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A NOTE ON THE DYNAMIC RESPONSE OF RIGID EMBEDDED FOUNDATIONS

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SUMMARY

The problem of the dynamic response of rigid embedded foundations subjected to the action of external forces and seismic excitation is analysed. It is shown that to calculate the response of rigid embedded foundations, or the response of flat rigid foundations subjected to non-vertically incident seismic waves, it is necessary to obtain not only the impedance matrix for the foundation, but also the forces induced by the incident seismic waves. Under these general conditions, rocking and torsional motion of the foundation is generated in addition to translation. The case of a two-dimensional rigid foundation of semi-elliptical cross-section is used as an example to illustrate the effects of the embedment depth and angle of incidence of the seismic waves on the response of the foundation.

INTRODUCTION

The problem of the dynamic response of a rigid foundation embedded in an elastic half-space and subjected to the action of both seismic waves and external forces plays an important role in the study of the soil-structure interaction effects for partially embedded structures.

Most present soil-structure interaction studies are based on several simplifying assumptions; namely, the foundation is assumed to be flat, rigid and placed at ground level and the seismic excitation is represented by a vertically incident plane wave.[1–3] These assumptions reflect the fact that no general solutions are available for the problem of the dynamic response of embedded foundations. In particular, the assumption of vertically incident seismic waves is introduced to avoid the study of the scattering of the incoming seismic waves by the foundation, whereas the requirement that the foundation is rigid is used to simplify the mathematical description of the coupling between the superstructure and the ground. In practice, however, most structures are partially embedded in the ground and the seismic excitation cannot be described solely on the basis of vertically incident seismic waves. In fact, analysis of strong-motion records has shown that surface waves, i.e. waves that propagate in a horizontal direction, contribute in a significant amount to the earthquake motion recorded at the soil surface.[4]

There are several qualitative differences between the response of an embedded foundation subjected to non-vertically incident seismic waves and that of a flat foundation excited by vertically incident plane waves. The objective of this note is to analyse these differences and to discuss the effects that these differences may have on the solution of the soil-structure interaction problem.

FORMULATION OF THE PROBLEM

For the purpose of this presentation it is convenient to consider first the steady-state motion of a rigid massless foundation embedded in an elastic half-space and subjected to the action of seismic waves and external forces. The motion of the foundation may be described by six co-ordinates corresponding to two orthogonal horizontal translations, a vertical translation, rocking rotations about two mutually perpendicular

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horizontal axes, and torsion about a vertical axis. The generalized displacement $\mathbf{u}e^{i\omega t}$ of the foundation, where $\mathbf{u}$ is a six-element vector including translations and rotations, may be represented as the superposition of two displacements

$$\mathbf{u} = \mathbf{u}^* + \mathbf{u}_0 \tag{1}$$

where $\mathbf{u}^* e^{i\omega t}$ corresponds to the displacement of the foundation under the action of the seismic waves and in the absence of external forces, while $\mathbf{u}_0 e^{i\omega t}$ corresponds to the displacement of the foundation under the action of the external forces and in absence of seismic excitation. In this work, the displacements $\mathbf{u}^*$ and $\mathbf{u}_0$ will be designated as 'foundation input motion' and 'relative displacement', respectively. In the case of a flat foundation placed on the ground surface and subjected to a vertically incident plane wave, the foundation input motion, $\mathbf{u}^*$, corresponds to a translation and it is equal to the free-field motion. For embedded foundations, or non-vertically incident excitation, the foundation input motion contains rotations as well as translations, and it is different from the free-field motion. The differences between these two cases are larger for short incident waves and for deep embedded foundations.

The relative displacement $\mathbf{u}_0 e^{i\omega t}$ generated by the interaction forces $\mathbf{F}_s e^{i\omega t}$ that the foundation exerts on the ground may be expressed by means of the following linear relationship

$$\mathbf{u}_0 = [\mathbf{C}_s]\mathbf{F}_s \tag{2}$$

where $[\mathbf{C}_s]$ is the compliance matrix for the embedded foundation. Conversely, the interaction forces may be expressed in terms of the relative displacement, $\mathbf{u}_0$, by

$$\mathbf{F}_s = [\mathbf{K}_s]\mathbf{u}_0 \tag{3}$$

where $[\mathbf{K}_s] = [\mathbf{C}_s]^{-1}$ is the impedance matrix for the embedded foundation. Substitution from equation (1) into equation (3) leads to the following alternative expression for the interaction forces

$$\mathbf{F}_s = [\mathbf{K}_s]\mathbf{u} - \mathbf{F}_s^* \tag{4}$$

where

$$\mathbf{F}_s^* = [\mathbf{K}_s]\mathbf{u}^* \tag{5}$$

The forces, $\mathbf{F}_s^*$, appearing in equations (4) and (5) will be called here 'driving forces' and correspond to the forces that the ground exerts on the embedded foundation when the foundation is kept fixed under the action of the seismic waves.[5] The term $[\mathbf{K}_s]\mathbf{u}$ appearing in equation (4) corresponds to the restraining forces acting on the foundation. The driving forces depend on the properties of the foundation and the soil, and also on the nature of the seismic excitation. The impedance matrix $[\mathbf{K}_s]$ depends only on the characteristics of the foundation and soil and on the frequency of the motion.

It is a common practice to separate the real and imaginary parts of the impedance matrix as follows

$$[\mathbf{K}_s] = [\mathbf{k}_s] + i\omega[\mathbf{c}_s] \tag{6}$$

where

$$[\mathbf{k}_s] = \mathrm{Re}\,[\mathbf{K}_s] \tag{7}$$

$$[\mathbf{c}_s] = (1/\omega)\,\mathrm{Im}[\mathbf{K}_s] \tag{8}$$

Thus, the $[\mathbf{k}_s]$ and $[\mathbf{c}_s]$ matrices may be thought of as frequency-dependent stiffness and damping matrices, respectively.

To solve the problem of the dynamic response of rigid embedded foundations, it is usually convenient to separate the problem in two parts: the first part corresponding to the determination of the restraining forces or, equivalently, the determination of the impedance matrix; the second part being the evaluation of the driving forces $\mathbf{F}_s^*$.[5] The foundation input motion $\mathbf{u}^*$ may then be obtained by inverting equation (5).

In the case of a flat foundation subjected to vertically incident seismic waves, the main components of the driving forces are axial forces, although small rocking and torsion moments may also be present because of small coupling terms in the impedance matrix $[\mathbf{K}_s]$.[5–7] For embedded foundations or for non-vertically incident seismic waves, the foundation input motion $\mathbf{u}^*$ in equation (5) already contains rocking and torsional effects and consequently the driving forces for this case will have larger rocking and torsional components.

The steady-state response of a rigid foundation having a mass matrix $[\mathbf{M}_0]$ and subjected to the external forces, $\mathbf{F}_{\text{ext}}\, e^{i\omega t}$, and to the seismic excitation may be easily obtained if the foundation impedance matrix and the driving forces, or, alternatively, the foundation input motion, are known. The equation of harmonic motion, for a co-ordinate system located at the centre of mass of the foundation, is given by

$$-\omega^2[\mathbf{M}_0]\,\mathbf{u} = -\mathbf{F}_s + \mathbf{F}_{\text{ext}}. \tag{9}$$

Substitution from equations (4) and (6) into equation (9) leads to the following equation in terms of the total displacements $\mathbf{u}$ of the foundation

$$(-\omega^2[\mathbf{M}_0] + i\omega[\mathbf{c}_s] + [\mathbf{k}_s])\,\mathbf{u} = \mathbf{F}_s^* + \mathbf{F}_{\text{ext}}. \tag{10}$$

Similarly, substitution from equations (1), (3) and (6) into equation (9) leads to an alternative equation of motion in terms of the relative displacements $\mathbf{u}_0$

$$(-\omega^2[\mathbf{M}_0] + i\omega[\mathbf{c}_s] + [\mathbf{k}_s])\,\mathbf{u}_0 = \omega^2[\mathbf{M}_0]\,\mathbf{u}^* + \mathbf{F}_{\text{ext}} \tag{11}$$

In the case of soil-structure interaction problems, the external forces $\mathbf{F}_{\text{ext}}$ correspond to the forces that the super-structure exerts on the foundation. Equations (10) and (11) justify the designation given to $\mathbf{u}_0$, $\mathbf{u}^*$ and $\mathbf{F}_s^*$ as relative displacement, foundation input motion and driving forces, respectively.

Most of the studies on the dynamic response of foundations have been directed toward the evaluation of the impedance matrices for different types of foundations. Numerical values for the impedance functions over a wide range of frequencies have been presented, among others, by Veletsos and Wei[6] and Luco and Westmann[7] for a flat rigid circular foundation and by Karasudhi *et al.*[8] Oien[9] and Luco and Westmann[10] for a flat rigid strip foundation. Impedance functions for rigid three-dimensional embedded foundations have been presented by Tajimi,[11] Masao and Tajimi,[12] Abe and Ang[13] and also by Novak and collaborators.[14–17] The impedance functions for embedded foundations obtained in these studies are based on various simplifying assumptions in regards to the effects that the layers of soil surrounding the foundation have on the total response.

In particular, the results obtained by Novak and Sachs[17] indicate that for low frequencies the impedance functions of a rigid cylindrical foundation of radius a embedded to a depth h in an elastic half-space are approximately in the following ratios with the corresponding coefficients for a flat foundation:

(a) torsional vibrations

$$\left.\begin{aligned} k_{\text{TT}}/k_{\text{TT}_0} &= 1 + 2{\cdot}38\,\delta \\ c_{\text{TT}}/c_{\text{TT}_0} &= 1 + 7{\cdot}7\,\delta \end{aligned}\right\} \tag{12}$$

(b) vertical vibrations

$$\left.\begin{aligned} k_{\text{VV}}/k_{\text{VV}_0} &= 1 + 0{\cdot}36\,\delta \\ c_{\text{VV}}/c_{\text{VV}_0} &= 1 + 0{\cdot}99\,\delta \end{aligned}\right\} \tag{13}$$

(c) horizontal vibrations

$$\left.\begin{aligned} k_{\text{HH}}/k_{\text{HH}_0} &= 1 + 0{\cdot}80\,\delta \\ c_{\text{HH}}/c_{\text{HH}_0} &= 1 + 3{\cdot}36\,\delta \end{aligned}\right\} \tag{14}$$

(d) coupled horizontal-rocking vibrations

$$\left.\begin{aligned} k_{\text{MH}}/a_0\, k_{\text{HH}_0} &= 0{\cdot}18\,\delta^2 \\ c_{\text{MH}}/a_0\, c_{\text{HH}_0} &= 1{\cdot}68\,\delta^2 \end{aligned}\right\} \tag{15}$$

(e) rocking vibrations

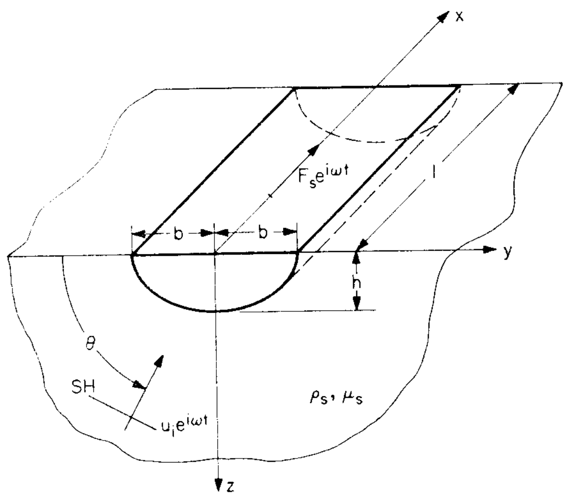
$$\left.\begin{aligned} k_{\text{MM}}/k_{\text{MM}_0} &= 1 + \delta + 0{\cdot}55\,\delta^3 \\ c_{\text{MM}}/c_{\text{MM}_0} &= 1 + 4{\cdot}2\,\delta + 8{\cdot}2\,\delta^3 \end{aligned}\right\} \tag{16}$$

where $\delta = h/a$ is the embedment ratio; k_{TT}, k_{VV}, k_{HH}, k_{MM} and c_{TT}, c_{VV}, c_{HH}, c_{MM} represent the equivalent stiffness and damping coefficients for torsional, vertical, horizontal and rocking vibrations, respectively; and k_{MH} and c_{MH} correspond to the equivalent coupling stiffness and damping coefficients, respectively. The impedance coefficients listed in (c), (d) and (e) correspond to coupled horizontal–rocking vibrations referred to the base of the embedded foundation. These results indicate a considerable increase in the radiation damping and a somewhat lower increase in the equivalent stiffnesses.

In order to study the dynamic response of embedded foundations, knowledge of the impedance matrix $[\mathbf{K}_s]$ and of the driving forces $\mathbf{F}_s^*$ (or, equivalently, the foundation input motion $\mathbf{u}^*$) is required. Unfortunately, very few studies have been directed toward the evaluation of the effects that the embedment and the angle of incidence of the seismic waves have on the driving forces.[9,11,18,19] In general, this aspect of the problem is avoided by assuming that the foundation input motion may be approximated by the free-field motion. A typical example of this situation may be found in a recent study of the dynamic behaviour of structures with embedded foundations conducted by Bielak.[20] In that study the effects of the embedment of the foundation on the impedance functions are considered while the associated effects on the driving forces are neglected. Salmon *et al.*[5] have found that in some cases the above-mentioned approximation leads to conservative estimates of the foundation response. It is important to note, however, that in common practice the free-field motion is defined in terms of translations only; since the foundation input motion contains rotations as well as translations, the torsional and rocking effects may not be properly accounted for by this approximation.

IMPEDANCE AND DRIVING FORCES FOR A TWO-DIMENSIONAL RIGID ELLIPTICAL FOUNDATION

The general solution of the dynamic response of embedded foundations subjected to external forces and to the action of seismic waves represents a very difficult mathematical problem. There are, however, some particular cases that lend themselves to closed-form or series solutions. Such is the case of an infinitely long rigid foundation of semi-elliptical cross-section subjected to antiplane excitation (Figure 1). This problem has been studied by Wong and Trifunac[21] in connection with the dynamic interaction between a shear wall and the soil. Similar studies for a foundation of semi-circular cross-section have been reported by Luco[22] and Trifunac.[23]



$F_s = (K_s \Delta - F_s^*)$
(Massless foundation)

Figure 1. Foundation model

This foundation model, in spite of its simplicity, provides an example by means of which a better understanding of the dynamic response of embedded foundations may be obtained. In particular, the effects of the embedment on the foundation impedance and the effects of the embedment and angle of incidence of the seismic excitation on the driving forces may be studied in detail. Following Wong and Trifunac,[21] the longitudinal impedance function per unit length for a rigid foundation of semi-elliptical cross-section is given by

$$\frac{\mathbf{K}_s}{\mu_s} = -2\pi \sum_{m=0}^{\infty} (A_0^{2m})^2 \frac{Mc_{2m}^{(4)'}(\xi_0, q)}{Mc_{2m}^{(4)}(\xi_0, q)} \tag{17}$$

The corresponding longitudinal driving force per unit length generated by a plane SH wave of amplitude $u_i e^{i\omega t}$ and angle of incidence θ is

$$\frac{F_s^*}{\mu_s u_i} = 4\pi \sum_{m=0}^{\infty} (-1)^m A_0^{2m} ce_{2m}(\theta, q) \frac{Mc_{2m}^{(1)'}(\xi_0, q)\, Mc_{2m}^{(4)}(\xi_0, q) - Mc_{2m}^{(1)}(\xi_0, q)\, Mc_{2m}^{(4)'}(\xi_0, q)}{Mc_{2m}^{(4)}(\xi_0, q)} \tag{18}$$

where μ_s is the shear modulus of the soil, $q = (\omega a/2\beta_s)^2$, $a = (b^2 - h^2)^{\frac{1}{2}}$ is the focal length of the ellipse $(b > h)$, β_s is the shear wave velocity in the soil, $\xi_0 = \frac{1}{2}\log(b+h/b-h)$ is the value of the radial co-ordinate corresponding to the perimeter of the foundation, b is the half-width of the foundation and h is the depth of the embedment. Further details on the Mathieu functions entering in equations (17) and (18) may be found in the above-mentioned study by Wong and Trifunac.[21]

The equivalent stiffness and damping coefficients associated with the real and imaginary parts of K_s are shown in Figure 2 for different embedment ratios h/b and for dimensionless frequencies $a_0 = \omega b/\beta_s$ in the range from 0 to 4. These results indicate that both the equivalent stiffness and damping coefficients are fairly constant for values of the dimensionless frequency larger than 0·5. For low frequencies, the stiffness coefficient

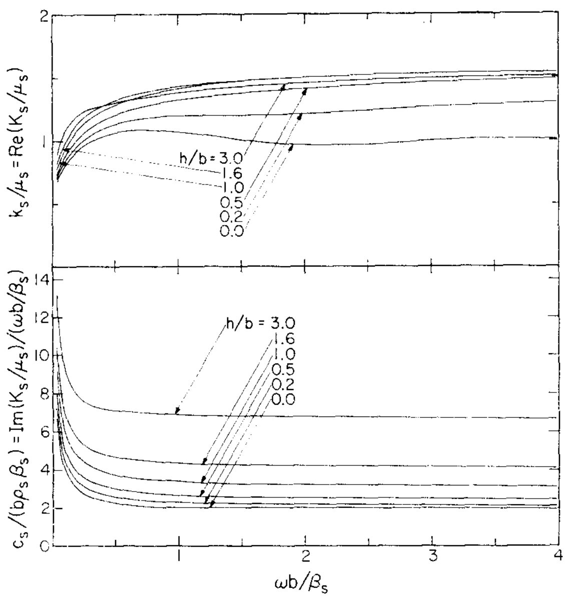


Figure 2. Real and imaginary parts of the longitudinal impedance function

increases with the embedment ratio; for high frequencies, however, the value of the stiffness coefficient lies between the values corresponding to a strip foundation[24] $(h/b = 0)$ and a semi-circular foundation[22,23] $(h/b = 1)$. This result is in agreement with the finding of Thau and Umek[19] that the longitudinal stiffness coefficient for a rectangular embedded foundation at high frequencies is only 20 per cent larger than the corresponding stiffness for a flat foundation. Figure 2 also shows that the damping coefficient is highly dependent on the embedment ratio; the values for high frequencies being directly proportional to the contact area between the foundation and soil. This result is thus in agreement with one's intuitive expectation that the efficiency of energy loss through radiation of waves would be increased for larger foundations.

The real and imaginary parts of the longitudinal driving force, per unit length, $F_s{}^*$ acting on the foundation are shown in Figures 3 and 4, respectively, for different angles of incidence and different embedment ratios. In particular, for a flat foundation ($h/b = 0$) and for vertical incidence ($\theta = 90$ degrees), equation (5) indicates

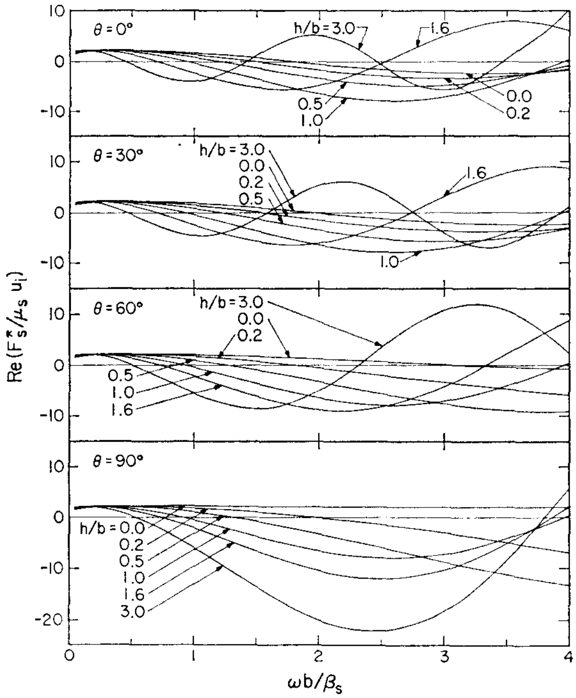


Figure 3. Real part of the longitudinal driving force

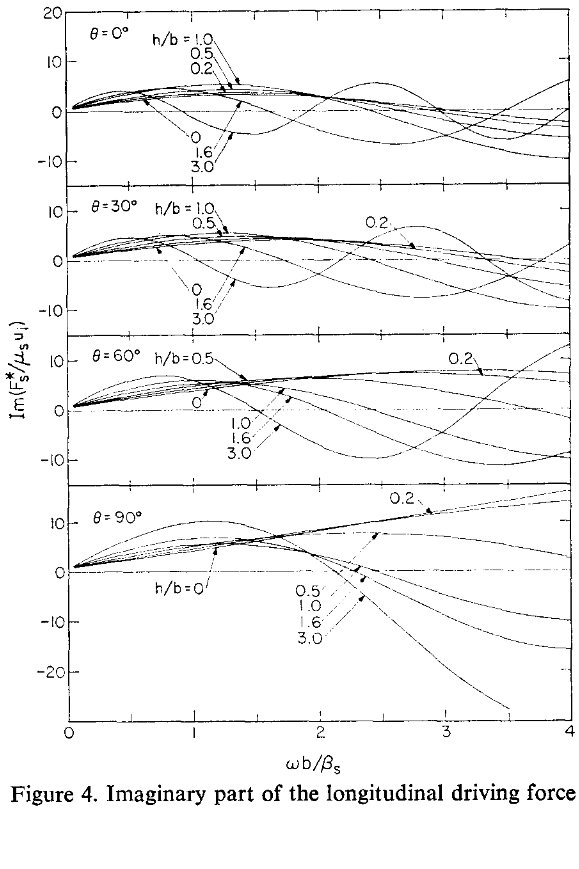


Figure 4. Imaginary part of the longitudinal driving force

that $F_s^* = 2K_s u_i$ ($u^* = 2u_i$). For other embedment ratios and angles of incidence not equal to $\theta = 90$ degrees, $F_s^* \neq 2K_s u_i$, as may be seen in Figure 3. The deviations from this simple case become more important for higher frequencies and for deeper embedments, indicating that the input motion $u^* = F_s^*/K_s$ may be quite different from the free-field motion $2u_i$. In general, these differences become apparent at values of $\omega b/\beta_s$ between 0·5 and 1 and correspond to wavelengths of the order of three to six times the total width of the foundation. The longitudinal impedance and driving force given by equations (17) and (18) may be used, together with equation (10), to study the soil-structure interaction problem for the SH type excitation.[21] In particular, if the superstructure is a shear wall with fundamental fixed-base natural frequency ω_1 and mass per unit length M_b, the force that the superstructure exerts on the foundation is

$$F_{\text{ext}}\, e^{i\omega t} = \omega^2 M_b \frac{\tan(\pi\omega/2\omega_1)}{(\pi\omega/2\omega_1)}\, u\, e^{i\omega t} \tag{19}$$

where u is the total longitudinal displacement of the foundation. Substitution from equation (19) into equation (10) leads to the following result for the total displacement of the foundation

$$u = F_s^* \left\{ k_s + i\omega c_s - \omega^2 \left[M_0 + M_b \frac{\tan(\pi\omega/2\omega_1)}{(\pi\omega/2\omega_1)} \right] \right\}^{-1} \tag{20}$$

where M_0 corresponds to the mass per unit length of the foundation. Equation (20) shows that, in this case, the displacement of the foundation may be obtained as the response of a one degree-of-freedom oscillator of stiffness k_s, damping constant c_s and mass given by the term inside the square bracket in equation (20), when excited by the driving force F_s^*. It should be mentioned that the equivalent stiffness, damping constant, and mass are frequency dependent.

An interesting aspect, usually not considered in soil-structure interaction studies, results from the action of non-vertically incident waves. In this case, the stresses generated in the contact area between the foundation and the soil are not symmetrical with respect to a vertical axis through the centre of the foundation (e.g. Figure 1). Consequently, torsional effects are generated. A measure of these torsional effects is provided by the torsional driving moment T_s^* acting on the foundation and generated by the action of the seismic waves when the foundation is kept fixed. For the foundation model shown in Figure 1, the torsional driving moment per unit length of foundation is given by

$$\frac{T_s^*}{2\mu_s b u_i} = -i\pi \sum_{m=0}^{\infty} (-1)^m A_1^{2m+1}\, ce_{2m+1}(\theta, q) \left[Mc_{2m+1}^{(1)\prime}(\xi_0, q) - Mc_{2m+1}^{(4)\prime}(\xi_0, q) \frac{Mc_{2m+1}^{(1)}(\xi_0, q)}{Mc_{2m+1}^{(4)}(\xi_0, q)} \right] \tag{21}$$

The real and imaginary parts of this moment, T_s^* are shown in Figures 5 and 6, respectively, for different angles of incidence and for different embedment ratios. It may be seen from these Figures that T_s^* becomes more important for shallower angles of incidence and for deeper embedments. For vertical incidence ($\theta = 90$ degrees) the moment T_s^* is, of course, zero for all embedment ratios and all frequencies. It is also seen that the torsional moment is highly dependent on the frequency of the excitation, high values being obtained for dimensionless frequencies in the range from 1 to 3, and corresponding to wavelengths of the order of one to three times the total width of the foundation. These results indicate that the torsional effects may be of importance for structures having large plan dimensions. It should be mentioned that for the particular type of foundation just considered, rocking driving moments about a horizontal axis (y-axis in Figure 1) are also generated. Even though the two-dimensional anti-plane model just considered allows obtaining of estimates of the torsional and rocking driving moments, the evaluation of the torsional and rocking response would require the solution of a three-dimensional problem.

CONCLUSIONS

A discussion of the dynamic response of rigid embedded foundations as compared with the corresponding response of flat rigid foundations has been presented. It has been shown that the evaluation of the response of embedded foundations subjected to the simultaneous action of external forces and seismic excitation requires the determination of the impedance matrix for the foundation as well as the determination of the driving forces, or, equivalently, the foundation input motion. It has been noted that most of the studies on the dynamic response of foundations have been directed toward the determination of the foundation impedance matrix, while little attention has been given to the equally important evaluation of the driving forces. Generally, this last aspect is avoided by assuming that the foundation input motion is equal to the free-field motion. The foundation input motion for embedded foundations or for flat foundations subjected to non-vertically incident seismic waves differs from the free-field motion in the sense that rocking and

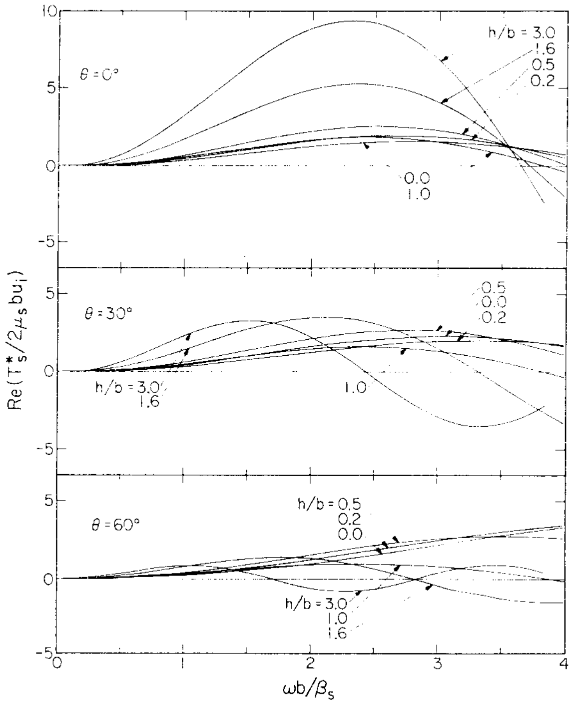


Figure 5. Real part of the torsion driving moment

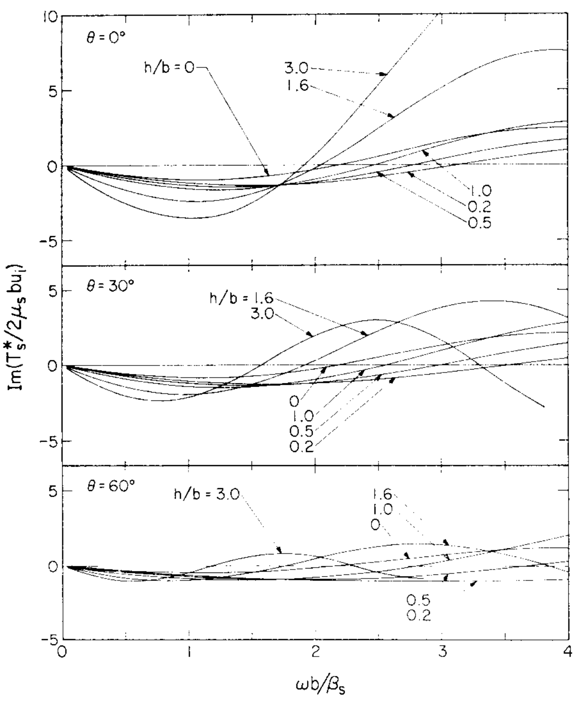


Figure 6. Imaginary part of the torsion driving moment

torsional components are present simultaneously with horizontal and vertical translations. These effects could play an important role in the earthquake response of structures placed on large foundations.

A rigid two-dimensional foundation of a semi-elliptical cross-section has been used as an example to study the effects of the embedment and angle of incidence of the seismic waves on the foundation response. It has been found that the foundation impedance, especially the part associated with the radiation damping, is highly dependent on the embedment ratio. The longitudinal driving force depends on both the embedment ratio and the angle of incidence and shows a strong dependence on the frequency of the excitation. Torsional and rocking driving moments are generated in addition to the longitudinal driving force. The torsional driving moment becomes more important for deeper foundations and for shallower angles of incidence and reaches maximum values when the wavelength is of the order of one to three times the total width of the foundation.

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