqrt{2}} + \frac{nN(nN-1)(nN-2)}{3!\sqrt{3}} + \dots (-1)^{nN-1} \frac{1}{\sqrt{nN}} \Bigg] \quad \dots\dots\dots (8)$$

TABLE 2.—$E(a^n)/\bar{a}$ from Eqs. 9 and 10 as a Function of ϵ, n, and N

	n									
N (1)	1 (2)	2 (3)	3 (4)	4 (5)	5 (6)	6 (7)	7 (8)	8 (9)	9 (10)	10 (11)
					(*a*) $\epsilon = 0.00$					
4	1.42	1.20	1.10	1.04						
6	1.56	1.35	1.25	1.19	1.15	1.12				
8	1.64	1.45	1.36	1.30	1.26	1.23	1.21	1.18		
10	1.71	1.52	1.43	1.38	1.34	1.31	1.29	1.27	1.25	1.24
100	2.28	2.13	2.07	2.03	2.00	1.98	1.95	1.95	1.94	1.93
1,000	2.74	2.61	2.56	2.53	2.51	2.49	2.48	2.46	2.45	2.45
					(*b*) $\epsilon = 0.20$					
4	1.32	1.09	0.98	0.92						
6	1.50	1.28	1.19	1.13	1.08	1.03				
8	1.60	1.40	1.31	1.25	1.21	1.18	1.15	1.13		
10	1.68	1.48	1.40	1.34	1.30	1.27	1.25	1.23	1.21	1.20
100	2.27	2.13	2.06	2.02	1.99	1.97	1.96	1.94	1.93	1.92
1,000	2.73	2.61	2.56	2.52	2.50	2.48	2.47	2.46	2.45	2.44
					(*c*) $\epsilon = 0.40$					
4	1.32	1.09	0.98	0.92						
6	1.50	1.28	1.19	1.13	1.08	1.03				
8	1.60	1.40	1.31	1.25	1.21	1.18	1.15	1.13		
10	1.68	1.48	1.40	1.34	1.30	1.27	1.25	1.23	1.21	1.20
100	2.26	2.11	2.05	2.01	1.98	1.96	1.94	1.93	1.91	1.90
1,000	2.72	2.60	2.54	2.51	2.49	2.47	2.46	2.45	2.44	2.43
					(*d*) $\epsilon = 0.60$					
4	1.32	1.09	0.98	0.92						
6	1.42	1.20	1.10	1.04	0.99	0.96				
8	1.55	1.35	1.25	1.19	1.15	1.12	1.09	1.07		
10	1.64	1.45	1.36	1.30	1.26	1.23	1.21	1.18	1.17	1.15
100	2.23	2.08	2.02	1.97	1.95	1.93	1.91	1.89	1.88	1.87
1,000	2.70	2.57	2.52	2.48	2.46	2.44	2.43	2.42	2.41	2.40
					(*e*) $\epsilon = 0.80$					
4	1.18	0.94	0.82	0.76						
6	1.32	1.09	0.98	0.92	0.87	0.83				
8	1.42	1.20	1.10	1.04	0.99	0.96	0.93	0.90		
10	1.55	1.35	1.25	1.19	1.15	1.12	1.09	1.07	1.05	1.04
100	2.17	2.01	1.94	1.90	1.87	1.85	1.83	1.82	1.81	1.80
1,000	2.64	2.52	2.46	2.43	2.40	2.39	2.37	2.36	2.35	2.34

For $n = 1$, this reduces to the result presented by Longuet-Higgins (see Ref. 2, Eq. 53), for $\epsilon = 0$.

Eq. 8 gives the exact values of $E(a^n)/a$ for n and N, but may be difficult to evaluate numerically for large N. It can be shown, however, that (see Appendix I):

$$\frac{E(a^n)}{\bar{a}} \approx n\left[(\ln N)^{1/2} + \frac{1}{2}\gamma(\ln N)^{-1/2}\right] - \frac{n(n-1)}{2!}\left[(\ln 2N)^{1/2}\right.$$

$$\vdots$$

$$\left.+\frac{1}{2}\gamma(\ln N)^{-1/2}\right](-1)^{n+1}\left[(\ln nN)^{1/2} + \frac{1}{2}\gamma(\ln nN)^{-1/2}\right] \quad \text{. . . . .} \quad (9)$$

The difference between Eqs. 8 and 9 is of the order of $(\ln N)^{-3/2}$, so that Eq. 9 represents an excellent approximation to Eq. 8 even for small values of N. For $n = 1$, Eq. 9 also reduces to the result previously presented by Longuet-Higgins (see Ref. 2, Eq. 59).

Fig. 2 and Table 1 present the exact values of $E(a^n)/\bar{a}$ for $n = 1, 2, \ldots, 7$ and 8, and for N between 2 and 1,000. It is seen that $E(a^n)/\bar{a}$ decreases most rapidly for small n. The difference between $E(a^i)/\bar{a}$ and $E(a^{i+1})/\bar{a}$ diminishes slowly with increasing N. Table 2 presents $E(a^n)/\bar{a}$ computed from Eq. 9, and shows that this approximation is quite good even for small N.

For $\epsilon \neq 0$, and neglecting the terms of the order of $\{\ln[(1-\epsilon)^{1/2} nN]\}^{-3/2}$, Eq. 9 can be generalized (see Appendix I) to

$$\frac{E(a^n)}{2} = n\left\{[\ln(1-\epsilon^2)^{1/2}N]^{1/2} + \frac{1}{2}\gamma\,[\ln(1-\epsilon^2)^{1/2}]^{-1/2}\right\}$$

$$-\frac{n(n-1)}{2!}\left\{\gamma\,[\ln(1-\epsilon^2)^{1/2}2N]^{1/2} + \frac{1}{2}\gamma\,[\ln(1-\epsilon^2)^{1/2}2N]^{-1/2}\right\}$$

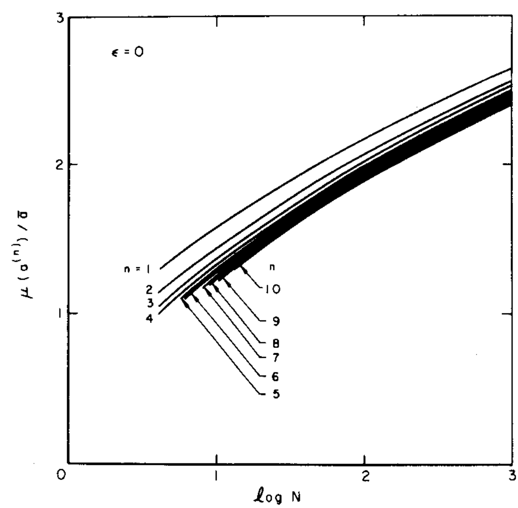

FIG. 3.—$\mu(a^n)/\bar{a}$ Versus $\log_{10} N$ for $\epsilon = 0$

$$\vdots$$

$$(-1)^n \left\{ [\ln (1-\epsilon^2)^{1/2} nN]^{1/2} + \frac{1}{2}\gamma\, [\ln (1-\epsilon^2)^{1/2} nN]^{-1/2} \right\} \quad \ldots\ldots (10)$$

Table 2 presents $E(a^n)/\bar{a}$ for ϵ = 0.00, 0.02, 0.04, 0.06, and 0.08, and for selected values of n and N. Though it contains values of $E(a^n)/\bar{a}$ for large

TABLE 3.—$\mu(a^n)/\bar{a}$ from Eqs. 12 and 13 as a Function of ϵ, n, and N

N	n									
	1	2	3	4	5	6	7	8	9	10
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(a) ϵ = 0.00										
4	1.29	1.13	1.04	0.99						
6	1.42	1.27	1.20	1.15	1.11	1.09				
8	1.52	1.37	1.30	1.25	1.22	1.19	1.17	1.15		
10	1.58	1.45	1.37	1.33	1.30	1.27	1.25	1.23	1.22	1.21
100	2.17	2.07	2.02	1.99	1.97	1.95	1.94	1.93	1.92	1.91
1,000	2.64	2.57	2.53	2.50	2.48	2.47	2.46	2.45	2.44	2.43
(b) ϵ = 0.20										
4	1.28	1.12	1.03	0.98						
6	1.41	1.26	1.19	1.14	1.10	1.08				
8	1.51	1.36	1.29	1.24	1.21	1.18	1.16	1.14		
10	1.58	1.44	1.36	1.32	1.29	1.26	1.24	1.23	1.21	1.20
100	2.17	2.07	2.02	1.99	1.96	1.95	1.93	1.92	1.91	1.90
1,000	2.64	2.56	2.52	2.50	2.48	2.47	2.45	2.45	2.44	2.43
(c) ϵ = 0.40										
4	1.26	1.09	1.00	0.95						
6	1.40	1.24	1.16	1.11	1.08	1.05				
8	1.49	1.34	1.27	1.22	1.18	1.16	1.14	1.12		
10	1.56	1.42	1.34	1.30	1.26	1.24	1.22	1.20	1.18	1.17
100	2.15	2.05	2.00	1.97	1.95	1.93	1.92	1.91	1.89	1.88
1,000	2.63	2.55	2.51	2.48	2.47	2.45	2.44	2.43	2.42	2.41
(d) ϵ = 0.60										
4	1.22	1.04	0.95	0.89						
6	1.36	1.19	1.11	1.05	1.01	0.98				
8	1.45	1.30	1.22	1.17	1.13	1.10	1.08	1.06		
10	1.52	1.37	1.30	1.25	1.21	1.19	1.16	1.14	1.13	1.11
100	0.12	2.02	1.97	1.94	1.91	1.89	1.88	1.87	1.86	1.85
1,000	2.60	2.52	2.48	2.46	2.44	2.42	2.41	2.40	2.39	2.39
(e) ϵ = 0.80										
4	1.13	0.93	0.82	0.75						
6	1.26	1.08	0.99	0.93	0.87	0.85				
8	1.36	1.19	1.10	1.05	1.00	0.97	0.94	0.92		
10	1.43	1.27	1.19	1.14	1.10	1.07	1.04	1.02	1.00	0.98
100	2.05	1.95	1.90	1.86	1.84	1.82	1.80	1.79	1.78	1.77
1,000	2.55	2.46	2.43	2.40	2.38	2.36	2.36	2.34	2.33	2.33

ϵ, for completeness of presentation, the accuracy of these approximate results diminishes with increasing ϵ and decreasing n and N.

Most Probable Value of a^n

The most probable value of a^n, denoted by $\mu(a^n)$, can be determined by finding the maximum of the probability density function of a^n with

$$\alpha = \frac{r^2}{\bar{a}^2} \Rightarrow \frac{\mu(a^n)}{\bar{a}} = \sqrt{\alpha} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (11)$$

and for $\epsilon = 0$, it can be shown (see Appendix II) that α is the solution of the following equation

$$\alpha = \ln\left\{N\left[1 - \frac{(n-1)(1-e^{-\alpha})^N}{1-(1-e^{-\alpha})^N}\right]\right\} - \ln\left[1 - \frac{1}{2\alpha}(1-e^{-\alpha})^N\right] \quad \ldots \quad (12)$$

Fig. 3 and Table 3 present $\mu(a^n)/\bar{a}$ for selected values of n and N. Comparison with corresponding entries in Table 1 show that the amplitudes of $E(a^n)/\bar{a}$ and $\mu(a^n)/\bar{a}$ for $\epsilon = 0$ are similar for small n and N. [Note: $\mu(a^n)/\bar{a} = \sqrt{\alpha}$].

For $\epsilon > 0$, but small, α in Eq. 12, can be approximately generalized to

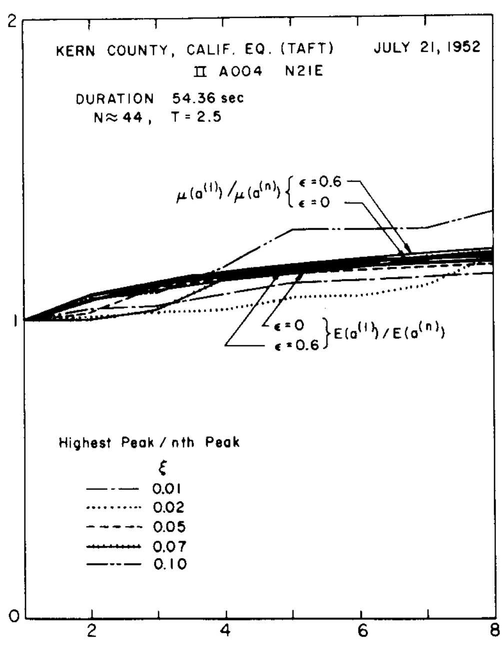

FIG. 4.—Ratio of Largest and nth Peaks [$\mu(a^1)/\mu(a^n)$ and $E(a^1)/E(a^n)$] for Accelerogram (A004, Component N21E) Recorded during July 21, 1952 Kern County Earthquake in California

$$\alpha = \ln\left(N(1-\epsilon^2)^{1/2}\left\{1 - \frac{(n-1)[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]^N}{1-[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]^N}\right\}\right)$$

$$-\ln\left\{1 - \frac{1}{2\alpha}[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]\right\} \quad \ldots\ldots\ldots\ldots \quad (13)$$

Table 3 presents $\mu(a^n)/\bar{a}$ for selected values of ϵ, n, and N.

Analysis

The preceding results are applicable to $f(t)$ which are stationary in time, and to the cases in which the local extrema of $f(t)$ are mutually independent. The success with which these results may then be able to describe the relative amplitudes of the local peaks of $f(t)$ will depend on the degree to which the problem at hand departs from these simple conditions. In earthquake engineering applications, e.g., in which $f(t)$ may represent the relative displacement of a

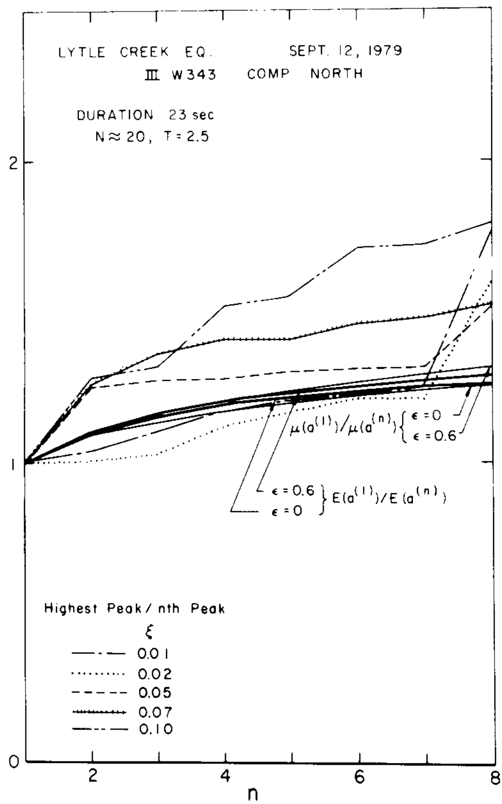

FIG. 5.—Ratio of Largest and nth Peaks [$\mu(a^1)/\mu(a^n)$ and $E(a^1)/E(a^n)$] for Accelerogram (W343, Component NORTH) Recorded during Sept. 12, 1979 Lytle Creek Earthquake in California

viscously-damped oscillator (see Fig. 1), excited by strong ground acceleration, these assumptions are not realized exactly. The nonstationary nature of strong shaking results in nonstationarity of the response, and the "memory" of the oscillator of the preceding excitation increases with decreasing fraction of critical damping. Thus, the relative response, $f(t)$, is neither stationary, nor are the local extrema mutually independent. From the practical viewpoint, however, it is of interest to find whether for some range of the variables describing relative response, the preceding analysis offers some useful information on the

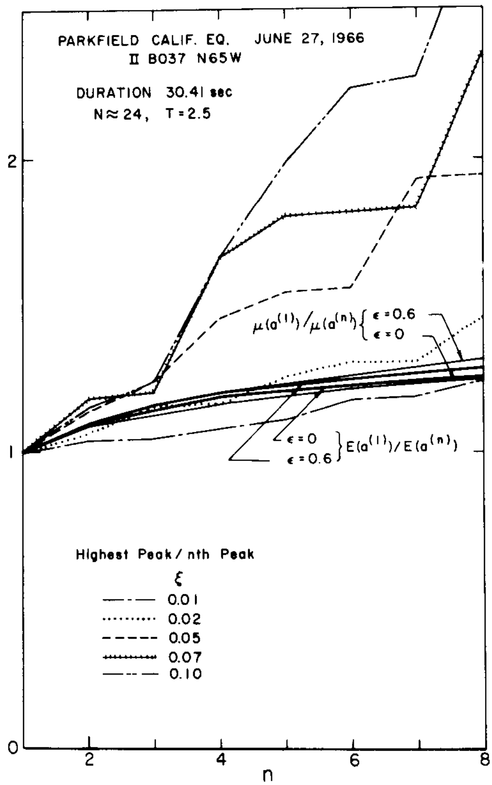

FIG. 6.—Ratio of Largest and nth Peaks [$\mu(a^1)/\mu(a^n)$ and $E(a^1)/E(a^n)$] for Accelerogram (B037, Component N65W) Recorded during June 27, 1966 Parkfield Earthquake in California

trends of the relative amplitudes of peak responses. To this end, a numerical experiment was carried out in which the peaks of the computed response, $f(t)$, were compared with the prediction from the stationary theory.

Figs. 4, 5, and 6 present typical examples. In these figures, the ratio of the highest and the nth peaks of relative displacement response are plotted versus n for selected fractions of critical damping ξ. To facilitate presentation, the discrete points at $n = 1, 2, 3, 4, \ldots$, are connected with lines. It is seen that the computed ratios a^1/a^n gradually increase in a manner not too different from the theoretical trend of $E(a^1)/E(a^n)$ for small n. As ξ, or n, or both, increase, the computed ratio a^1/a^n increases more rapidly than $E(a^1)/E(a^n)$, suggesting that the theoretical model may be adequate only for small n. The ratios $\mu(a^1)/\mu(a^n)$ are very close to the ratios of $E(a^1)/E(a^n)$. As n increases past 4 and 5, and for a longer nonstationary record, Fig. 5 suggests that both $\mu(a^n)$ and $E(a^n)$ may cease to be adequate estimates of a^n amplitudes.

Figs. 7, 8, and 9 compare $E(a^n)/\bar{a}$ and $\mu(a^n)/\bar{a}$ from Eqs. 10 and 13 with computed response to selected recorded strong motion accelerations.

Table 2 predicts that, on the average (for $\epsilon \sim 0$), the largest peak of the relative response amplitude is from about 5% (for $N \sim 1{,}000$) to about 20% ($N \sim 4$) greater than the second largest peak. Table 3 shows that the most probable largest peak is from about 3% (for $N \sim 1{,}000$) to about 14% (for

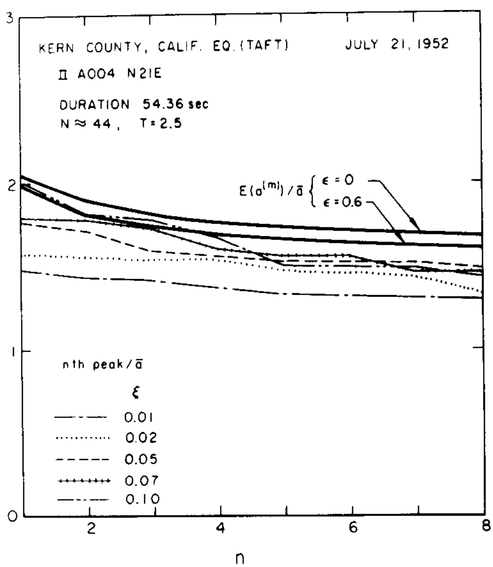

FIG. 7.—$E(a^n)/\bar{a}$ and $\mu(a^n)/\bar{a}$ Versus n from Eqs. 10 and 13 Compared with Computed Ratios from Accelerogram (A004, Component N21E) Recorded during July 21, 1952 Kern County Earthquake in California

$N \sim 4$) greater than the most probable amplitude of the second largest peak. This suggests, e.g., that by reducing the high frequency end of the relative response spectral amplitudes by ~5%, the low frequency end by about 14%–20%, and the intermediate spectral amplitudes by the appropriate intermediate percent reduction, one can determine the expected or the most probable spectral amplitudes which will be exceeded only once throughout the shaking corresponding to the original unreduced spectrum. Continuing further in this direction, by reducing the high frequency spectrum amplitudes by about 10%, the low frequency spectral amplitudes by about 30%–50%, and the intermediate spectral amplitudes by the intermediate spectral amplitudes by the appropriate intermediate reduction, one arrives at the expected or most probable response spectrum

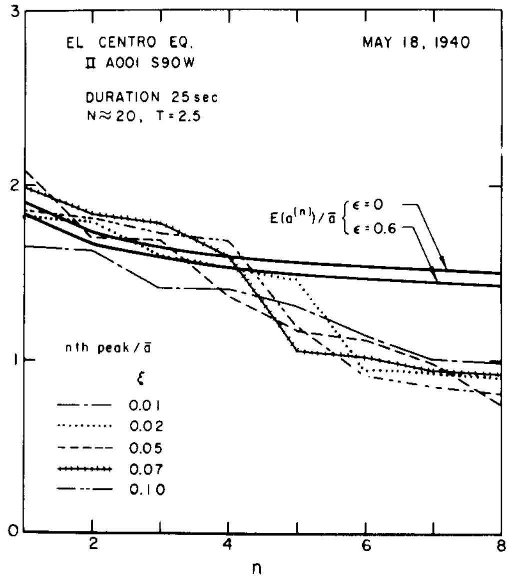

FIG. 8.—$E(a^n)/\bar{a}$ and $\mu(a^n)/\bar{a}$ Versus n from Eqs. 10 and 13 Compared with Computed Ratios from Accelerogram (A001, Component S90W) Recorded during May 18, 1940 Imperial Valley (El Centro) Earthquake in California

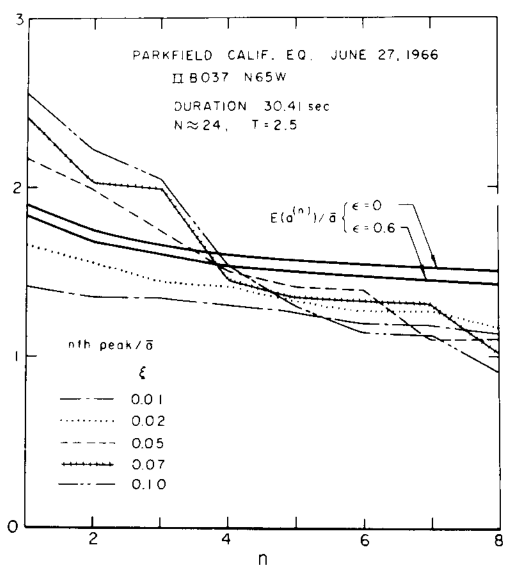

FIG. 9.—$E(a^n)/\bar{a}$ and $\mu(a^n)/\bar{a}$ Versus n from Eqs. 10 and 13 Compared with Computed Ratios from Accelerogram (B037, Component N65W) Recorded during June 27, 1966 Parkfield Earthquake in California

amplitudes which will be exceeded only twice during excitation, corresponding to the originally considered spectral amplitudes. In this analysis, it is assumed that the frequency dependent duration of strong shaking is such that for a typical excitation, high-frequency oscillator experiences 100 cycles–1,000 cycles, while the low-frequency oscillator vibrates through several cycles only (5,6).

The foregoing analysis applies to the rough, overall trends of the response of structural systems which can be characterized by a narrow-band (i.e., $\epsilon \sim 0$) function of the form of $f(t)$ used here. In specific examples, however, departure from the average or most probable trend may be significant as suggested by Figs. 4–9.

Appendix I.—Computation of $E(a^n)/\bar{a}$

Cartwright and Longuet-Higgins (1) show that with

$$\eta = \frac{f(t)}{m_0^{1/2}} \quad (14)$$

and $$\epsilon^2 = 1 - \frac{m_2^2}{m_0 m_4} \quad (15)$$

the probability density function of the heights of maxima of $f(t)/m_0^{1/2}$ is given by

$$p(\eta) = \frac{1}{\sqrt{2\pi}}\left[\epsilon \exp\left(-\frac{1}{2}\frac{\eta^2}{\epsilon^2}\right) + (1-\epsilon^2)^{1/2}\eta \exp\left(-\frac{1}{2}\eta^2\right)\int_{-\infty}^{\eta(1-\epsilon^2)^{1/2}/\epsilon} \exp\left(-\frac{1}{2}x^2\right) dx\right] \quad (16)$$

The cumulative probability $\phi(\eta)$ defined by

$$\phi(\eta) = \int_{\eta}^{\infty} p(\eta)\, d\eta \quad (17)$$

is then given by (1):

$$\phi(\eta) = \frac{1}{\sqrt{2\pi}}\left[\int_{\eta/\epsilon}^{\infty} e^{-1/2x^2}\, dx + (1-\epsilon^2)^{1/2} e^{-1/2\eta^2}\int_{-\infty}^{\eta(1-\epsilon^2)^{1/2}/\epsilon} e^{-1/2x^2}\, dx\right] \quad (18)$$

For consistency with the notation of Longuet-Higgins (2) note that for $\bar{a}$, as defined by Eq. 5

$$\bar{a} = \sqrt{2}\, a_{\mathrm{rms}} \equiv \sqrt{2}\, m_0^{1/2} \quad (19)$$

For $\epsilon = 0$, Eq. 5 becomes

$$\phi(r) = e^{-r^2/\bar{a}^2} \quad (20)$$

in which $\phi(r)$ = the probability that a should exceed a certain value, r; thus

$$p(a \geq r) = e^{-r^2/\bar{a}^2} \quad (21)$$

Then, the probability that every a in the sample of N will be less than r (assuming independence of extrema) becomes:

$$p(a_i \leq r \mid i = 1, 2, \ldots, N) = (1 - e^{-r^2/\bar{a}^2})^N \quad (22)$$

The probability that at least one of N peaks shall exceed r is (4):

$$p(a_i \geq r \mid i = 1, 2, \ldots, N) = 1 - (1 - e^{-r^2/\bar{a}^2})^N \quad (23)$$

Assuming independence of all local extrema and using Eq. 23, the probability that n of a''s (denoted by a^n) shall exceed r as given by:

$$p(a_i^n \geq r \mid i = 1, 2, \ldots, N) = [1 - (1 - e^{-r^2/\bar{a}^2})^N]^n \quad (24)$$

Using the aforementioned results, the probability that a shall lie between r and $r + dr$ becomes

$$p(r \leq a_i^n \leq r + dr) = p(a_i^n \geq r \mid i = 1, 2, \ldots, N)$$

$$- p(a_i^n \geq r + dr \mid i = 1, 2, \ldots, N) \quad (25)$$

or $$p(r \leq a_i^n \leq r + dr) = -d\{[1 - (1 - e^{-r^2/\bar{a}^2})^N]^n\} \quad (25a)$$

From Eq. 25a, the probability distribution of a^n becomes

$$p(r) = \frac{2nNr}{\bar{a}^2} e^{-r^2/\bar{a}^2}(1 - e^{-r^2/\bar{a}^2})^{N-1}[1 - (1 - e^{-r^2/\bar{a}^2})^N]^{n-1} \quad (26)$$

The expected value of a^n, $E(a^n)$ is then

$$E(a^n) = -\int_0^\infty rd\{[1 - (1 - e^{-r^2/\bar{a}^2})^N]^n\} \quad (27)$$

First, integrating by parts, with $\theta = r^2/\bar{a}^2$, or both

$$\frac{E(a^n)}{\bar{a}} = \frac{1}{2}\int_0^\infty [1 - (1 - e^{-\theta})^N]^n \theta^{-1/2} d\theta \quad (28)$$

Next, using binomial expansion and

$$1 - n + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \ldots (-1)^n = 0 \quad (29)$$

there follows

$$[1 - (1 - e^{-\theta})^N]^n = n\left[Ne^{-\theta} - \frac{N(N-1)}{2!}e^{-2\theta} + \ldots (-1)^{N+1}e^{-N}\right]$$

$$- \frac{n(n-1)}{2!}\left[2Ne^{-\theta} - \frac{2N(2N-1)}{2!}e^{-2\theta} + \ldots + (-1)^{2N-1}e^{-2N\theta}\right]$$

$$\vdots$$

$$- (-1)^n\left[nNe^{-\theta} - \frac{nN(nN-1)}{2!}e^{-2\theta} + \ldots + (-1)^{nN-1}e^{-nN\theta}\right] \quad (30)$$

Using Eq. 30 and considering the integral (2):

$$\int_0^\infty e^{-n'\theta}\,\theta^{1/2}\,d\theta = \left(\frac{\pi}{n'}\right)^{1/2} \quad \dots \quad (31)$$

we find

$$\frac{E(a^n)}{\bar{a}} \equiv \frac{n\sqrt{\pi}}{2}\left[N - \frac{N(N-1)}{2!\sqrt{2}} + \frac{N(N-1)(N-1)}{3!\sqrt{3}} + \dots + (-1)^{N+1}\frac{1}{\sqrt{N}}\right]$$

$$-\frac{n(n-1)\sqrt{\pi}}{(2!)2}\left[2N - \frac{2N(2N-1)}{2!\sqrt{2}} + \frac{2N(2N-1)(2N-2)}{3!\sqrt{3}}\right.$$

$$\left. + \dots + (-1)^{2N-1}\frac{1}{\sqrt{2N}}\right] + \frac{n(n-1)(n-2)}{3!}\,\frac{\sqrt{\pi}}{2}\left[2N - \frac{2N(2N-1)}{2!\sqrt{2}}\right.$$

$$\left. + \frac{2N(2N-1)(2N-2)}{3!\sqrt{3}} + \dots + (-1)^{3N+1}\frac{1}{\sqrt{3N}}\right] - (-1)^n\frac{\sqrt{\pi}}{2}\left[nN\right.$$

$$\vdots$$

$$\left. - \frac{nN(nN-1)}{2!\sqrt{2}} + \frac{nN(nN-1)(nN-2)}{3!\sqrt{3}} + \dots + (-1)^{nN-1}\frac{1}{\sqrt{nN}}\right] \quad \dots \quad (32)$$

Using Eqs. 28 and 29 one can write

$$\frac{E(a^n)}{\bar{a}} = \frac{1}{2}\left\{\int_0^\infty [n - n(1-e^{-\theta})^N]\,\theta^{-1/2}\,d\theta\right.$$

$$-\int_0^\infty\left[\frac{n(n-1)}{2!} - \frac{n(n-1)}{2!}(1-e^{-\theta})^{2N}\right]\theta^{-1/2}\,d\theta$$

$$\left. + \int_0^\infty\left[\frac{n(n-1)(n-2)}{3!} - \frac{n(n-1)(n-2)}{3!}(1-e^{-\theta})^{3N}\right]\theta^{-1/2}\,d\theta\right\}$$

$$\vdots$$

$$+\int_0^\infty [(-1)^{n-1} - (-1)^{n-1}(1-e^{-\theta})^{nN}]\,\theta^{-1/2}\,d\theta \quad \dots \quad (33)$$

Then, making use of the approximation suggested by Longuet-Higgins (2), which for the case of $n = 1$ states that

$$\frac{1}{2}\int_0^\infty [1 - (1-e^{-\theta})^N]\,\theta^{-1/2}\,d\theta = (\ln N)^{1/2} + \frac{1}{2}\gamma(\ln N)^{-1/2}$$

$$+ 0(\ln N)^{-1/2} \quad \dots \quad (34)$$

one can derive $E(a^n)/\bar{a}$ for any n, as

$$\frac{E(a^n)}{a} \approx n\left[(\ln N)^{1/2} + \frac{1}{2}\gamma(\ln N)^{-1/2}\right]$$

$$-\frac{n(n-1)}{2!}\left[(\ln N)^{1/2}+\frac{1}{2}\gamma(\ln 2N)^{-1/2}\right]$$

$$\vdots$$

$$(-1)^{n+1}\left[(\ln nN)^{1/2}+\frac{1}{2}\gamma(\ln nN)^{-1/2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

The difference between $E(a^n)/\bar{a}$ in Eqs. 35 and 32 is of the order of $(\ln N)^{-3/2}$ and does not affect the approximate result even for small values of N (4) when $n = 1$.

To generalize these results to the case when $\epsilon \neq 0$, we follow the same procedure as in the case of $\epsilon = 0$ in deriving the expected value $E(a^n)$. The probability distribution of a^n is

$$-d(\{1-[1-\phi(\eta)]^N\}^n) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

$$\text{or} \quad p(\eta) = -\frac{d}{d\eta}(\{1-[1-\phi(\eta)]^N\}^n) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (37)$$

and the expected value of η^n becomes

$$E(\eta^n) = -\int_{-\infty}^{\infty}\eta\, d\{1-[1-\phi(\eta)]^N\}^n \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

Separating the aforementioned integral in two parts we have

$$E(\eta^n) = +\int_{-\infty}^{0} -\eta\frac{d}{d\eta}\{1-[1-\phi(\eta)]^N\}^n d\eta$$

$$+\int_{0}^{\infty} -\eta\frac{d}{d\eta}\{1-[1-\phi(\eta)]^N\}^n d\eta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

Then expanding the term $\{1-[1-\phi(\eta)]^N\}^n$ as:

$$[1-(1-\phi)^N]^n = 1 - n(1-\phi)^N + \frac{n(n-1)}{2!}(1-\phi)^{2N}$$

$$-\frac{n(n-1)(n-2)}{3!}(1-\phi)^{3N} + \ldots + (-1)^n(1-\phi)^{nN} \quad \ldots\ldots\ldots\ldots \quad (40)$$

and substituting Eq. 40 into Eq. 39, there follows

$$E(\eta^n) = \int_{-\infty}^{0} +\eta\frac{d}{d\eta}\left[n(1-\phi)^N - \frac{n(n-1)}{2!}(1-\phi)^{2N}\right.$$

$$\left.+\ldots+(-1)^{n+1}(1-\phi)^{nN}\right]d\eta + \int_{0}^{\infty}\eta\frac{d}{d\eta}\left[n(1-\phi)^N\right.$$

$$\left.-\frac{n(n-1)}{2!}(1-\phi)^{2N}+\ldots+(-1)^{n+1}(1-\phi)^{nN}\right]d\eta \quad \ldots\ldots\ldots\ldots \quad (41)$$

To simplify Eq. 41 we first consider $n = 1$. Then

$$E(\eta^1) = \int_{-\infty}^{0} + \eta \frac{d}{d\eta}(1-\phi)^N d\eta + \int_0^{\infty} \eta \frac{d}{d\eta}(1-\phi)^N d\eta \quad \ldots \ldots \quad (42)$$

Integrating the first integrals by parts yields

$$E(\eta^1) = \eta(1-\phi)^N \Big]_{-\infty}^{0} - \int_{-\infty}^{0} (1-\phi)^N d\eta + \eta(1-\phi)^N \Big]_0^{\infty}$$

$$- \int_0^{\infty} (1-\phi)^N d\eta \quad \ldots \ldots \quad (43)$$

Note that $\eta(1-\phi)^n]_{-\infty}^{0} \approx 0$. Also note that one can write

$$\eta(1-\phi)^N \Big]_0^{\infty} = \int_0^{\infty} 1\, d\eta \quad \ldots \ldots \quad (44)$$

Thus, from Eqs. 43 and 44 there follows

$$E(\eta^1) = -\int_{-\infty}^{0} (1-\phi)^N d\eta + \int_0^{\infty} [1-(1-\phi)^N]\, d\eta \quad \ldots \ldots \quad (45)$$

With the use of Eq. 45 and writing Eq. 41 as the sum of terms like

$$E(\eta^i) = \int_{-\infty}^{0} + \eta \frac{d}{d\eta}(1-\phi)^{iN} d\eta + \int_0^{\infty} \eta \frac{d}{d\eta}(1-\phi)^{iN} d\eta \quad \ldots \ldots \quad (46)$$

in which $i = 1, \ldots, n$, we can write

$$E(\eta^n) = n\left\{-\int_{-\infty}^{0} (1-\phi)^N d\eta + \int_0^{\infty} [1-(1-\phi)^N]\, d\eta\right\}$$

$$-\frac{n(n-1)}{2!}\left\{-\int_{-\infty}^{0} (1-\phi)^{2N} d\eta + \int_0^{\infty} [1-(1-\phi)^{2N}]\, d\eta\right\}$$

$$+\frac{n(n-1)(n-2)}{3!}\left\{-\int_{\infty}^{0} (1-\phi)^{3N} d\eta + \int_0^{\infty} [1-(1-\phi)^{3N}]\, d\eta\right\}$$

$$\vdots$$

$$(-1)^{n+1}\left\{-\int_{-\infty}^{0} (1-\phi)^{nN} d\eta + \int_0^{\infty} [1-(1-\phi)^{nN}]\, d\eta\right\} \quad \ldots \ldots \quad (47)$$

When N is large, $[1-\phi(\eta)]^N$ is very small unless $\phi(\eta)$ is of the order of $1/N$. Thus, Eq. 47 can be written as

$$E(\eta^n) = n\int_0^{\infty} [1-(1-\phi)^N]\, d\eta + \frac{(-n)(n-1)}{2!}\int_0^{\infty} [1-(1-\phi)^{2N}]\, d\eta$$

$$+ \ldots + (-1)^{n+1}\int_0^{\infty} [1-(1-\phi)^{nN}]\, d\eta \quad \ldots \ldots \quad (48)$$

Note that as x tends to infinity

$$\int_x^\infty e^{-1/2x^2}\,dx = e^{-1/2x^2}\left[\frac{1}{x} + 0\left(\frac{1}{x^3}\right)\right] \quad (49)$$

Using Eq. 49 in approximating Eq. 18, we find

$$\phi(\eta) = (1-\epsilon^2)^{1/2} e^{-1/2\eta^2} + 0\left[\left(\frac{1}{\eta^3}\right) e^{-1/2\eta^2/\epsilon^2}\right] \quad (50)$$

for large values of η and when $0 \le \epsilon < 1$. If ϕ is of the order $1/N$, then η is of the order $(\ln N)^{1/2}$. Therefore, neglecting terms of order $(\ln N)^{-3/2}$, we have (1):

$$\phi(\eta) = (1-\epsilon^2)^{1/2} e^{-1/2\eta^2} \quad (51)$$

Letting $\theta = 1/2\eta^2$ in Eq. 51 and substituting into Eq. 48 there results

$$E(\eta^n) = \frac{1}{2^{1/2}}\left\{n\int_0^\infty [1-(1-\epsilon^2)^{1/2}e^{-\theta}]^N\,d\eta\right.$$

$$\frac{1}{2^{1/2}}\left\{\frac{n(n-1)}{2!}\int_0^\infty \{1-[1-(1-\epsilon^2)^{1/2}e^{-\theta}]^{2N}\}\,d\eta\right.$$

$$\vdots$$

$$\frac{1}{2^{1/2}}\left\{(-1)^n\int_0^\infty \{1-[1-(1-\epsilon^2)^{1/2}e^{-\theta}]^{nN}\}\,d\eta\right. \quad (52)$$

To evaluate this integral, consider the last integral only

$$\frac{1}{2^{1/2}}\int_0^\infty \{1-[1-(1-\epsilon^2)^{1/2}e^{-\theta}]^{nN}\}\,d\eta \quad (53)$$

Cartwright and Longuet-Higgins (1) show that the aforementioned integral is equal to

$$e^{1/2}\left[\theta_0^{1/2} + \frac{1}{2}\gamma\theta_0^{-1/2} + 0(\theta_0^{-3/2})\right] \quad (54)$$

in which $\theta_0 = \ln\,[(1-\epsilon^2)^{1/2} iN]$ (55)

and $i = 1, 2, \ldots, n$. Neglecting the terms of the order $\{\ln\,[(1-\epsilon^2)^{1/2} iN]\}^{-3/2}$ and using Eq. 55, one can write an approximate result as

$$E(\eta^n) = 2^{1/2}\left(n\left\{[\ln(1-\epsilon^2)^{1/2}N]^{1/2} + \frac{1}{2}\gamma\,[\ln(1-\epsilon^2)^{1/2}N]^{-1/2}\right\}\right.$$

$$-\frac{n(n-1)}{2!}\left\{[\ln(1-\epsilon^2)^{1/2}2N]^{1/2} + \frac{1}{2}\gamma\,[\ln(1-\epsilon^2)^{1/2}2N]^{-1/2}\right\}$$

$$\vdots$$

$$\left.(-1)^{n+1}\left\{[\ln(1-\epsilon^2)^{1/2}nN]^{1/2} + \frac{1}{2}\gamma\,[\ln(1-\epsilon^2)^{1/2}nN]^{-1/2}\right\}\right) \quad (56)$$

Appendix II.—Computation of $\mu(a^n)/\bar{a}$

The probability distribution of a^n from Eq. 26 in Appendix I is (for $\epsilon = 0$)

$$p(r) = \frac{2nNr}{\bar{a}^2} e^{-r^2/\bar{a}^2}(1 - e^{-r^2/a^2})^{N-1}[1 - (1 - e^{-r^2/\bar{a}^2})N]^{n-1} \quad (57)$$

The most probable value of a^n which is denoted here by $\mu(a^n)$ is derived by finding the maximum value of $p(r)$.

Let $\alpha = r^2/\bar{a}^2$, then

$$\frac{dp}{d\alpha} = e^{-\alpha}(1 - e^{-\alpha})^{N-2}[1 - (1 - e^{-\alpha})^N]^{n-2}\left\{\frac{(1 - e^{-\alpha})[1 - (1 - e^{-\alpha})^N]}{2\sqrt{\alpha}}\right.$$
$$- \sqrt{\alpha}(1 - e^{-\alpha})[1 - (1 - e^{-\alpha})^N] + (N - 1)\sqrt{\alpha}e^{-\alpha})[1 - (1 - e^{-\alpha})^N]$$
$$\left. - N(n - 1)\alpha e^{-\alpha}(1 - e^{-\alpha})^N\right\} \quad (58)$$

Setting the terms inside { } equal to zero and dividing by $2\alpha(1 - e^{-\alpha})[1 - (1 - e^{-\alpha})^N]$ results in

$$e^{\alpha} = \frac{N\left[1 - \dfrac{(n - 1)(1 - e^{-\alpha})^N}{1 - (1 - e^{-\alpha})^N}\right]}{1 - \dfrac{1}{2\alpha}(1 - e^{-\alpha})} \quad (59)$$

or $$\alpha = \ln N\left[1 - \frac{(n - 1)(1 - e^{-\alpha})^N}{1 - (1 - e^{-\alpha})^N}\right] - \ln\left[1 - \frac{1}{2\alpha}(1 - e^{-\alpha})^N\right] \quad (60)$$

For $\epsilon \neq 0$, $p(r)$ in Eq. 57 is of the form

$$p(\eta) = -\frac{d}{d\eta}\{[1 - (1 - \phi)^N]^n\} \quad (61)$$

Letting $\eta^2/2 = \alpha$, taking the derivative of $p(\eta)$ and setting it equal to zero, we find

$$(1 - \epsilon^2)^{1/2}e^{-\alpha}[1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]^{N-2}\{1 - [1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]^N\}^{n-2}$$
$$\left(\frac{[1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]\{1 - [1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]^N\}}{2\sqrt{\alpha}}\right.$$
$$- \alpha[1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]\{1 - [1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]^N\}$$
$$+ (N - 1)\sqrt{\alpha}(1 - \epsilon^2)^{1/2}e^{-\alpha}\{1 - [1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]^N\}$$
$$\left. - N(n - 1)\sqrt{\alpha}(1 - \epsilon^2)^{1/2}e^{-\alpha}[1 - (1 - \epsilon^2)^{1/2}e^{-\alpha}]^N\right) = 0 \quad (62)$$

This results in

$$e^{\alpha} = \frac{N(1-\epsilon^2)^{1/2}\left\{1 - \dfrac{(n-1)[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]^N}{1-[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]^N}\right\}}{1 - \dfrac{1}{2\alpha}[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]} \qquad (63)$$

or $$\alpha = \ln N(1-\epsilon^2)^{1/2}\left\{1 - \frac{(n-1)[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]^N}{1-[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]^N}\right\}$$

$$- \ln\left\{1 - \frac{1}{2\alpha}[1-(1-\epsilon^2)^{1/2}e^{-\alpha}]\right\} \qquad (64)$$

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Appendix IV.—Notation

The following symbols are used in this paper:

$\bar{a}$ = rms amplitude of the peaks of $f(t)$;
a_{rms} = rms of $f(t)$;
a^n = nth highest peak of $f(t)$;
E = expected value;
$E(\omega)$ = energy spectrum;
$f(t)$ = random function;
m_0 = zero moment of energy spectrum $E(\omega)$;
m_2 = second moment of energy spectrum $E(\omega)$;
m_4 = fourth moment of energy spectrum $E(\omega)$;
m_n = nth moment of energy spectrum $E(\omega)$;
N = total number of peaks of response;
P = probability function;
r = amplitude of $f(t)$;
T_n = natural period in seconds ($T_n = 2\pi/\omega_n$);

γ = Euler's constant (0.5772);
ϵ = measure of the width of $E(\omega)$ spectrum;
ξ = damping;
μ = most probable value;
$\phi(\eta)$ = cumulative probability distribution; and
ω_n = natural circular frequency.

16050 DISTRIBUTION IN EARTHQUAKE RESPONSE

KEY WORDS: Design; **Displacement; Distribution; Dynamics; Earthquakes; Linear systems;** Narrowband; Probability theory

ABSTRACT: In the response spectrum approach to earthquake-resistant design, it is assumed that: (1)The structure remains linear or can be modeled by an equivalent linear system; and (2)vibrations can be described by the largest relative (or absolute) response amplitude. From the viewpoint of understanding the progressing damage, however, it is useful to determine other response characteristics which, for example, relate duration of strong shaking with all, not just the largest, relative response amplitude. A generalization of the theory of Cartwright and Longuet-Higgins is presented to describe the expected and the most probable amplitudes of local response peaks in terms of: (1)Root-mean-square amplitude of the response; (2)a measure, (ϵ), of the frequency "width" of the response spectrum; and (3) total number of peaks of response.

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