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Influence of Canyon on Soil-Structure Interaction

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Introduction

In the search for different wave propagation phenomena, which might prove effective in reducing the adverse effects associated with seismic excitation of structures, several researchers have investigated the possibility of placing structures in the shadow zones that are created by some canyon-like surface topographic features. In 1962, Barkan (3) reviewed this problem and concluded that such topographic features seem to be useless from the practical point of view. In 1968, Woods (14) examined this problem from an experimental point of view, and in 1971 Brown (4), who studied several theoretical results, which had been derived for surface Love (2,5) waves and Rayleigh (8,9) waves, concluded that when the shear wave velocity of the ground material is low, the shielding effect created by a trench may be important. A numerical procedure applicable to two-dimensional antiplane vibrations in a layer placed over a rigid half space and in the presence of a trench has been presented by Lysmer and Waas (7). They found that the presence of a trench can decrease as well as increase the amplitudes of foundation motion and that this amplitude dependence is highly frequency dependent.

Several exact solutions for simple (11,12) and irregular (13) trench cross sections are now available in the literature for excitation consisting of plane SH-waves, but no exact solution has yet been presented for the trench-soil-foundation structure interaction. The purpose of this paper is therefore to present such a solution for a model which is capable of illustrating some characteristics of wave scattering and diffraction around a semicylindrical trench near which is a structure erected on a rigid semicylindrical foundation. Though the geometry

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of this model is far too simple to be of significance in real applications, it is believed that its investigation should help in clarifying to what extent similar geometries may act as shields against incident plane SH-waves. Finally, it appears that the availability of such an exact solution may prove to be useful for the checking of finite element, finite difference, and other approximate schemes that can be developed to handle irregular geometries.

Model, Input Motion, and Solution of Problem

Consider the model shown in Fig. 1. It consists of a semicircular cylindrical foundation with radius a_2, which is placed to the right of a semicircular canyon with radius a_1. The rigid foundation is assumed to be welded to the underlying half space which is characterized by the shear wave velocity, β, and the rigidity, μ.

Since the wave scattering objects are assumed to be of semicircular cross sections, it is convenient to define two polar coordinates, (r_1, ϕ_1) and (r_2, ϕ_2), with their origins located at the centers of the canyon and the foundation,

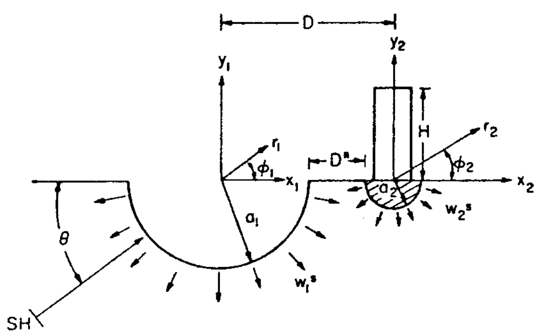

FIG. 1.—Canyon, Shear Wall, Foundation, and Soil

respectively. The two origins are separated by a distance, D, which must be greater than $a_1 + a_2$.

The boundary conditions for this problem are:

$$\sigma_{yz}|_{y=0} = 0 \qquad (1)$$

$$\sigma_{rz}|_{r=a_1} = 0 \qquad (2)$$

and $$w_1 + w_2 + w^i + w^r|_{r=a_2} = \Delta e^{i\omega t} \qquad (3)$$

in which w_1 and w_2 are the scattered waves from the canyon and the foundation, respectively; w^i = the incident wave; w^r = the reflected wave from the half-space boundary; and Δ = the displacement amplitude of the rigid foundation.

These antiplane displacements, w, satisfy the Helmholtz equation in polar coordinates

$$\frac{\partial^2 w}{\partial r_i^2} + \frac{1}{r_i}\frac{\partial w}{\partial r_i} + \frac{1}{r_i^2}\frac{\partial^2 w}{\partial \phi_i^2} + k^2 w = 0; \quad i = 1, 2 \qquad (4)$$

in which $k = \omega/\beta$ = the wave number in the soil medium.

The incident plane SH-wave w^i is

$$w^i = e^{i\omega t}\left[e^{-i(\omega/\beta)(x_2\cos\theta + y_2\sin\theta)}\right] \qquad (5)$$

Since $x_2 = x_1 + D$, the incident wave expressed in terms of (x_1, y_1) is

$$w^i(x_1, y_1) = e^{i\omega t}\, e^{-i(\omega/\beta)D\cos\theta}\left[e^{-i(\omega/\beta)(x_1\cos\theta + y\sin\theta)}\right] \qquad (6)$$

Thus, there is a phase shift between the two locations which is equal to $e^{-i(\omega/\beta)D\cos\theta}$. The wave reflected from the half-space surface, w^r, has the same form as w^i except that θ is replaced by $-\theta$.

Because of the geometry selected for this model (Fig. 1), it is convenient to express w^i and w^r in polar coordinates. The plane wave expansion is made in terms of Bessel and harmonic functions. At (r_2, ϕ_2), this expansion is

$$w^i + w^r = 2\sum_{m=0}^{\infty} i^m E_m J_m(kr_2)\cos m\phi_2 \cos m\theta \qquad (7)$$

in which $E_o = 1$, $E_m = 2$ for $m \neq 1$. The analogous expression for (r_1, ϕ_1) can be obtained by comparing Eqs. 5 and 6.

The unknown in this problem is Δ, the displacement amplitude of the rigid foundation. It depends on the equilibrium of forces exerted on the foundation by the superstructure, on the incident wave, and the inertia force of the foundation itself. The corresponding unknown inhomogeneous boundary condition, Eq. 3, can be separated into two parts, which can be formulated as follows:

Problem A: $$w_1^A + w_2^A + w^i + w^r|_{r=a_2} = 0 \qquad (8a)$$

and Problem B: $$(w_1^B + w_2^B)|_{r=a_2} = 1e^{i\omega t} \qquad (8b)$$

The boundary condition in Eq. 3 can then be retrieved by adding Eq. 8a and Δ times Eq. 8b and by defining

$$w_i = w_i^A + w_i^B\Delta; \quad i = 1, 2 \qquad (9)$$

The new boundary conditions, Eqs. 8a and 8b, do not involve any unknowns, and problems A and B can now be solved.

Problem A—Contribution from Incident Wave.—The physical interpretation of the boundary condition in Eq. 8a is that the rigid foundation is held fixed while subjected to incident seismic waves w^i and w^r. The resulting force, which is exerted on the foundation by the soil, is called the "driving force." It is the primary source of excitation in this problem.

We assume that the scattered waves have the form

$$\left.\begin{aligned} w_1^A(r_1, \phi_1) &= \sum_{n=0}^{\infty} c_n^A H_n^{(2)}(kr_1)\cos n\phi_1 \\ w_2^A(r_2, \phi_2) &= \sum_{n=0}^{\infty} f_n^A H_n^{(2)}(kr_2)\cos n\phi_2 \end{aligned}\right\} \qquad (10)$$

in which c_n^A and f_n^A are the unknown coefficients. The cosine functions are chosen because they satisfy the boundary condition in Eq. 1 automatically, i.e., $\partial w_1^A/\partial\phi_1|_{\phi_1=0,-\pi} = 0$ and $\partial w_2^A/\partial\phi_2|_{\phi_2=0,-\pi} = 0$. The Hankel function of the second kind, $H_n^{(2)}(kr_1)$, is chosen because it corresponds to the outgoing wave at infinity and it satisfies the scalar wave equation in polar coordinates.

Since the scattered waves, w_1^A and w_2^A, are expressed in their respective coordinates, the series must be transformed to the (r_1, ϕ_1) coordinates to satisfy the boundary condition in Eq. 2 and to the (r_2, ϕ_2) coordinates to satisfy boundary condition in Eq. 8*a*. This can be done by employing the Addition Theorem (1)

$$\left.\begin{aligned} H_n^{(2)}(kr_1)\cos n\phi_1 &= \sum_{m=0}^{\infty}(-1)^m \frac{E_m}{2} K_m^n(kD)\, J_m(kr_2)\cos m\phi_2 \\ H_n^{(2)}(kr_2)\cos n\phi_2 &= (-1)^n \sum_{m=0}^{\infty}\frac{E_m}{2} K_m^n(kD)\, J_m(kr_1)\cos m\phi_1 \end{aligned}\right\} \quad (11)$$

in which $K_m^n(kD) = H_{n+m}^{(2)}(kD) + (-1)^m H_{n-m}^{(2)}(kD)$; $E_o = 1$; $E_m = 2$ for $m \neq 1$; and k = the wave number. Transforming w_2^A to the (r_1, ϕ_1) coordinates and imposing the boundary condition in Eq. 2 gives

$$-2e^{i(\omega/\beta)D\cos\theta}\sum_{m=0}^{\infty} i^m E_m [J_m(kr_1)]|_{r_1=a_1}\cos m\phi_1 \cos m\theta$$

$$= \sum_{m=0}^{\infty} c_m^A [H_m^{(2)}(kr_1)]|_{r_1=a_1}\cos m\phi_1 + \sum_{n=0}^{\infty} f_n^A(-1)^n$$

$$\sum_{m=0}^{\infty}\frac{E_m}{2} K_m^n(kD)[J_m(kr_1)]|_{r_1=a_1}\cos m\phi_1 \quad (12)$$

Similarly, the boundary condition in Eq. 8*a* gives

$$-2\sum_{m=0}^{\infty} i^m E_m [J_m(ka_2)]\cos m\phi_2 \cos m\theta = \sum_{m=0}^{\infty} f_m^A H_m^{(2)}(ka_2)\cos m\phi_2$$

$$+ \sum_{n=0}^{\infty} c_n^A \sum_{m=0}^{\infty}(-1)^m \frac{E_m}{2} K_m^n(kD)\, J_m(ka_2)\cos m\phi_2 \quad (13)$$

Since the cosine functions are orthogonal, Eqs. 12 and 13 can be separated into two equations for each harmonic $\cos m\phi_j$:

$$2c_m^A\left[\frac{ka_1 H_{m-1}^{(2)}(ka_1) - mH_m^{(2)}(ka_1)}{ka_1 J_{m-1}(ka_1) - mJ_m(ka_1)}\right] + E_m\sum_{n=0}^{\infty} f_n^A(-1)^n K_m^n(kD)$$

$$= -4E_m i^m e^{i(\omega/\beta)D\cos\theta}\cos m\theta \quad (14)$$

$$2f_m^A\left[\frac{H_m^{(2)}(ka_2)}{J_m(ka_2)}\right] + (-1)^m\sum_{n=0}^{\infty} E_m c_n^A K_m^n(kD) = -4E_m i^m \cos m\theta;$$

$$m = 0, 1, 2, \ldots \quad (15)$$

Eqs. 14 and 15 constitute an infinite matrix for the unknown coefficients, c_m^A, f_m^A, $m = 0, 1, 2, \ldots$. These equations can be rearranged into a matrix form

$$\mathbf{M}\mathbf{q}^A = \mathbf{p}^A \quad (16)$$

in which $\{q^A\}^T = \{c_0^A, f_0^A, c_1^A, f_1^A, \ldots, c_m^A, f_m^A, \ldots\}$ (17)

$$\left.\begin{aligned} p_{2m+1}^A &= -4E_m i^m \cos m\theta\, e^{i(\omega/\beta)D\cos\theta} \\ p_{2m+2}^A &= -4E_m i^m \cos m\theta \end{aligned}\right\} \quad (18)$$

and $M_{2m+1,2n+1} = 2\left[\dfrac{ka_1 H_{m-1}^{(2)}(ka_1) - mH_m^{(2)}(ka_1)}{ka_1 J_{m-1}(ka_1) - mJ_m(ka_1)}\right]\delta_{mn}$;

$$M_{2m+2,2n+2} = 2\left[\frac{H_m^{(2)}(ka_2)}{J_m(ka_2)}\right]\delta_{mn};\quad M_{2m+1,2n+2} = (-1)^n E_m K_m^n(kD);$$

$$M_{2m+2,2n+1} = (-1)^m E_m K_m^n(kD);\quad m, n = 0, 1, 2, \ldots \quad (19)$$

The infinite matrix, Eq. 16, can be solved approximately by considering a finite system which closely approximates it. The number of coefficients required to give accurate results depends on the ratio, a_1/a_2. If $a_1 >> a_2$, the number of harmonics required to expand the displacement distribution around a_1 is large because the scattered wave from the foundation has become almost a point source. For small values of k and a_1/a_2, the coefficients, c_m^A and f_m^A, decrease rapidly; only a small matrix will give results that are accurate to within a few percent at the lower frequencies.

When the coefficients, c_m^A and f_m^A, are determined, the "driving force," F_s^*, induced by the incident SH waves can be expressed as

$$\left.\begin{aligned} F_s^* &= +\mu\int_{-\pi}^{0}\left.\frac{\partial(w^i + w^r + w_1^A + w_2^A)}{\partial r_2}\right|_{r_2=a_2} a_2\, d\phi_2 \\ F_s^* &= -\mu\pi ka_2 J_1(ka_2)\left\{f_o^A\frac{H_1^{(2)}(ka_2)}{J_1(ka_2)} + \sum_{n=0}^{\infty} c_n^A H_n^{(2)}(kD) + 2\right\} \end{aligned}\right\} \quad (20)$$

in which the infinite sum of c_n^A represents the contribution from the canyon. Therefore, as $D \to \infty$, the effect of the canyon becomes negligible because the amplitude of $H_m^{(2)}(kD)$ decays as $D^{-1/2}$ for large D.

Problem B—Impedance of Foundation-Soil System.—The physical interpretation of the boundary condition in Eq. 8*b* is that the rigid foundation is forced to move harmonically with maximum displacement amplitude equal to one. The force resisting this motion is the foundation impedance. Assuming that

$$\left.\begin{aligned} w_1^B(r_1, \phi_1) &= \sum_{n=0}^{\infty} c_n^B H_n^{(2)}(kr_1)\cos n\phi_1 \\ \text{and}\quad w_2^B(r_2, \phi_2) &= \sum_{n=0}^{\infty} f_n^B H_n^{(2)}(kr_2)\cos n\phi_2 \end{aligned}\right\} \quad (21)$$

and substituting Eqs. 21 into the boundary condition, Eq. 8*b*, we have

$$2c_m^B\left[\frac{ka_1 H_{m-1}^{(2)}(ka_1) - mH_m^{(2)}(ka_1)}{ka_1 J_{m-1}(ka_1) - mJ_m(ka_1)}\right] + E_m\sum_{n=0}^{\infty} f_n^B(-1)^n K_m^n(kD) = 0 \quad (22)$$

$$2f_m^B\left[\frac{H_m^{(2)}(ka_2)}{J_m(ka_2)}\right] + (-1)^m\sum_{n=0}^{\infty} E_m c_n^B K_m^n(kD) = \frac{2\delta_{mo}}{J_m(ka_2)} \quad (23)$$

In matrix form, Eqs. 22 and 23 become $\mathbf{M}\mathbf{q}^B = \mathbf{p}^B$, in which $q^B_{2m+1} = c^B_m$; $q^B_{2m+2} = f^B_m$; $p^B_{2m+1} = 0$; and $p^B_{2m+2} = 2\delta_{m0}/J_m(ka_2)$; $m, n = 0, 1, 2, \ldots$ Matrix **M** is identical to that of Eqs. 19. The impedance, K_s, is then

$$\left.\begin{aligned} K_s &= +\mu \int_{-\pi}^{0} \frac{\partial}{\partial r_2}(w_1^B + w_2^B)\Big|_{r_2=a_2} a_2\, d\phi_2 \\ K_s &= -\mu\pi ka_2 J_1(ka_2)\left[f_0^B \frac{H_1^{(2)}(ka_2)}{J_1(ka_2)} + \sum_{n=0}^{\infty} c_n^B H_n^{(2)}(kD)\right] \end{aligned}\right\} \quad (24)$$

The infinite sum represents the contribution from the existence of the canyon. As in Eqs. 20, when $D \to \infty$, this contribution becomes negligible and the impedance reduces to the result for a rigid foundation embedded in an infinite half space (6).

Interaction Equation for Δ

For simplicity, we assume that the building is represented by a linear elastic shear-beam whose modulus of rigidity is μ_b and shear wave velocity is β_b. For this system, when excited by the harmonic base motion, $\Delta e^{i\omega t}$, the base shear, i.e., the force the building exerts on the foundation per unit length, is (6)

$$F_b = +\omega^2 M_b \left[\frac{\tan(k_b h)}{k_b h}\right]\Delta \quad (25)$$

in which $k_b = \omega/\beta_b$; $M_b = (\mu_b/\beta_b^2)2a_2 h$; and h = the height of the building (shown as H in Fig. 1). Introducing $M_s \equiv (\mu/\beta^2)(\pi a_2^2/2)$, which is the mass of the soil replaced by the circular foundation, and writing the dynamic equation of equilibrium for the foundation mass, M_0

$$-(-\omega^2 M_0 \Delta) + K_s \Delta = F_s^* - F_b \quad (26)$$

there follows

$$\Delta = \frac{-J_1(ka_2)\left[f_0^A \dfrac{H_1^{(2)}(ka_2)}{J_1(ka_2)} + \displaystyle\sum_{n=0}^{\infty} c_n^A H_n^{(2)}(kD) + 2\right]}{\dfrac{ka_2}{2}\left(\dfrac{M_0}{M_s} + \dfrac{M_b}{M_s}\dfrac{\tan k_b h}{k_b h}\right) - J_1(ka_2)\left[f_0^B \dfrac{H_1^{(2)}(ka_2)}{J_1(ka_2)} + \displaystyle\sum_{n=0}^{\infty} c_n^B H_n^{(2)}(kD)\right]} \quad (27)$$

Dependence of Δ on Different Model Parameters

To investigate the characteristics of response of a rigid foundation and shear wall structure system, the amplitude of the foundation motion $|\Delta|$ has been plotted versus the dimensionless frequencies, $\omega a_1/\beta$ and $\omega a_2/\beta$ (Figs. 2, 3, 4, and 5). Without any loss of generality, the foundation radius, a_2, is set equal to 1, while a_1 is varied to show the effects the canyon size may have on the response amplitudes. Since the presence of the canyon causes $|\Delta|$ to be θ-dependent, the five cases in which $\theta = 0°, 45°, 90°, 135°$, and $180°$ have been studied.

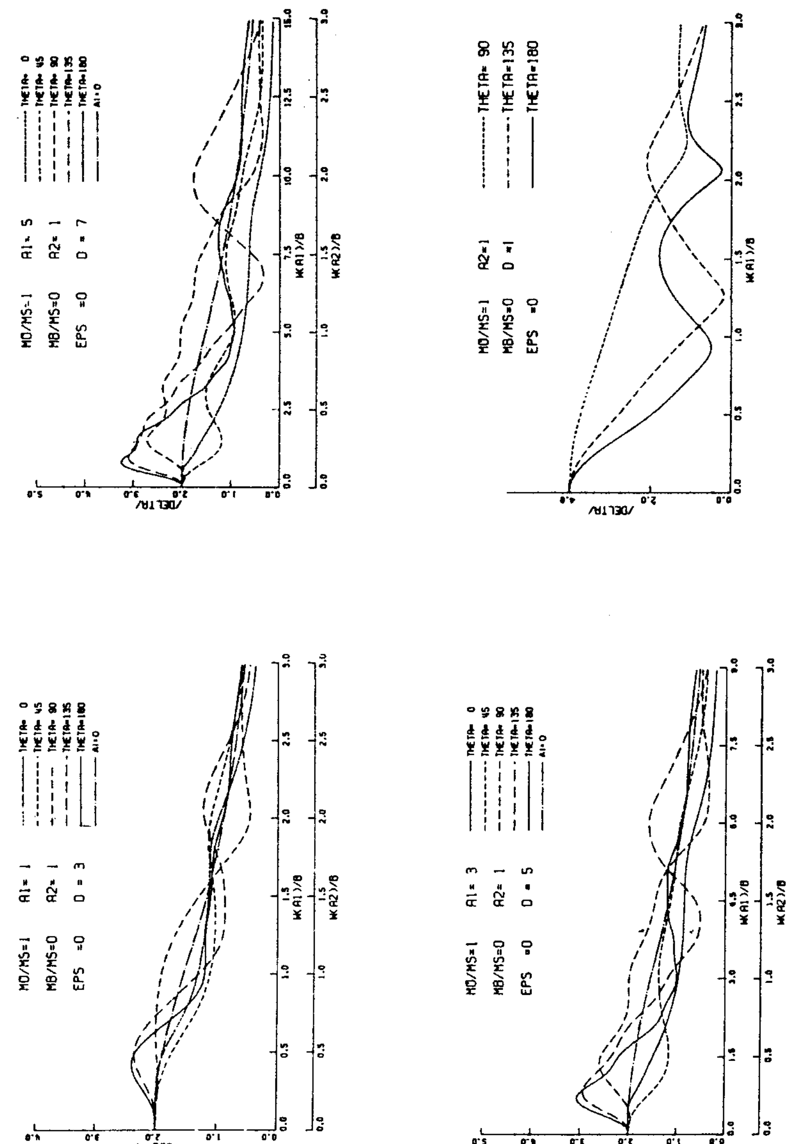

FIG. 2. Foundation Motion $|\Delta|$ as Function of Dimensionless Frequency; $M_0/M_s = 1$, $M_b/M_s = 0$, $\epsilon = 0$, $D^* = 1$: (a) $a_1/a_2 = 1$; (b) $a_1/a_2 = 3$; (c) $a_1/a_2 = 5$; (d) $a_1/a_2 = \infty$, Quarter-Space Solution

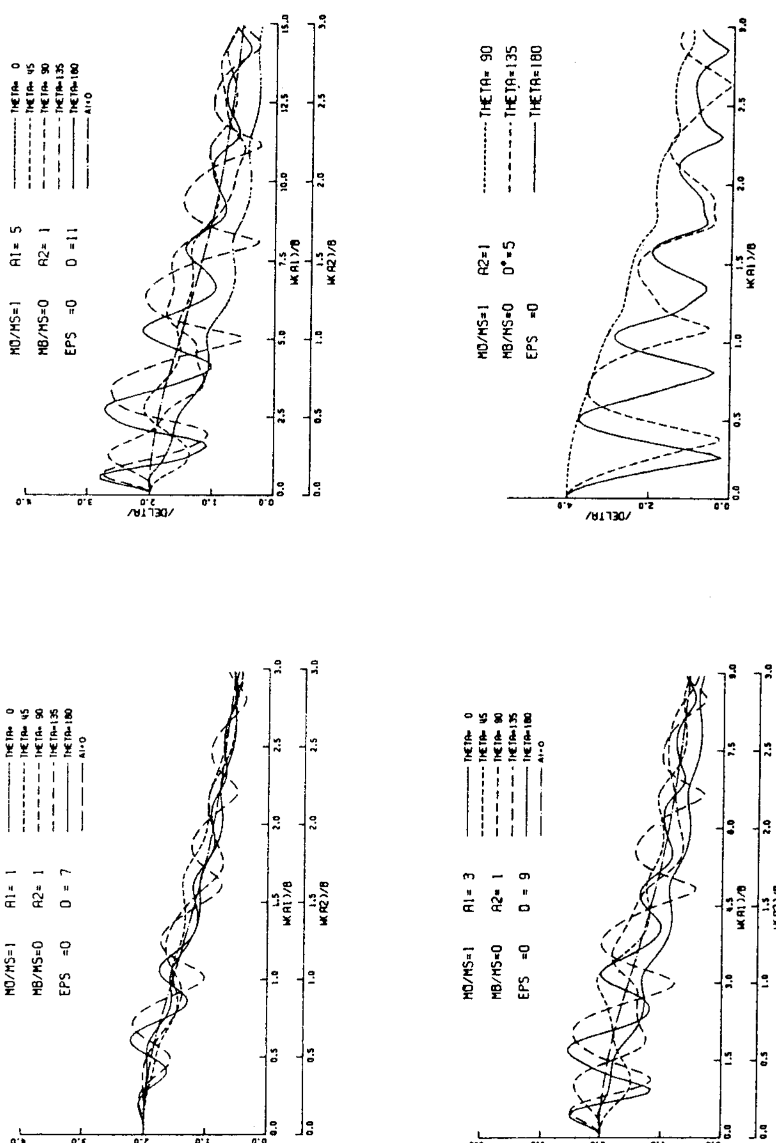

FIG. 3.—Foundation Motion $|\Delta|$ as Function of Dimensionless Frequency; $M_o/M_s = 1$, $M_b/M_s = 0$, $\epsilon = 0$, $D^* = 5$: (a) $a_1/a_2 = 1$, (b) $a_1/a_2 = 3$; (c) $a_1/a_2 = 5$; (d) $a_1/a_2 = \infty$, Quarter-Space Solution

Figs. 2 and 3 show the response of a rigid foundation without a superstructure for the foundation density equal to that of the soil medium, i.e., $M_0/M_s = 1$. This arrangement was made so that the effect of the canyon can be studied without the interaction with the superstructure. In each of these figures, the cases in which $a_1 = 1$, 3, 5, and ∞ are shown in parts (a), (b), (c), and (d), respectively. Also plotted on these graphs, for comparison, is the case in which $a_1 = 0$, i.e., the half-space solution without a canyon (10). This response curve is represented by the dast-dot line. Since different sizes of canyons are dealt with, it is convenient to select the distance so that $D^* = D - (a_1 + a_2)$ is a constant for all (a), (b), (c), and (d) parts of each figure: D^* is the distance between the two nearest edges of the foundation and the canyon. In Fig. 2, $D^* = 1$; while in Fig. 3, $D^* = 5$.

While examining Figs. 2 and 3, one of the most important questions one may ask is whether there are any significant shielding effects for foundation motion that may result from the existence of the canyon. To answer this question, consider first the horizontally incident waves for $\theta = 0°$. The foundation response $|\Delta|$ is then as represented by a fine dashed line. As shown in these figures, this response is very similar to that of the half-space solution when $a_1 = 1$. In fact, the results are nearly identical up to $\omega a_2/\beta = 0.5$. Physically, this means that the low frequency waves which have long wavelengths are not greatly altered by small scattering objects. To scatter away some of these longer waves, the size of the canyon must be increased, as is clearly shown by parts (b) and (c) in the figures. The shielding by the canyon with $a_1 = 5$ is effective for $\omega a_2/\beta$ down to 0.25; for $\omega a_2/\beta > 0.75$, the amplitude is reduced nearly 50%. Therefore, a canyon in front of a foundation does scatter a certain fraction of the horizontally incident wave energy, which corresponds to the wavelengths less than the width of the canyon. However, some consequences caused by nonhorizontally incident waves must also be considered carefully.

For incident waves with $\theta \neq 0°$, the shadow zone behind the canyon approximately extends up to its projection onto the half-space surface. Therefore, the shielding diminishes if the structure is moved further away from the canyon. For example, consider the case when $\theta = 45°$ and $D^* = 1$ in Fig. 2; the foundation is shielded for $a_1 = 3$ and 5, but the foundation is already out of the shadow area if $a_1 = 1$. If the distance, D^*, is increased to 5, the foundation is no longer protected for incident angles greater than 45°. In fact, the response amplitude may exceed that of the half-space solution for certain frequencies because part of the energy is trapped in between the two scatterers and the wave amplitudes may interfere constructively.

The nature of response rapidly changes as the incident angle increases. For $\theta > 90°$, the canyon plays a role of a wave source as part of the incident waves are reflected and focused back towards the foundations. As shown by Figs. 2 and 3, the response amplitude can increase by more than 50% over the half-space solution. Of course, this amplification is less pronounced if the canyon is further away, because in that case the amplitude of the reflected wave goes down as $1/\sqrt{D}$. Using the same reasoning as before, the longer waves are not reflected by the smaller canyon, therefore, the amplitude equal to 2 for low frequencies. With the presence of a canyon, the longer waves are also partially reflected, while all waves are reflected for the limiting case, $a_1 = \infty$ [Figs. 2(d) and 3(d)]. For $a_1 = \infty$, the half space has become the

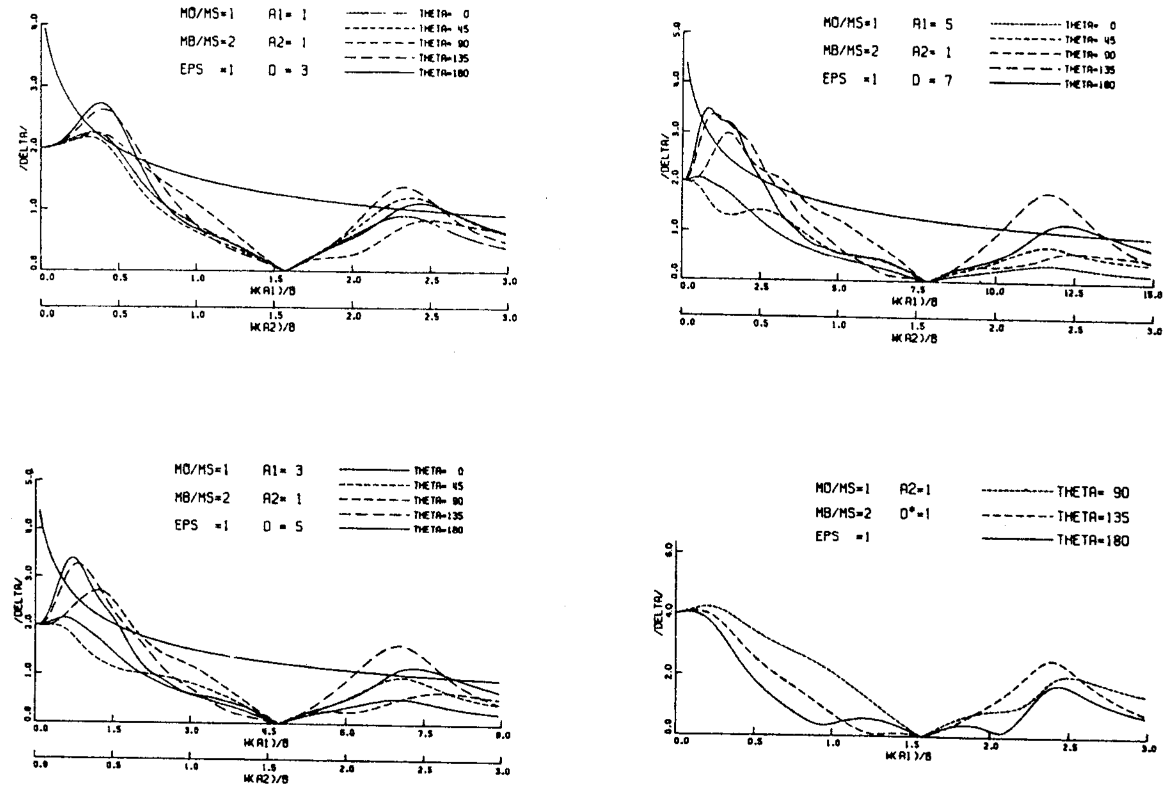

FIG. 4.—Foundation Motion $|\Delta|$ as Function of Dimensionless Frequency; $M_o/M_s = 1$, $M_b/M_s = 2$, $\epsilon = 1$, $D^* = 1$: (*a*) $a_1/a_2 = 1$; (*b*) $a_1/a_2 = 3$; (*c*) $a_1/a_2 = 5$; (*d*) $a_1/a_2 = \infty$, Quarter-Space Solution

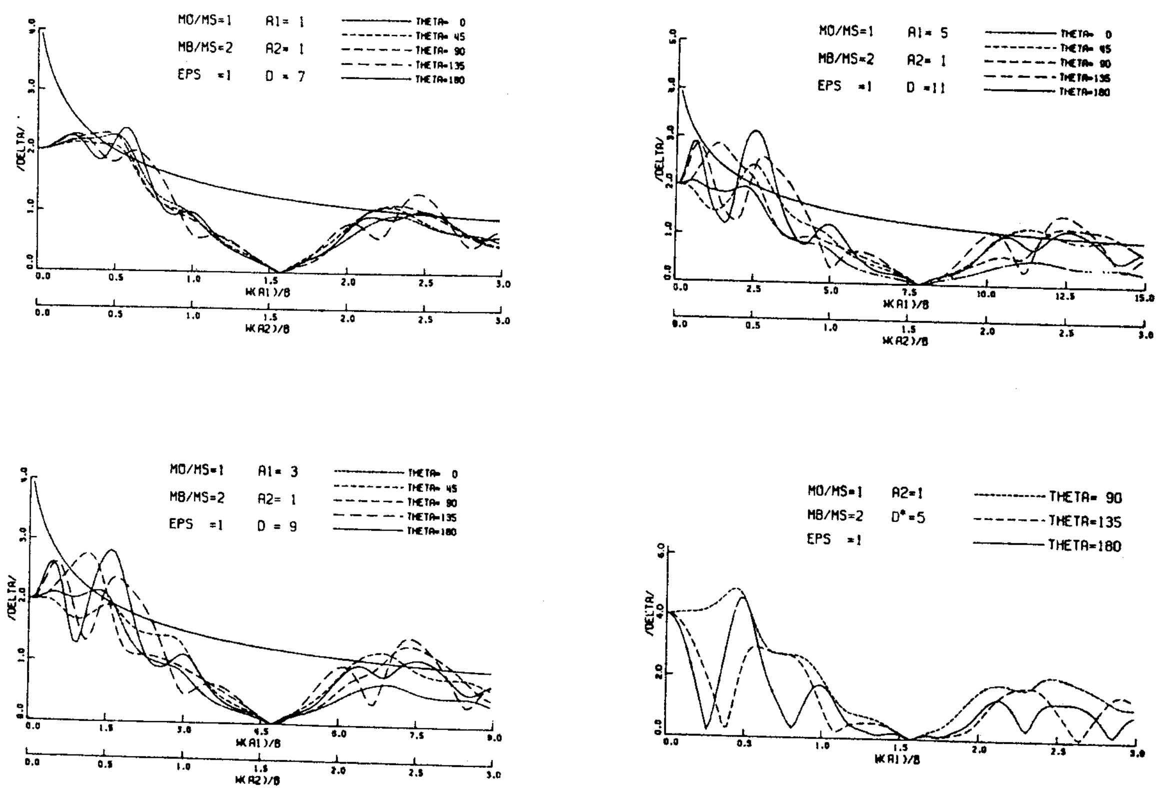

FIG. 5.—Foundation Motion $|\Delta|$ as Function of Dimensionless Frequency; $M_o/M_s = 1$, $M_b/M_s = 2$, $\epsilon = 1$, $D^* = 5$: (*a*) $a_1/a_2 = 1$; (*b*) $a_1/a_2 = 3$; (*c*) $a_1/a_2 = 5$; (*d*) $a_1/a_2 = \infty$, Quarter-Space Solution

quarter space and the free surface amplitude becomes 4 for a unit input excitation.

The massiveness of the building and foundation system (M_b/M_s large) may lead to more than average contribution from inertial forces in Eq. 27. For low-frequency, the $|\Delta|$ versus $\omega a_2/\beta$ curve may then resemble the steady-state response of a single degree-of-freedom oscillator with viscous damping (10). When the canyon is located in the immediate vicinity of the rigid foundation, the shapes of $|\Delta|$ curves become similar to those that correspond to the half-space solution (10) for $a_1 = 0$.

As the distance between the foundation and the canyon increases, there is a better chance for the waves scattered from the canyon and the foundation to interfere. This introduces numerous local peaks and troughs that are superimposed on the overall trends of $|\Delta|$ curves.

Figs. 4 and 5 show examples of interaction when the shear wall is flexible ($\epsilon \neq 0$). The zeros of $|\Delta|$ at $\omega a_2/\beta = [(2n+1)/2](\pi/\epsilon)$ for $n = 0, 1, 2, \ldots$ correspond to the natural frequencies of the shear wall (10). At these frequencies the base shear force per unit length is canceled by the input driving forces and the foundation does not move. From Eq. 27, it is seen that the frequency spacing and the dimensionless frequencies, $\omega a_2/\beta$, in which $|\Delta| = 0$, are not affected by the existence of the canyon. However, since the existence of the canyon does influence the shape of $|\Delta|$ curves, the relative response of the shear wall (10) is affected by the existence of the canyon. Figs. 4 and 5 show the effect of separation distance D and indicate that as D increases the number of local peaks and troughs superimposed on the overall trends of $|\Delta|$ curves increases and becomes more complicated.

When the angle of incident SH-waves is greater than 90°, a significant amount of wave energy may be reflected from the canyon and can excite the foundation to vibrate with amplitudes that may be even greater than those for a uniform half space. To show how large these amplitudes may be for the model parameters studied in Figs. 4 and 5, the envelope of $|\Delta|$ curves for the soil-structure interaction in the uniform half space (10) has also been plotted in these figures by a heavy full line and careful comparison of this envelope with the curves computed from Eq. 27 shows that for small values of $\omega a_2/\beta$, amplitudes of $|\Delta|$ for all angles of incidence may be larger in the presence than in the absence of a canyon. This occurs for small values of D and apparently results from the reduced effective spring constant of the half space when the canyon is present. Removal of a portion of the half space by the canyon reduces its impedance and increases the driving forces for low frequencies. This because the existence of a canyon creates scattered waves that add to the driving forces. These same scattered waves bring back a fraction of energy radiated from the rigid foundation and thus decrease the amount of radiative damping relative to that for the uniform half-space solution. Consequently, for low values of $\omega a_2/\beta$ and for small D, amplitudes of $|\Delta|$ computed from Eq. 27 can be larger than those in the absence of a canyon even for $\theta = 0°$. When $\omega a_2/\beta$ and D increase, this effect is reduced and the amplitudes of $|\Delta|$ from Eq. 27 appear to be closer to the half-space solution. Also, for larger $\omega a_2/\beta$ the shielding (for $\theta = 0°$) and the amplifying effects ($\theta = 180°$) created by the canyon seem to be more pronounced for intermediate values of D, while for large values of D diffraction (for $\theta = 0°$) and reflection (for $\theta = 180°$) from the canyon make the values of $|\Delta|$ from Eq. 27 approach the solution in the uniform half space (10).

Conclusions

The foregoing analysis suggests that for the two-dimensional wave propagation problem consisting of a canyon and a shear wall erected on a rigid semicircular foundation which are excited to vibrate by incident plane SH-waves, the shielding effects of a canyon on the response of a shear wall are not significant. For certain directions of wave incidence, for large canyon dimensions, high frequencies, and for small velocity of shear waves in the half space, the shielding effect of a canyon may indeed be quite significant. However, for realistic values of these parameters and because the waves do not necessarily always arrive from one direction, the presence of a canyon near a foundation of a large and long building may also amplify the amplitudes of shear wall response relative to the motions that would occur in the absence of a canyon. Although the results based on this study of a simple mathematical model cannot be generalized to apply to other, or perhaps even similar geometries, it appears that our results, as well as the results of some previous investigators (3,7,14), suggest that the shielding of structures by canyons or trenches may be useless for many practical applications.

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

a_1 = radius of canyon;
a_2 = radius of foundation;
c_n^A, c_n^B = expansion coefficients for scattered waves from canyon;
D = distance between origins of two coördinate systems (x_1, y_1) and (x_2, y_2);
D^* = distance between two nearest edges of canyon and foundation;
F_b = external force: force exerted on foundation by shear building;
F_s^* = driving force: force induced by incident wave onto foundation;
f_n^A, f_n^B = expansion coefficients for scattered waves from foundation;
$H_n^{(2)}(kr)$ = Hankel function of second kind, nth order and with argument kr;
h = height of shear wall;
$J_m(kr)$ = Bessel function of mth order with argument kr;
$K_m^n(kD)$ = linear combination of Hankel functions = $H_{n+m}^{(2)}(kD) + (-1)^m H_{n-m}^{(2)}(kD)$;
K_s = impedance function of foundation-soil-canyon system;
k = ω/β = wave number of soil medium;
M_0, M_b = masses of foundation and shear building, respectively;
r_j = radial coordinate for the jth polar coordinate system;
w^i = displacement field of incident plan SH-wave;
w^r = reflection of plane wave from half-space surface;
w_1 = scattered waves from canyon;
w_2 = scattered waves from foundation;
w_1^A, w_1^B = scattered waves from canyon for problems A and B, respectively;
w_2^A, w_2^B = scattered waves from foundation for problem A and B, respectively;
β, β_b = shear wave velocity in soil and in building, respectively;
$\Delta e^{i\omega t}$ = motion of foundation induced by combined loading;
E_m = $\mathrm{E}_m = 1$, for $m = 1$ and $\mathrm{E}_m = 2$, otherwise;
ϵ = $k_b h / k a_2$ = measure of relative stiffness and height of shear wall;
θ = incident angle of plane SH-wave;
μ, μ_b = shear moduli of soil and shear building, respectively;
σ_{rz} = shear stresses in z direction;
ϕ_j = angular coordinate for jth polar coordinate system; and
ω = angular frequency.