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A NOTE ON THE ACCURACY OF COMPUTED GROUND DISPLACEMENTS FROM STRONG-MOTION ACCELEROGRAMS

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ABSTRACT

In this paper the accuracy of routine methods for processing strong-motion earthquake accelerograms (Trifunac, 1971, 1972; Hudson *et al.*, 1971) has been tested by comparing displacement curves computed from the twice-integrated accelerograph recordings with displacement curves computed from displacement-meter measurements. The displacement meters have transducers with natural periods typically several seconds long. Agreement is found to be very good, suggesting that the methods used for routine processing of strong-motion accelerograms are quite accurate.

INTRODUCTION

Accurate data on strong earthquake ground motions are essential for gaining a better understanding about the damaging potential of earthquakes. To measure the ground motions, a high-frequency transducer ($f_n = 10$ to 30 Hz) is typically used. An acceleration transducer is selected for two important reasons: (1) It results in a rugged instrument that can withstand severe shaking and (2) the absolute ground acceleration represents the most direct information needed for calculations of structural response to ground shaking. In many applications, however, when information on the intermediate and low frequencies of the earthquake spectrum is required, data on ground velocity or displacement may be preferred. For example, for long bridges excessive relative motion of the abutments may prove to be a key factor in the kinematic stability of the structural system, or quasi-static axial compression and/or extension of long pipes, tunnels, and similar structures may be the governing factor in their response to strong ground shaking. In source mechanism studies, for example, to avoid the adverse effects of the propagation path and surface topography on the recorded motions, the emphasis is frequently placed on ground displacements which by their nature bring out the long-wave low-frequency components of ground motions that are not very sensitive to these propagational effects (Trifunac, 1974; Trifunac and Udwadia, 1974). It thus appears that all forms of strong ground motion, and not only the recorded acceleration, are needed for a complete description and evaluation of near-field earthquake phenomena.

The frequencies one can recover with a given recording-processing system depend on the combined noise level of all procedures in the processing chain, on the signal amplitude in the frequency band of interest, and finally on the type of transducer used. For acceleration transducers, for example, ground displacements of the form $y = A \sin \omega t$ are recorded as $-A\omega^2 \sin \omega t$. This means that at some long period ($\omega \rightarrow 0$) the transducer output, which is proportional to ω^2, will become too small to separate from the recording and processing noise. The frequency at which this happens will then represent the low-frequency limit down to which the data can be resolved. The usable frequency band may further be restricted at the low-frequency end by the requirements for signal accuracy specified in terms of the signal-to-noise ratio. Because this ratio depends on the amplitude of the recorded signal, we find that the usable frequency band varies from one record to another. In strong-motion seismology, this means that the accelerograms recorded close to the epicenter or during larger earthquakes will contain, relatively speaking, more accurate long-period data than those accelerograms recorded at greater distances or during smaller magnitude shocks. Although the optimum usable frequency band could be determined separately for every recorded accelerogram and this may be feasible for specialized research applications, in routine processing of vast amounts of strong-motion data we use an average long-period limit. Ideally, this limit results in a conservative choice which leads to elimination of some still usable long-period data. Occasionally this average long-period limit may allow for some low-frequency noise to remain unfiltered from data that have an unusually small signal-to-noise ratio.

The amplitudes and the frequency distribution of long-period recording and processing errors have been studied in considerable detail at the Earthquake Engineering Research Laboratory of the California Institute of Technology (e.g., Trifunac *et al.*,

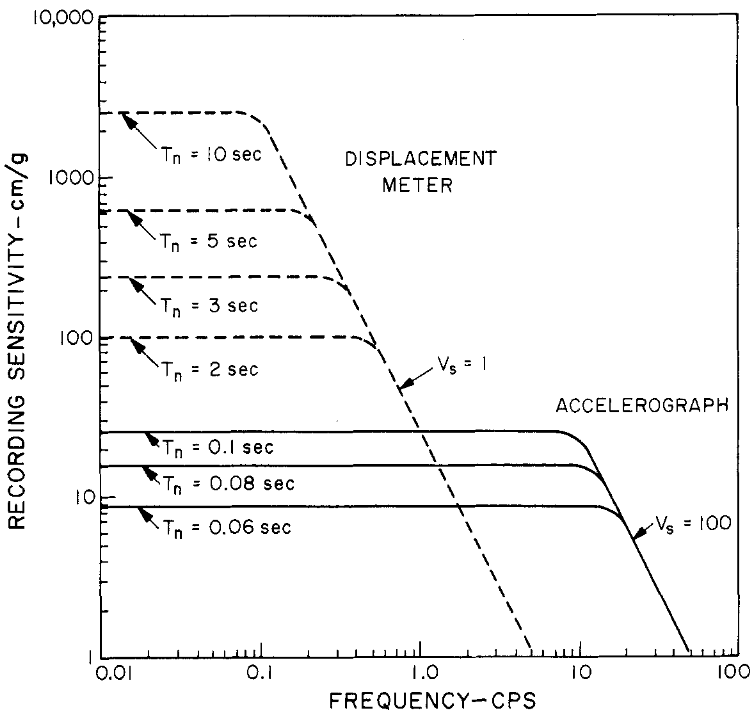

FIG. 1. Typical transducer sensitivities for USCGS strong-motion instrument.

1973a, b). These studies have indicated that under typical conditions the resulting displacement amplitudes can be uncertain to within several centimeters at the selected cut-off period of 16 sec. T. C. Hanks (see Hanks, 1974; Trifunac *et al.*, 1973c) has recently examined numerous accelerograph recordings obtained during the San Fernando earthquake. By comparing the computed ground displacements at stations separated only by a small fraction of the recorded wavelengths, he has been able to provide an independent check of the accuracies of ground displacements computed from accelerograms. His results indicate that for a recording sensitivity of about 8 cm/g (Figure 1), the estimated errors are less than 1 cm in the period range of 5 to 8 sec, approximately 2 cm at periods close to 10 sec, and several centimeters in the period range of 10 to 15 sec.

The purpose of this paper is to present some additional information on the accuracy of long-period displacement measurements, to evaluate the significance of the long-period ground motions that have been filtered out by routine processing schemes ($T \gtrsim 16$ sec), and to examine the usefulness of long-period ($T_n = 2$ to 10 sec) transducers for recording strong ground motion. These long-period transducers, frequently referred to as "displacement meters," have been used essentially as a supplement to the USCGS strong-motion accelerograph since the beginning of the strong-motion measurement program in the United States, but have so far received little or no attention. In what follows, we will use the records from these transducers to check the accuracy of the ground displacements computed from acceleration transducers measuring the same component of ground motion. We will also examine the usefulness of these "displacement meters" for measuring the absolute ground acceleration and the frequency band in which this can be done.

Accuracy of Displacement Computed from Acceleration Record

To test the computed displacements obtained by double integration of strong-motion accelerograms (Hudson *et al.*, 1971), we analyze here the records from the standard USCGS strong-motion instruments. Some of these instruments have been equipped with "displacement meters," i.e., long-period transducers with $T_n = 2$ to 10 sec, recording in the same direction as the acceleration transducers. Typically these acceleration transducers have a static magnification, V_s, of about 100, whereas the "displacement meters" have a static magnification of about 1 (Figure 1). Both transducers operate as single-degree-of-freedom oscillators and can be characterized by the standard differential equation of motion

$$\ddot{x} + 2\omega_n \zeta \dot{x} + \omega_n^2 x = -V_s \ddot{z} \tag{1}$$

where

x = relative displacement of the transducer mass
ω_n = natural frequency of the oscillator
ζ = fraction of critical damping, usually about 60%
V_s = static magnification
$\ddot{z}$ = absolute support acceleration.

By definition, for a static accleration $\ddot{z}$ equal to $1g$, the amplitude of the recorded trace offset represents the recording acceleration sensitivity S. From equation (1) it is seen that the static magnification V_s and the recording sensitivity S are related by

$$V_s = \omega_n^2 \frac{S}{g}. \tag{2}$$

For typical USCGS strong-motion instruments the acceleration and displacement meter sensitivities are shown in Figure 1. From this figure we find that depending on the natural period $T_n = 2\pi/\omega_n$ of the transducer, the displacement meter can have a recording acceleration sensitivity between one and two orders of magnitude larger than that of the acceleration transducer for the ground-motion periods greater or equal to the period of the transducer. The contribution of the high frequencies to the ground displacements is, of course, negligible because of the effective $1/\omega^2$ filtering which results from the double integration. This means that the signal to the digitization noise ratio for the displacement meter transducer is also one to two orders of magnitude higher at the low-frequency end of the recorded spectrum. Consequently, the accuracy of the computed displacements from such a transducer is much higher than that computed from the accelerograph record and can be used to test the accuracy of the displacements calculated from accelerograph recordings.

To test the accuracy of routine methods for computing ground displacements (Hudson *et al.*, 1971), in Figures 2 and 3 we compare six displacement curves computed from accelerograms with the displacement curves calculated from displacement meter recordings. To compute ground displacement either from accelerograms or from a displacement meter record the same steps that are involved in routine processing of accelerations (Trifunac and Lee, 1973) were followed. These steps involve the instrument correction

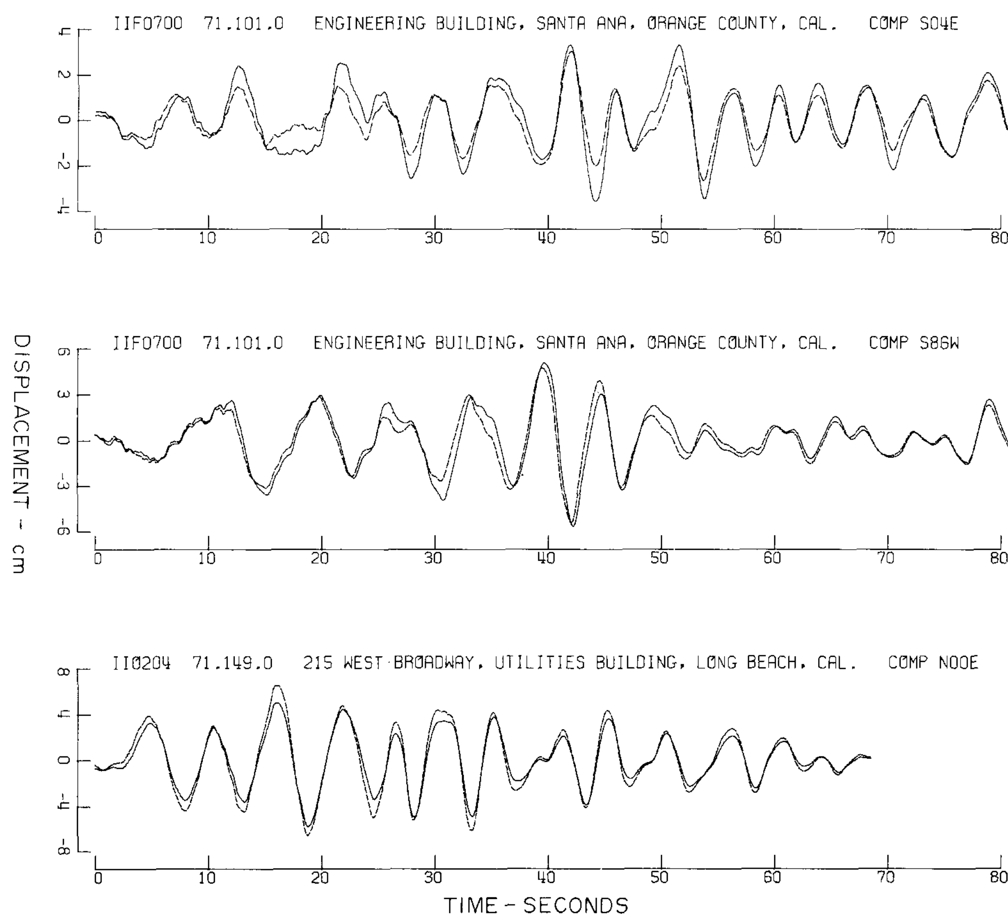

FIG. 2. High-pass filtered ground displacement, in the San Fernando earthquake of February 9, 1971, computed: - - - from displacement meter record and ——— from accelerograph record (f_c = 0.07 Hz; f_T = 0.05 Hz).

(Trifunac, 1972) and the base-line correction (Trifunac, 1971) followed by double integration and the high-pass filtering of velocity and displacement data (Hudson *et al.*, 1971). For the six records presented in Figures 2 and 3, acceleration and displacement meter transducers recorded the same components of ground motion, so that, ideally, the high-pass filtered displacements computed from the different transducers should be identical. Although some differences exist, the agreement between the two sets of curves is quite good. The discrepancies that exist between some of the curves apparently result from inaccurate sensitivity information available for several transducers which affects the differences only in a linear manner. We made no attempt to recalibrate the transducer sensitivities but used the sensitivities provided by the Seismic Engineering Branch of the USGS, inasmuch as the linear scaling factors do not in any way affect the results of the present analysis. The instrumental constants used for calculations in this paper are summarized in Table 1.

The data in Figures 2 and 3 have been high-pass filtered using an Ormsby filter (Trifunac, 1970) with the cut-off frequency $f_c = 0.07$ Hz and a roll-off termination frequency $f_T = 0.05$ Hz so that both the accelerogram and the displacement meter records contain only periods shorter than about 16 sec. Although this comparison demonstrates that the point-by-point accuracy of the displacement computed from accelerograms is good to

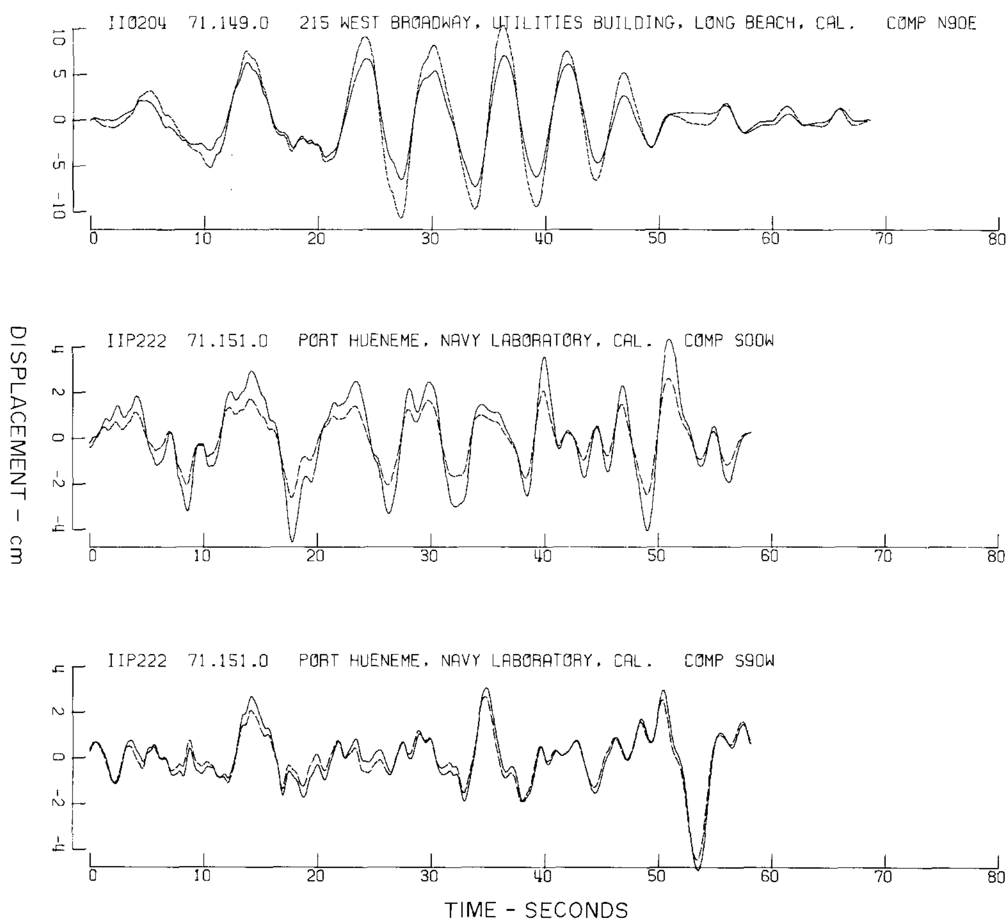

FIG. 3. High-pass filtered ground displacement, in the San Fernando earthquake of February 9, 1971, computed: – – – from displacement meter record and ——— from accelerograph record ($f_c = 0.07$ Hz; $f_T = 0.05$ Hz).

within several centimeters for this frequency band ($f > 0.07$ Hz) in agreement with other independent estimates of the same accuracy (Trifunac, 1970; Hanks, 1974), it is interesting to examine the effects of this limited frequency band on the presented displacement curves. From the physical nature of strong earthquake ground motion we know that close to an earthquake source permanent ground displacements do occur. Because the D.C. components of ground motion cannot be measured with acceleration transducers having finite dynamic range, the resulting high-pass filtered displacement data might then present a seriously distorted picture of actual ground motions. At greater distances, where the static field contributes negligible amounts to the permanent ground displacements, the dynamic field may still contribute to ground motions in the frequency range outside the frequency band that can be extracted accurately from strong-motion accelerograms, and the high-pass filtered ground displacement, again, may present a distorted picture of actual ground movements. To examine this point the S04E component of ground motion at Santa Ana was further studied. In figures 4 and 5 the ground displacements computed from accelerograms and high-pass filtered to include only periods shorter than about 16 sec ($f_c = 0.07$ Hz, $f_T = 0.05$ Hz) are compared with ground displacements computed from the displacement-meter record high-pass filtered to include only periods shorter than 8, 10, 15, 20, 25, 30, 40, and 50 sec ($f_c = 1/8, 1/10, \ldots, 1/40, 1/50$ Hz and $f_T = f_c + 0.02$ Hz). Small changes in the displacement curves occur as the cut-off period is shifted from 8 to 50 sec. These variations are uniformly distributed over the record duration relative to the displacement computed from the accelerogram for the entire range of the cut-off periods examined and are not larger than the expected

TABLE 1

CONSTANTS OF ACCELEROGRAPHS AND DISPLACEMENT METERS

Station and Instrument	Direction	T_n	ζ Damping	V_s Static Magnification
Station 272, Port Hueneme				
Accelerographs	Up	0.081	0.59	118
	South	0.080	0.57	120
	West	0.081	0.59	119
Displacement meters	South	2.33	0.59	1.0
	East	2.50	0.59	1.0
Station 131, L. B. Utilities Building				
Accelerographs	Up	0.066	0.55	118
	North	0.066	0.59	121
	East	0.066	0.59	119
Displacement meters	North	2.01	0.59	0.9
	West	2.21	0.59	0.9
Station 281, Engineering Building, Santa Ana				
Accelerographs	Up	0.064	0.57	112
	South	0.063	0.57	117
	West	0.064	0.57	113
Displacement meters	SO4E	4.7	0.59	1.0
	N86E	4.74	0.59	1.0

uncertainties in the displacements computed from accelerograph records (Figures 2 and 3). Inasmuch as this nature of the long-period components of measured ground motions is undoubtedly related to the particular source mechanism, transmission path, and the source-to-station distance (about 80 km), it is interesting to observe that for this particular site ground motions calculated from the accelerograph record and processed by the methods used for routine processing of all strong-motion accelerograms (Trifunac and Lee, 1973) are apparently quite representative of ground motions in a much broader period range.

To illustrate the relative changes effected by different cut-off frequencies of the high-pass filtered data, all displacement curves from Figures 4 and 5 computed from the displacement meter record have been plotted with common scales in Figure 6. The observed differences, only slightly exceeding about 1.5 cm, clearly suggest that the content of long-period motions in the S04E direction at this station has been very small.

Displacement Meter as an "Accelerograph"

Just as the relative displacement response of the high-frequency acceleration transducer can be corrected at high frequencies (higher than the transducer's natural frequency) to compute the absolute ground acceleration (Trifunac, 1972), the same correction procedure can be carried out for the displacement meter transducer. The only limitation in

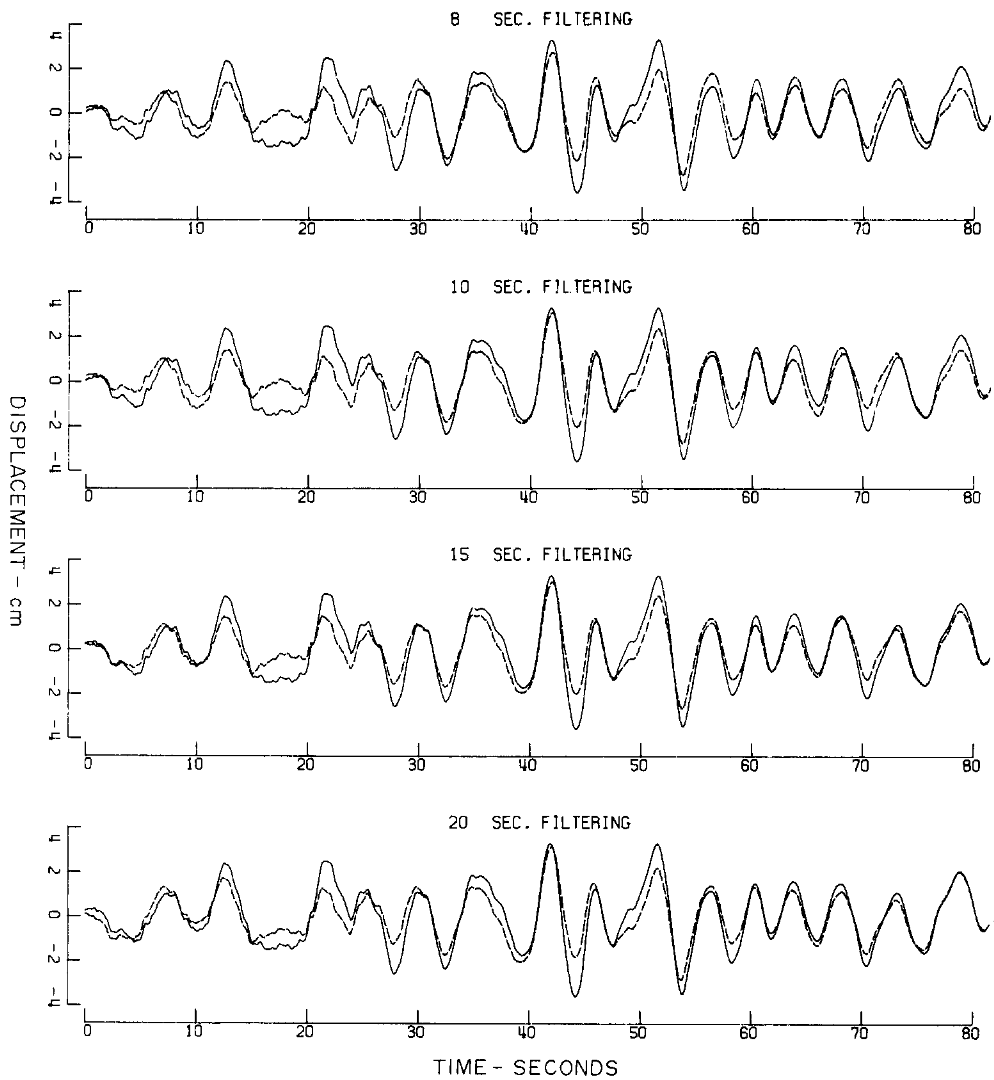

Fig. 4. Ground displacement in the San Fernando earthquake of February 9, 1971, at the Engineering Building, Santa Ana, Orange County, California (Comp. S04E), computed: - - - from displacement meter record and ——— from accelerograph record ($f_c - f_T = 0.02$ Hz).

both correction procedures results from the signal-to-noise ratio at high frequencies which limits the usable high-frequency band of the transducer output.

As an accelerometer the displacement meter transducer is characterized by a transfer function (Figure 1) which attenuates the signal amplitudes of frequencies higher than the natural frequency like $(\omega_n/\omega)^2$. Consequently it is of interest to find what is the highest input acceleration frequency that can be recovered from a displacement meter record. To find this we studied again the S04E component of ground motion in the Engineering

Building, Santa Ana. Both accelerograph and displacement meter data were corrected for instrument response and low-pass filtered by series of Ormsby filters. By comparing the ground acceleration recorded by the corresponding accelerograph and the acceleration computed from the displacement meter record, it has been possible to find that this particular displacement meter (T_n = 4.7 sec, ζ = 59%, V_s = 1) can give accurate information on the absolute ground acceleration up to the frequency of about 2 Hz. As shown

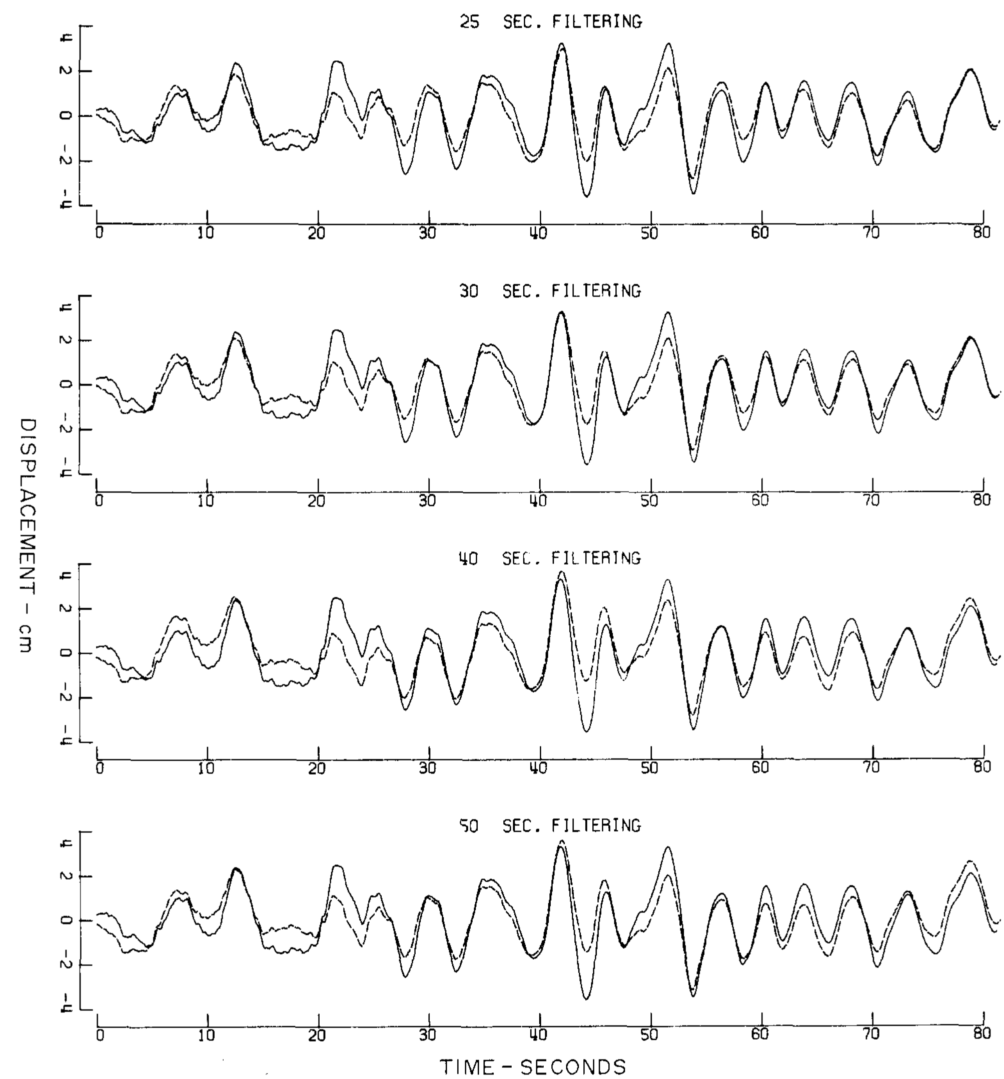

FIG. 5. Ground displacement in the San Fernando earthquake of February 9, 1971, at the Engineering Building, Santa Ana, Orange County, California (Comp. S04E), computed: --- from displacement meter record and ——— from accelerograph record ($f_c - f_T$ = 0.02 Hz).

in Figure 7 for the cut-off frequency f_c = 3 Hz, the high-frequency digitization noise begins to affect the computed acceleration seriously.

This example shows that it is possible to resolve the input acceleration from a displacement meter record up to a frequency which is about an order of magnitude higher than the natural frequency of the transducer. This of course depends on the amplitude of the digitization noise which is governed by the digitization system used. For the typical

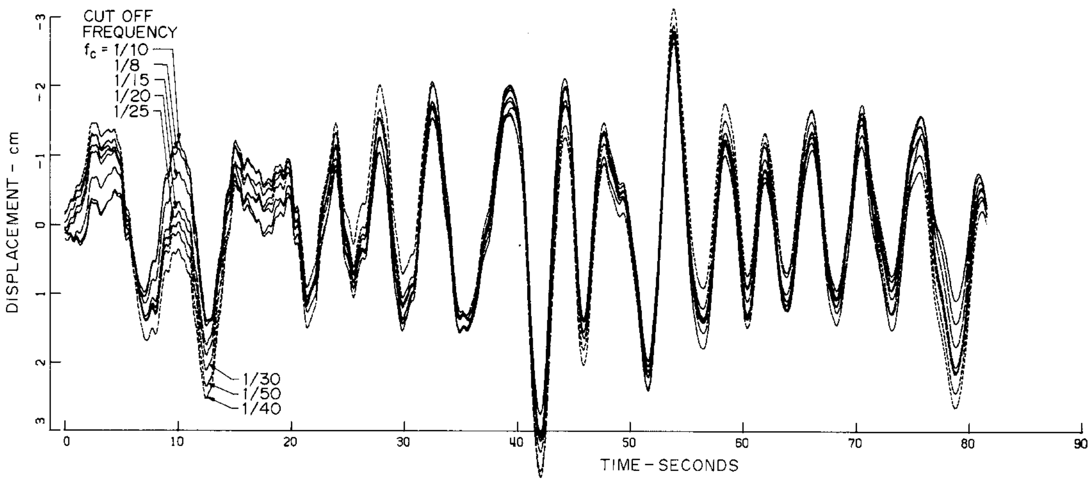

FIG. 6. High-pass filtered ground displacement, in the San Fernando earthquake of February 9, 1971, at the Engineering Building, Santa Ana, Orange County, California (Comp. S04E), computed from displacement meter record ($f_c - f_T$ = 0.02 Hz).

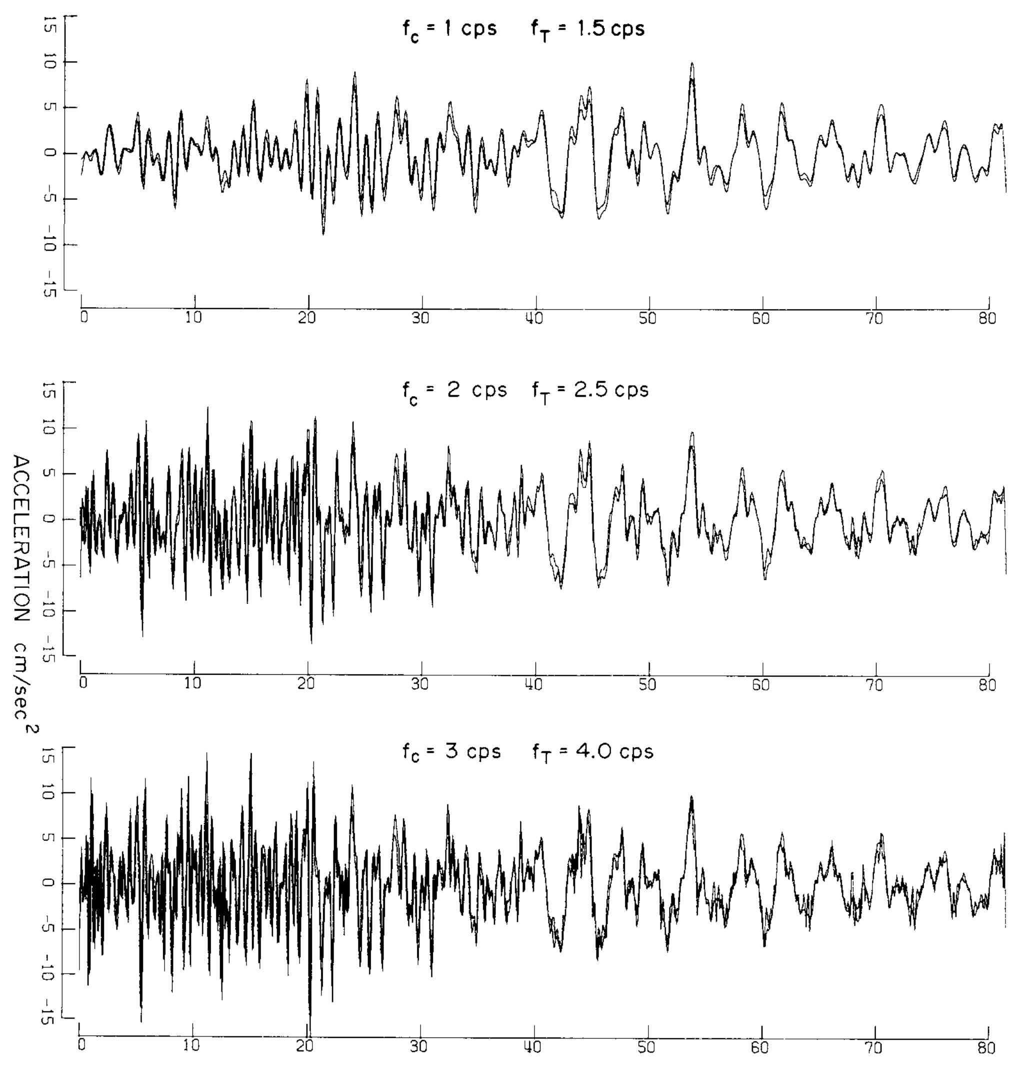

FIG. 7. Low-pass filtered ground acceleration in the San Fernando earthquake of February 9, 1971, at the Engineering Building, Santa Ana, Orange County, California (Comp. S04E) computed: - - - from displacement meter record and ——— from accelerograph record.

operator the standard deviation of the nearly Gaussian distributed digitization noise is about 1/300 cm (using the data-reducer system at the Earthquake Engineering Research Laboratory of the California Institute of Technology). From the *bottom trace* in Figure 7 we estimate that the apparent standard deviation of the digitization noise is about 1 cm/sec^2. In the coordinate system of Figure 1 this would lead to the digitization noise level at about $S = 1/3$ cm/g, and for the low-pass cut-off frequency of 3 Hz, to the signal-to-noise ratio of about 10:1 at the same frequency, which is in agreement with the *bottom trace* in Figure 7.

Conclusions

The main results of the above outlined study can be summarized as follows:

1. The high-pass filtered ground displacements ($f_c = 0.07$ Hz, $f_T = 0.05$ Hz) computed by routine processing methods (Trifunac and Lee, 1973) from typical (Trifunac *et al.*, 1973c; Hanks, 1974) strong-motion accelerograms are accurate to within a few centimeters. This has been demonstrated in this paper by comparing the ground displacements calculated from acceleration and displacement meter records.

2. At a station approximately 80 km distant from the earthquake source, the ground displacement computed from the recorded accelerogram and containing periods shorter than about 16 sec is essentially the same as the broader band displacement computed from the displacement meter record and containing periods shorter than 50 sec. This result suggests that at comparable distances and for similar magnitude earthquakes the ground displacements calculated by routine data processing methods (Trifunac and Lee, 1973; Hudson *et al.*, 1971) are an accurate approximation to actual ground motions.

3. A typical displacement meter record can be corrected for instrument response by using the single-degree-of-freedom model of its transducer (Trifunac, 1972). This permits the derivation of the absolute ground acceleration for frequencies as high as several Hertz.

4. The above analysis points to the fact that there is a wealth of information on ground motion obtained on the recordings of the standard USCGS instruments. To date, these basic data have hardly been examined, but in the light of modern processing techniques this is worth carrying out.

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