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A NOTE OF SURFACE STRAINS ASSOCIATED WITH INCIDENT BODY WAVES

MIHAILO D. TRIFUNAC
University of Southern California, Los Angeles, California

ABSTRACT

Surface strains in the elastic half-space associated with incident plane body waves can be expressed exactly in terms of 1) corresponding horizontal displacement, 2) incident wave number, 3) Poisson's ratio for the medium, and 4) the angle of incident waves. Application of these results in earthquake engineering is considered.

INTRODUCTION

Analyses of the earthquake-induced damage to engineering structures show that, by far, the most prominent and widespread cause of destruction is associated with strong ground shaking. There are, however, other examples of damage which result from differential types of motion like large strains associated with ground shaking. In some cases, these strains may be superimposed on the dynamic response and thus only contribute to the resulting total vibrations. In other cases when the characteristic frequencies of the system differ from the principal frequency content of strong motion, local strains may affect the structural systems in a quasi-static manner. Long underground pipelines and railroad tracks may buckle, bridges may collapse because of the excesive support motions and extended structures may crack, to name only a few examples, all from excessive local strains associated with earthquake ground shaking (Okamoto, 1973).

To estimate the amplitudes of local strain, it is necessary to understand the nature of incoming waves (type, amplitude and direction of propagation) as well as the characteristics of the medium through which these waves propagate. Since the strong shaking can be represented by the body and surface wave motion and since the surface waves represent interference patterns of body waves, it is seen that the understanding of strains associated with body waves will facilitate the interpretation of the strains accompanying the complete time history of excitation. To this end, in this paper, we examine the nature of the strains associated with body waves reflecting off the stress-free half-space boundary. We also explore the possibility of evaluating the strain amplitudes from recorded translational motions.

For nonlinear analyses of dynamic response of a soil deposit using essentially linear one-dimensional techniques, some methods employ nonlinear stress-strain relationships to "modify" soil properties according to the computed strain levels. In this respect, the aim of this paper is to point out how near surface strains should vary with amplitudes and approach angle of incident waves in three dimensions. Since the strain amplitudes will decay with depth as cos $[\omega x_2/c_L \cos\theta_0]$, for most θ_0 and for typical frequencies, it is sufficient to discuss surface strain amplitudes only.

INCIDENT P-WAVE

The coordinate system (x_1, x_2, x_3) is chosen so that the ray of incident P-wave, with amplitude A_0 and angle of incidence θ_0 is in the $x_3 = 0$ plane (Figure 1). Upon reflecting from the free surface ($x_2 = 0$), this wave generates two reflected waves: 1) P-wave (amplitude A_1, angle θ_1) and 2) SV-wave (amplitude A_2, angle θ_2). For incident P-wave with frequency ω and wave velocity c_L, at

$x_2 = 0$, there follows (Achenbach, 1973).

$$u_1 = (A_O \sin\theta_O + A_1 \sin\theta_1 + A_2 \cos\theta_2)$$

$$\exp[ik_O (x_1 \sin\theta_O - c_L t)] \tag{1}$$

and

$$u_2 = (A_O \cos\theta_O - A_1 \cos\theta_1 + A_2 \sin\theta_2)$$

$$\exp[ik_O (x_1 \sin\theta_O - c_L t)]. \tag{2}$$

The strains at $x_2 = 0$ are then

$$\epsilon_{x_1} = \frac{\partial u_1}{\partial u_1} = ik_O \sin\theta_O u_1 \ . \tag{3}$$

$$\epsilon_{x_2} = \frac{\partial u_2}{\partial u_2} = ik_O \left(1 - \frac{k^2}{k}\right)(A_O + A_1)$$

$$\exp[ik_O (x_1 \sin\theta_O - c_L t)] \tag{4}$$

and

$$\gamma_{x_1,x_2} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} = 0\,. \tag{5}$$

In these expressions, u_1 and u_2 represent displacements in x_1 and x_2 directions,

$$k_O = \omega / c_L \ , \tag{6}$$

$$k = \left[\frac{2(1-\upsilon)}{1-2\upsilon}\right]^{1/2} \tag{7}$$

θ_O, θ_1 and θ_2 are the angles of incident and two reflected waves (Figure 1), υ is the Poisson's ratio, and t is time. The amplitudes A_1 and A_2 are given by (Achenbach, 1973)

$$\frac{A_1}{A_O} = \frac{\sin 2\theta_O \sin 2\theta_2 - k^2 \cos^2 2\theta_2}{\sin 2\theta_O \sin 2\theta_2 + k^2 \cos^2 2\theta_2}\,, \tag{8}$$

and

$$\frac{A_2}{A_O} = \frac{2k \sin 2\theta_O \cos 2\theta_2}{\sin 2\theta_O \sin 2\theta_2 + k^2 \cos^2 2\theta_2}\,. \tag{9}$$

Figures 2 and 3 show ϵ_{x_1} and ϵ_{x_2} versus θ_O, for selected values of υ and for $A_O = 1$.

INCIDENT SV WAVE

Figure 4 shows the coordinate system and the positive motions A_1 and A_2 for incident SV-wave of amplitude A_O. The incident and reflected rays are in the plane $x_3 = 0$. The two nonzero motions are (at $x_2 = 0$)

$$u_1 = (-A_O \cos\theta_O + A_1 \sin\theta_1 + A_1 \cos\theta_2)$$

$$\exp[ik_O (x_1 \sin\theta_O - c_T t)] \tag{10}$$

and

$$u_2 = (A_O \sin\theta_O - A_1 \cos\theta_1 + A_2 \sin\theta_2)$$

$$\exp[ik_O (x_1 \sin\theta_O - c_T t)] \ , \tag{11}$$

where c_T is the shear wave velocity. At $x_2 = 0$ the corresponding strains are

$$\epsilon_{x_1} = \frac{\partial u_1}{\partial x_1} = ik_O \sin\theta_O u_1 \ , \tag{12}$$

$$\epsilon_{x_2} = \frac{\partial u_2}{\partial x_2} = ik_O \left(\frac{2-k^2}{2k}\right) A_1 \ , \tag{13}$$

and

$$\gamma_{x_1,x_2} = \frac{\partial u_2}{\partial x_2} + \frac{\partial u_1}{\partial x_2} = 0\,. \tag{14}$$

with (Achenbach, 1973)

$$\frac{A_1}{A_O} = \frac{-k \sin 4\theta_O}{\sin 2\theta_O \sin 2\theta_1 + k^2 \cos^2 2\theta_O} \tag{15}$$

and

$$\frac{A_2}{A_O} = \frac{\sin 2\theta_O \sin 2\theta_1 - k^2 \cos^2 2\theta_O}{\sin 2\theta_O \sin 2\theta_1 + k^2 \cos^2 2\theta_O} \tag{16}$$

Figures 5 and 6 present ϵ_{x_1} and ϵ_{x_2} versus θ_O, for selected values of υ and for $A_O = 1$. It is seen that at critical angle $\theta_O = \theta_{cr}$, where $k \sin\theta_O = 1$, strain amplitudes experience maxima.

INCIDENT SH-WAVES

For incident SH-waves (Figure 7) $A_2 = A_o$ and $\theta_2 = \theta_o$. The only nonzero motion is (at $x_2 = 0$)

$$u_3 = 2A_o \exp\left[ik_o(x_1 \sin\theta_o - c_T t)\right] \quad , \tag{17}$$

$$\epsilon_{x_1} = \epsilon_{x_2} = 0 \; ,$$

and

$$\gamma_{x_1,x_3} = \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} = ik_o \sin\theta_o 2A_o \exp$$

$$\left[ik_o\,(x_1 \sin\theta_o - c_T t)\right] \tag{18}$$

DISCUSSION

In earthquake engineering and strong motion seismology, measurements are made of strong motion amplitudes u_1, u_2 and u_3, but no recording is now available on the associated strains. From the preceding results, it is possible, however, to calculate the strains accompanying u_1, u_2 and u_3, provided something is known or can be assumed about (i) the type of incident wave, and (ii) the angle of its incidence. With further improvements in observational analyses of strong ground motion in the interpretation of these motions (Trifunac, 1971), it will become easier to estimate the strains associated with recorded translational motions of strong shaking.

For incident P-waves, it can be seen that

$$\frac{\epsilon_{x_1}}{k_o u_1} = i \sin\theta_o \; . \tag{19}$$

Furthermore, it can be shown that

$$\frac{\epsilon_{x_2}}{k_o u_1} = i \;\; 1 - \frac{k^2}{2} \;\; \frac{2}{k^2} \; \sin\theta_o \tag{20}$$

and

$$\frac{\epsilon_{x_2}}{k_o u_2} = i \left(1 - \frac{k^2}{2}\right) \frac{4}{k} \; \frac{\sin^2\theta_o \sqrt{1 - (\sin\theta_o/k)^2}}{k^2 - 2\sin^2\theta_o} \tag{21}$$

Figure 8 shows the modulus of $\epsilon_{x_2}/(k_o u_2)$ for different values of υ.

For incident SV-waves, from equation (12), there follows

$$\frac{\epsilon_{x_1}}{k_o u_1} = i \sin\theta_o \; . \tag{22}$$

from equations (13), (15) and (16), it can be shown that

$$\frac{\epsilon_{x_2}}{k_o u_1} = i \left(\frac{2-k^2}{2k}\right) \frac{2\sin\theta_o}{k} \tag{23}$$

and

$$\frac{\epsilon_{x_2}}{k_o u_2} = i \left(\frac{2-k^2}{2k}\right) \frac{-\cos 2\theta_o}{(1 - k^2 \sin^2\theta_o)^{1/2}} \; . \tag{24}$$

Figure 9 presents the amplitudes of $\epsilon_{x_2}/(k_o u_2)$. It is seen that at $\theta_o = \theta_{cr}$, equal to 41.8°, 37.8°, 32.2° and 24.1°, for $\upsilon = .1$, .2, .3 and .4, the normalized ϵ_{x_2} in (24) becomes infinite. This diminishes the usefulness of the ratio $\epsilon_{x_2}/(k_o u_2)$ for estimation of ϵ_{x_2} in terms of u_2, since it becomes too sensitive to the errors in estimating the amplitudes of u_2 near these angles of incidence. Thus, whenever it is possible, equation (23) should be employed instead to estimate ϵ_{x_2}.

For incident SH waves, the only nonzero strain can be written as

$$\frac{\gamma_{x_1,x_3}}{k_o u_3} = i \sin\theta_o \; . \tag{25}$$

CONCLUSIONS

It has been shown that the surface strains ($x_2 = 0$) in the infinite half-space, associated with incident plane P and SV waves, can be expressed in the simplest form by using the amplitudes of the associated horizontal displacements as follows:

$$\frac{\epsilon_{x_j}}{k_o u_1} = iA\sin\theta_o \tag{26}$$

where

$$A = 1 \qquad \text{for } j = 1 \tag{27}$$

and

$$A = \frac{2}{k^2} - 1 \qquad \text{for } j = 2\,, \tag{28}$$

Mutatis mutandis $A = 1$ for incident SH waves.

These strain amplitudes depend on (i) the angle of incident waves, and (ii) the Poisson's ratio υ. The strains lag the horizontal motion $u_1 (u_3)$ by $\pi/2$.

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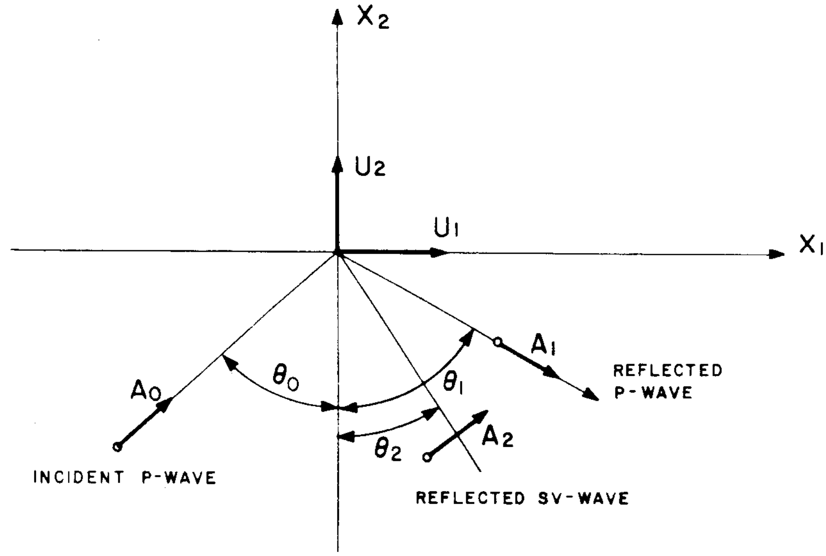

Fig. 1. Coordinate System and Particle Motions for Incident P-Wave (A_0), Reflected P-Wave (A_1) and Reflected SV-Wave (A_2). Ground Motion is given by u_1 and u_2.

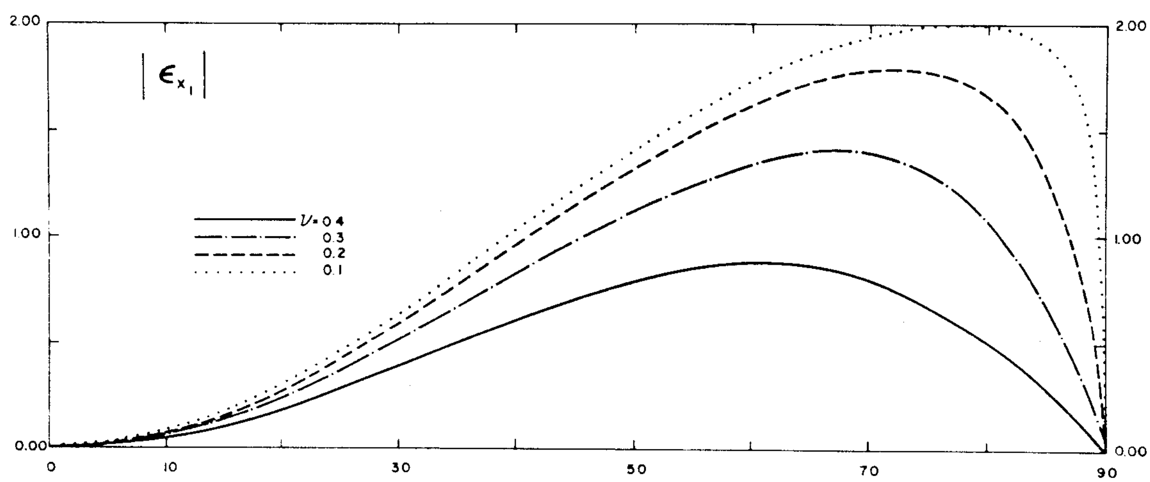

Fig. 2. Normalized ϵ_{x_1} (for $k_oA_o = 1$) versus Incidence Angle θ_o of P-Waves

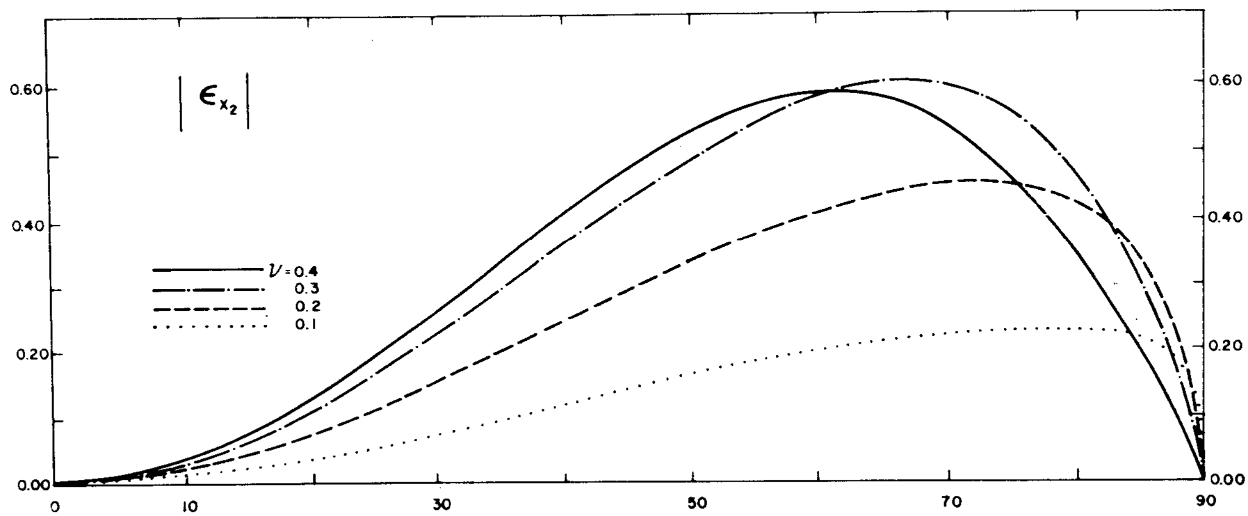

Fig. 3. Normalized ϵ_{x_2} (for $k_0A_0 = 1$) versus Incidence Angle θ_0 of P-Waves

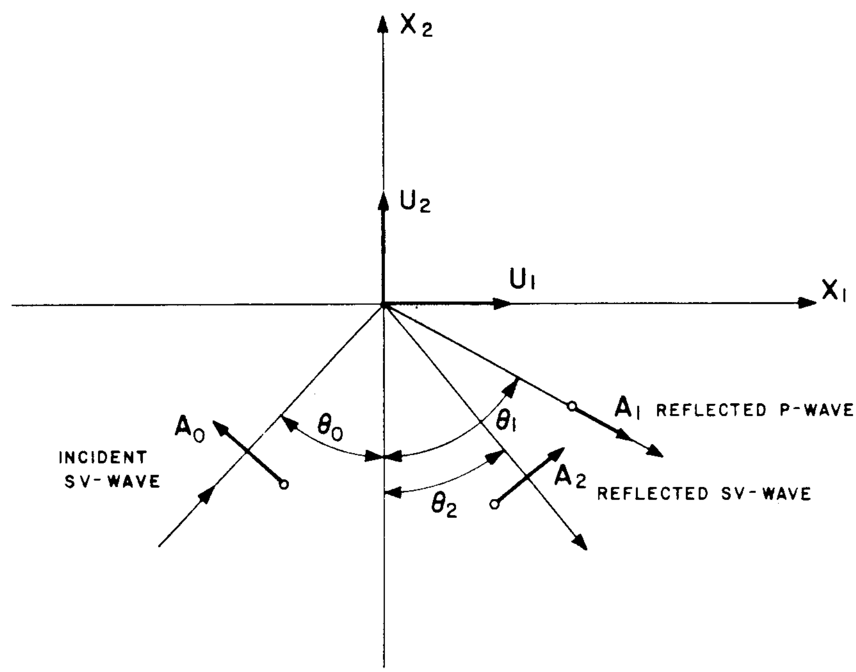

Fig. 4. Coordinate Systems and Particle Motions for Incident SV-Wave (A_0), Reflected P-Wave (A_1), and Reflected SV-Wave (A_2). Ground motions are given by u_1 and u_2

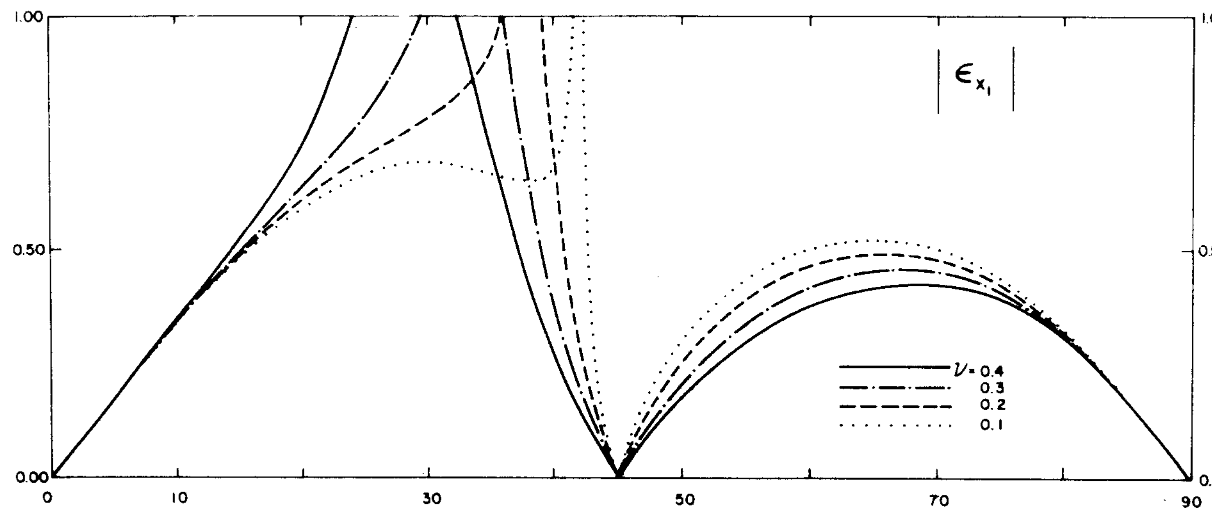

Fig. 5. ϵ_{x_1} (for $k_oA_o = 1$) versus Incidence Angle θ_o of SV-Waves

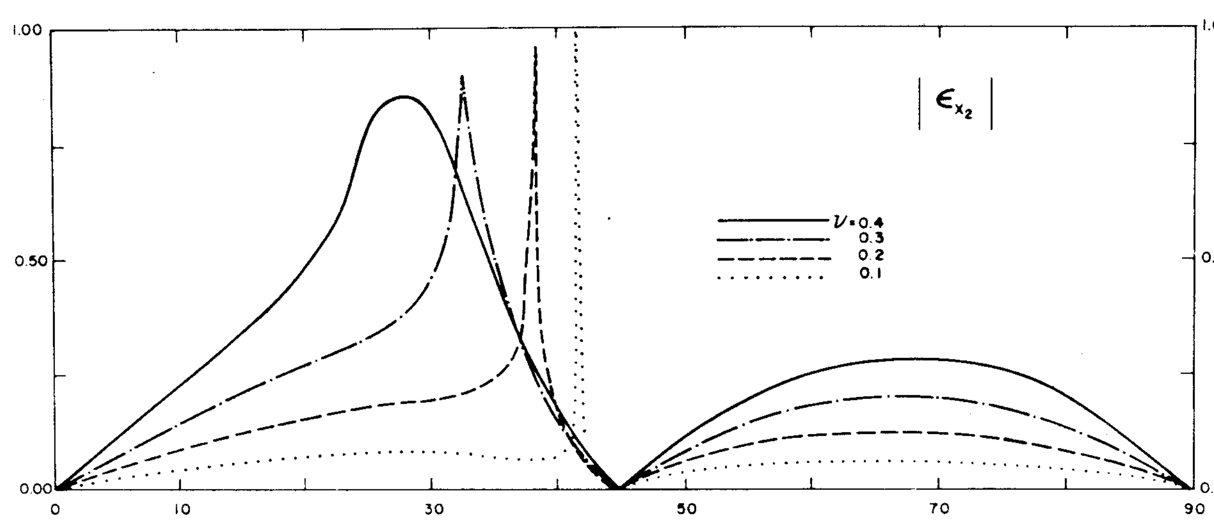

Fig. 6. ϵ_{x_2} (for $k_oA_o = 1$) versus Incidence Angle θ_o of SV-Waves

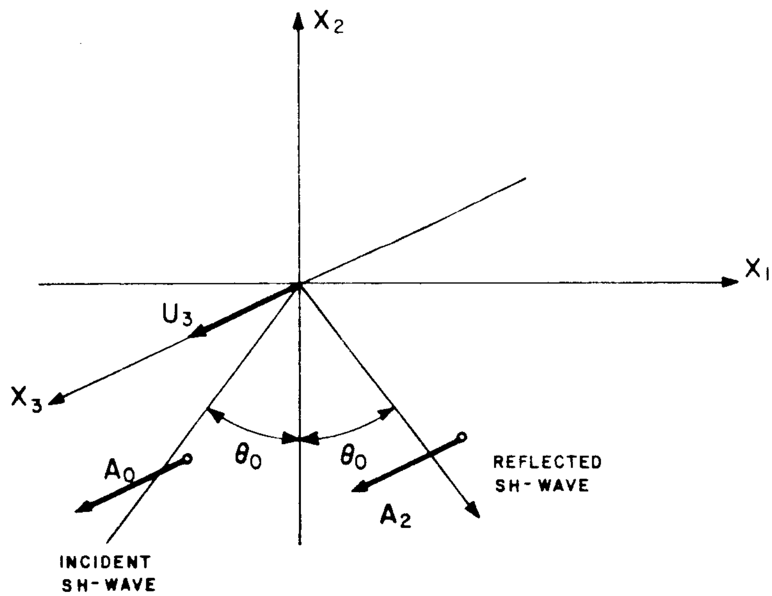

Fig. 7. Coordinate System and Particle Motions for Incident SH-Wave (A_o) and Reflected SH-Wave (A_2). Ground motion is given by u_3

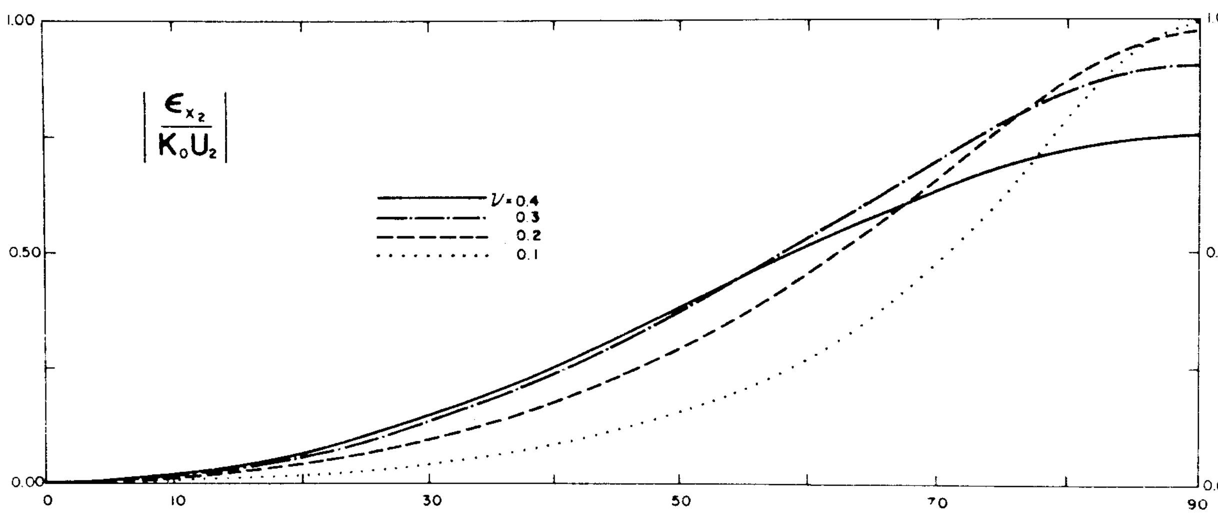

Fig. 8. Normalized ϵ_{x_2} (for $k_o A_o = 1$) versus θ_o for Incident P-Wave

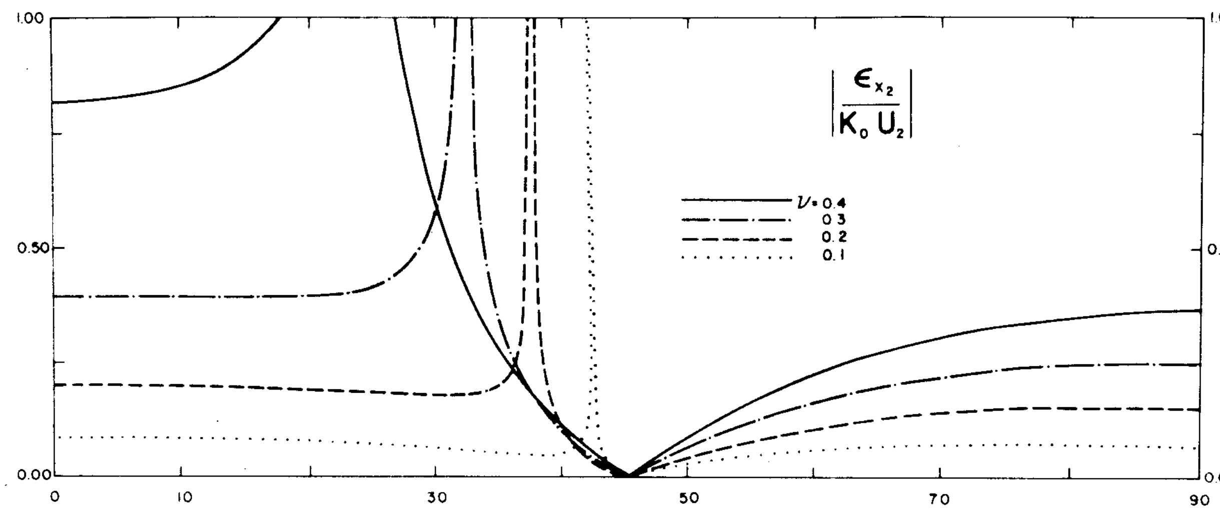

Fig. 9. Normalized ϵ_{x_2} (for $k_0A_0 = 1$) versus θ_0 for Incident SV-Wave