

**FIFTH WORLD CONFERENCE ON EARTHQUAKE ENGINEERING**  
**ROME 1973**

**RECENT DEVELOPMENTS IN DATA PROCESSING AND ACCURACY  
EVALUATIONS OF STRONG MOTION ACCELERATION MEASUREMENTS**

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# RECENT DEVELOPMENTS IN DATA PROCESSING AND ACCURACY EVALUATIONS OF STRONG MOTION ACCELERATION MEASUREMENTS

by

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## SYNOPSIS

Strong-motion accelerograph systems have been examined in detail from the standpoint of the recovery of the maximum amount of information from the record. This has involved a detailed evaluation of errors, including such factors as transducer properties, high and low frequency digitization errors, various distortions in photographic processing, and accuracy considerations of computing procedures. It is shown that existing standard accelerograph systems consisting of optical-recording analog accelerographs, photographic processing, semi-automatic hand digitization of analog records, and computer digital filtering and correction techniques form a reasonably compatible system from the standpoint of accuracy limits of the various components. Standard correction techniques are described, and overall frequency limits and accuracy estimates are presented.

## INTRODUCTION

Basic information on the detailed time history of close-in destructive earthquake ground motions comes from strong motion accelerographs that record three mutually perpendicular components of ground acceleration<sup>(1)</sup>. Instruments of this kind were first deployed in early 1933, and the Long Beach, California, earthquake of 1933 provided the first strong motion accelerograms. Since that time, numerous invaluable strong motion accelerograms have been recorded, mainly in the western United States. With the data most recently acquired during the San Fernando, California, earthquake of February 9, 1971, there are presently over 500 recorded accelerograms of excellent quality.

In the late 1960's, an extensive program of strong motion data processing was initiated at the Earthquake Engineering Research Laboratory of the California Institute of Technology. The principal objective for initiating this program was to provide all investigators with the basic data in a form as accurate as possible. The first volume, IA, which appeared in 1969<sup>(2)</sup> contained "uncorrected" accelerograms, and it was soon followed by the first parts of the volumes IIA, IIIA and IVA which

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presented corrected accelerograms and integrated velocity and displacement curves<sup>(3)</sup>, response spectra<sup>(4)</sup> and Fourier amplitude spectra<sup>(5)</sup>. Numerous subsequent parts (B, C, D, etc.) of all volumes are now being issued.

Perhaps the most important recent development in the experimental strong-motion seismology has been the improvement of the methods of data processing of strong-motion accelerograms emerging from the work on the above mentioned project. By using the modern techniques of trace digitization and the digital computer processing, it has become possible to significantly increase the accuracy and the amount of information that can be extracted from the recorded ground motions.

In this paper we summarize our present knowledge of the errors and corrective procedures which are associated with accelerogram data processing and attempt to outline briefly the state of the art in computation of velocity and displacement curves. This summary is based mainly on instruments recording on paper or film and on the semi-automatic digitization process. Therefore, many aspects of the outlined accuracy evaluations and error corrections may change with the development of new technology for recording and data processing. Nevertheless, the experience summarized here will remain essential for future processing of paper and film records.

Modern strong-motion instruments with optical recording are capable of measuring ground accelerations up to about  $1g$  with a resolution of the order of  $0.001g$ . This corresponds to a dynamic range of about 60 db. Although, theoretically, the usable frequency range for these instruments is from D.C. to about 25 cps, the low amplitudes of measured acceleration at long periods can be used only for frequencies higher than about 0.06 cps since the digitization noise becomes significant at longer periods. Since the recorded amplitude of long-period waves is a function of earthquake magnitude and the hypocentral distance, it is clear that this cut-off frequency may vary from one record to the other. However, for the routine accelerogram processing it is impractical to change its value from one accelerogram to another and the optimum cut-off frequency must be selected. This frequency is about 0.06 cps for a typical accelerogram and the overall processing accuracy at the Earthquake Engineering Research Laboratory. Other equipment and different processing procedures will, of course, lead to other cut-off frequencies. The high frequency limit for data retrieval which is currently at 25 cps results from the signal modifications imposed by the natural frequency of the transducer and the semi-automatic digitization procedures. With modern image processing methods and increased speed of record motion in the instrument (now between 1 and 2 cm/sec.), this frequency will increase in the future.

Accelerographs recording on analog magnetic tape are commercially available and have 30-35 db nominal dynamic range, but presently they do not offer an improvement over high quality photographic instruments.

## ERRORS IN DIGITIZED ACCELEROGRAMS

The modification of harmonic input amplitudes and phases by the strong-motion accelerographs are caused by the relatively low natural frequency of transducer element (Figure 1), usually between 10 and 30 cps for most of the mechanical-optical strong-motion accelerographs.

The relative motion  $X(t)$  of the transducer mass is described by the differential equation of motion  $\ddot{X}(t) + 2\omega_0 \zeta_0 \dot{X}(t) + \omega_0^2 X(t) = -a(t)$  (Figure 9), where  $\zeta_0$  is the fraction of critical damping,  $\omega_0$  is the natural frequency ( $\omega_0 = 2\pi f_0$ ),  $a(t)$  is the absolute base acceleration, and  $t$  is the time coordinate. For acceleration transducers, the largest possible  $\omega_0$  is chosen so that the term  $\omega_0^2 X(t)$  dominates on the left hand side of the differential equation. For input frequencies  $\omega$  that are several times smaller than  $\omega_0$ ,  $\ddot{X}(t)$  and  $2\omega_0 \zeta_0 \dot{X}(t)$  are small and  $\omega_0^2 X(t)$  is therefore nearly same as  $-a(t)$  (Figures 1 and 9). For higher input frequencies, both amplitudes and phases are significantly modified and the correction terms involving  $\ddot{X}(t)$  and  $2\omega_0 \zeta_0 \dot{X}(t)$  may not be neglected.

To simplify instrument response interpretation, the strong-motion acceleration transducers are usually designed to represent a single-degree-of-freedom viscously damped oscillator (Figure 2). Unfortunately, it is not always possible to design an ideal single-degree-of-freedom system. For example, the transducer mass supported by two leaf springs (Figure 2) is meant to vibrate in its fundamental transverse mode only. However, in addition to the higher modes in the transverse direction, the transducer configuration allows torsional vibrations as well. These torsional vibrations may be excited, for example, by a slight eccentricity of the electromagnetic damping force<sup>(9)</sup>.

There may be various other instrumental errors present in the recorded accelerogram, such as the transverse play of the recording paper, nonuniform velocity of record-driving mechanism, inaccurate timing, misalignment of the three transducers, cross-axis sensitivity of transducers, waveform clipping, etc. However, these will not be discussed here for a more detailed account on these errors may be found in our previous papers<sup>(6)</sup>.

Errors due to the warping of film negatives and translucent Mylar copies are caused by chemical processing and later by aging. Other photographic processing errors may be caused by the lens imperfections, or by the stretching of the film negatives while duplicating long rolls continuously. These errors are almost entirely eliminated by subtracting the digitized fixed trace from the accelerogram and by the proper scaling<sup>(6)</sup>.

The digitizing errors come from two principal sources: (1) an imperfect mechanical traverse mechanism of the digitizer cross hair system, and (2) the random errors generated by the operators. Whereas the errors from the first group can be eliminated in most cases by simultaneous digitization of the acceleration and fixed traces, the random digitization errors remain in the digitized data as "noise".

Through the detailed analysis of the above mentioned errors, it has been possible to show that the minimum resolution of a digitizer should be better than about  $1/300$  cm - the variance of an almost normally distributed human reading error, and, that the discretization process in the digitizer does not seriously add to the human error<sup>(6)</sup>. Perhaps another intuitively clear result indicates that a greater number of digitized data points decreases the amplitude of the digitization noise spectrum but extends the same spectrum to higher frequencies<sup>(6)</sup>. Finally, it has been possible to show that the unequally spaced digitized points in time which are connected with straight line segments define a new continuous function which has all Fourier spectrum amplitudes the same as those of the original function up to the average Nyquist frequency of digitization. In other words, discrete digitized points interconnected with straight lines play the role of a low pass filter with the cut-off frequency equal to the average Nyquist frequency of digitization.

### OVERALL EFFECTS OF ERRORS IN DIGITIZED ACCELEROGRAMS

While the detailed studies of different errors have led to the determination of their properties and helped develop procedures for elimination of many of them at their source<sup>(6)</sup>, the ultimate objective of such an error analysis has been to find their combined overall effect and to determine the frequency band in which the digitized data accurately represent ground acceleration. One can specify this frequency band by defining the cut-off frequencies  $f_L$  and  $f_H$ , where L stands for low and H for high frequencies. Ideally, the frequency  $f_L$  should be greater than all frequencies associated with long-period recording and digitizing errors whose spectral amplitudes and phases cannot be corrected, and the frequency  $f_H$  should be smaller than all frequencies representing high-frequency recording and digitizing errors. These frequencies change from one instrument to another, and depend on instrument recording speed and sensitivity<sup>(6)</sup>. For the standard acceleration correction,  $f_L$  and  $f_H$  are chosen to represent the average for all records.

The density of digitized points varies from one operator to another, and has been illustrated in Fig. 3 by plotting the average number of digitized points per one cycle versus period. The least-square-fitted straight lines show a tendency towards the increase in the number of points for longer periods. Certain distinct features of these curves distinguish this kind of hand-digitized data from equally spaced hand or machine digitization<sup>(6)</sup>. For example, a dashed line is plotted corresponding to the  $\Delta t = 0.1$  sec. This line intersects the level of 2 points per cycle at the period of 0.2 seconds in agreement with the Nyquist frequency criterion (5 cps). On the other hand, the lines calculated for the four typical acceleration digitizations with unequally spaced data never go below the level of 2 points per cycle.

Fig. 4 gives a histogram for the lowest average period present in the early 48 hand digitized paper records<sup>(2)</sup>. From this histogram, we observe that the lowest average period picked over one record is greater than 0.04 seconds and corresponds to the highest frequency being less than 25 cps. Since this histogram has been obtained from the data averaged over one second and from the operator's own

judgment of the number of points to be digitized according to the local frequency content, the highest frequency actually resolved in a short segment of record is substantially above the average. Numerous other records recently hand-digitized contain on the average about 40 or more points per second, and they are well represented by the steepest curve in Fig. 3. For these records, the average Nyquist frequency is well above 20 to 25 cps. The fundamental frequencies of most instruments fall between 10 and 30 cps. Recorded signals with frequencies higher than 30 cps have a low signal-to-noise ratio and are distorted by the higher modes of vibration of the instrument transducers (Figs. 1 and 2). Therefore, it appears that most hand-digitized paper accelerograms contain information on the frequencies up to about 25 cps.

From the analysis of random digitization noise, it has been found that the expected average errors in the ground displacement are relatively small up to periods of about 16 seconds<sup>(6)</sup> and then rapidly build up thereafter for longer periods. Thus, it was decided to filter out these long-period errors starting at about a 16 sec period. Whether this period should be chosen as the limit period beyond which the errors are to be considered as serious is a delicate question that depends on the expected use of the accelerogram, and in particular, on the required accuracy of the computed ground motion. If one could expect long-period ground displacements of several meters, then errors of several centimeters would certainly be acceptable and it would be possible to extend the validity of twice-integrated accelerograms up to, perhaps, 30 or 50 sec periods.

To examine the validity of  $f_L = 1/16$  cps as determined from the analysis of random digitization errors alone<sup>(6)</sup>, four experiments have been performed. A typical accelerograph was mounted on a horizontal table constrained to move along a horizontal line. The table was moved by hand following approximately a single cycle of a sinusoid, with the fundamental period increasing in successive tests from about 10 sec to about 35 sec. The absolute table motion has been monitored by a laboratory displacement meter which records on a Brush pen recorder.

The displacements from the four experiments were calculated from accelerations recorded on the 70 mm film. The negatives were enlarged to prints 24 inches long, and were digitized in the same way as the standard uncorrected accelerograms. A sloping zero acceleration baseline was inserted in each of the four digitized accelerograms by minimizing the root mean square of the acceleration, and the accelerations were double integrated in the usual way to obtain displacement. The measured and calculated displacements were plotted on the same graphs (with ordinates displaced by 5 in.) to allow comparison (Figs. 5, 6, 7 and 8).

The agreement between the recorded and double-integrated accelerograms and the measured displacement curves for periods of about 10 sec (Fig. 5) and 12 sec (Fig. 6) is excellent. The agreement for the period close to 20 sec (Fig. 7) is still very good, but it indicates small long-period drifts of several inches in amplitude which are very similar to the fluctuations caused by the random digitization errors<sup>(6)</sup>.

In experiment No. 4 (Fig. 8) with a predominant period of about 35 sec, the agreement between computed and measured table displacements is poor. These tests indicate that good agreement between computed and recorded displacement may be obtained for waves with periods shorter than 10 to 12 seconds and 10-20 inch amplitudes, with digitization errors becoming noticeable at about a 20 sec period.

### CORRECTED ACCELEROGRAMS

The standard processing of corrected accelerograms first involves instrument correction<sup>(8)</sup> and then baseline correction<sup>(3, 7)</sup>. From uncorrected accelerograms digitized at unequally spaced intervals, equally spaced data with 50 points per second are generated after low-pass filtering the original data. This process eliminates the high-frequency instrumental digitization errors and aliasing. The filtering is performed by using an Ormsby filter with a cut-off frequency  $f_C = 25$  cps and a roll-off termination frequency  $f_T = 27$  cps.

Instrument correction is performed by using a differential equation of motion for the single-degree-of-freedom system (Fig. 9). The instrumental constants  $\omega_0$  and  $\zeta_0$  required for this correction are determined from the calibration tests of each accelerograph component<sup>(2)</sup>.

Accelerograms corrected for instrument response are next baseline corrected by high-pass filtering with an Ormsby filter (Fig. 10). This process filters out periods longer than about 16 sec and eliminates the long-period errors introduced by recording, digitization, and record processing. The final result is a corrected accelerogram that accurately represents the absolute acceleration of instrument base in the frequency band between 0.06 cps and 25 cps.

Recent experiments have shown that small errors, which are associated with the extension of the accelerogram by a zero outside the interval 0 to T, a procedure required for digital filtering, and those resulting from the estimation of  $v_0$  and  $a_0$  (step 2 in Fig. 10), may lead to small but undesirable long-period components of the computed ground displacement. These errors are a consequence of applying the least square fitting method, which, from the point of view of the physical nature of the accelerogram, is only a mathematical fiction. To eliminate these effects, a new refinement has been added to the standard baseline correction procedures<sup>(7)</sup> which consists of extending the accelerogram as an even function outside the interval 0 to T<sup>(3)</sup>.

When a straight line  $v_0 + a_0 t$  is fitted to the velocity  $v(t)$  (step 2 in Fig. 10), small errors  $\Delta v_0$  and  $\Delta a_0$  are introduced in the estimates of  $v_0$  and  $a_0$ . Upon integration  $\Delta v_0$  and  $\Delta a_0$  lead to  $\Delta v_0 t + 1/2 \Delta a_0 t^2 + \text{const.}$ , an error in the computed displacements that possesses periods longer than 16 sec. Therefore, the second refinement added to the standard baseline correction procedures<sup>(7)</sup> consists of high-pass filtering the velocity and displacement curves (steps 8, 10 and 12 in Fig. 10).

For routine processings of strong-motion accelerograms, the above mentioned procedures are believed to adequately restore the

basic data for typical research work. For special applications when greater accuracy is required, further improvements in the above described methods may be called for. In such cases uncorrected accelerograms can be used as a starting point in the analysis.

Figs. 11 to 14 summarize the standard format of data presentation in Vols. II, III and IV<sup>(10)</sup>. Fig. 11 consists of the corrected accelerogram and computed velocity and displacement curves<sup>(3)</sup>. Fig. 12 presents the tripartite logarithmic plot of pseudo relative velocity spectrum; the corresponding true velocity spectrum (SV) and the Fourier amplitude spectrum (FS) are shown in Fig. 13<sup>(4)</sup>. Fig. 14 shows the Fourier amplitude spectrum with the 95% confidence level<sup>(5)</sup>.

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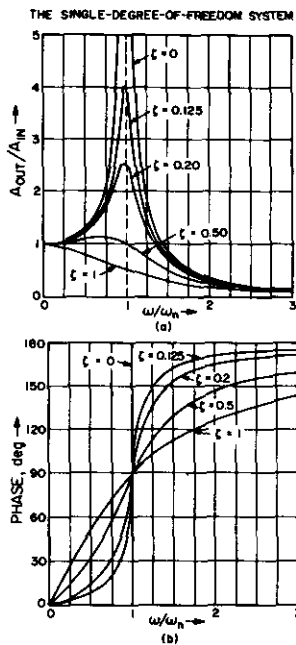


Figure 1

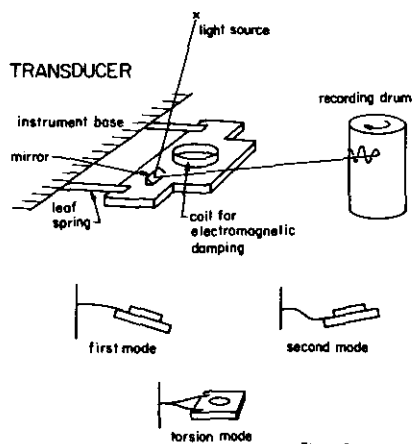


Figure 2

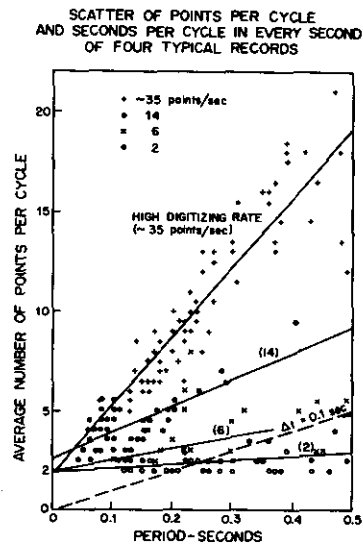


Figure 3

FREQUENCY OCCURRENCE OF THE SHORTEST AVERAGE PERIOD IN 48 HAND DIGITIZED ACCELEROGRAMS

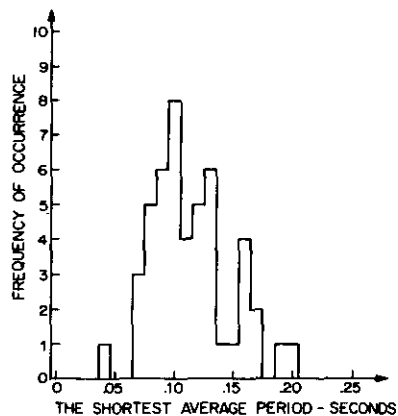


Figure 4

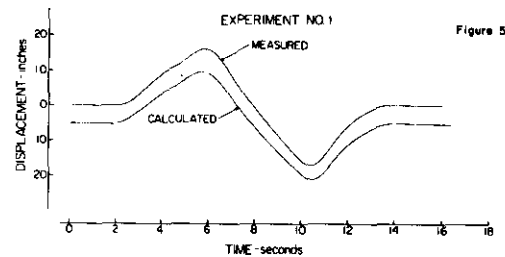


Figure 5

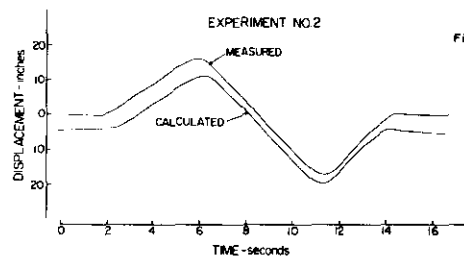


Figure 6

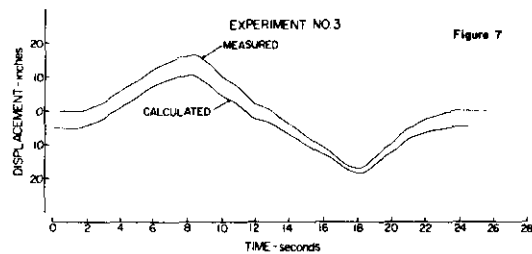


Figure 7

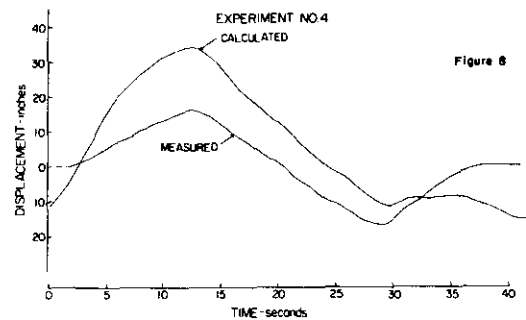


Figure 8

# FLOW CHART FOR ACCELEROGRAM INSTRUMENT CORRECTION

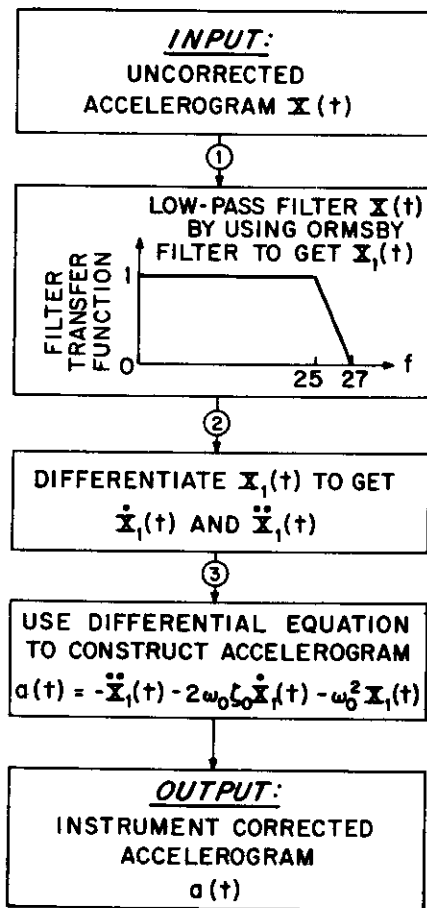


Figure 9

# FLOW CHART FOR BASELINE CORRECTION

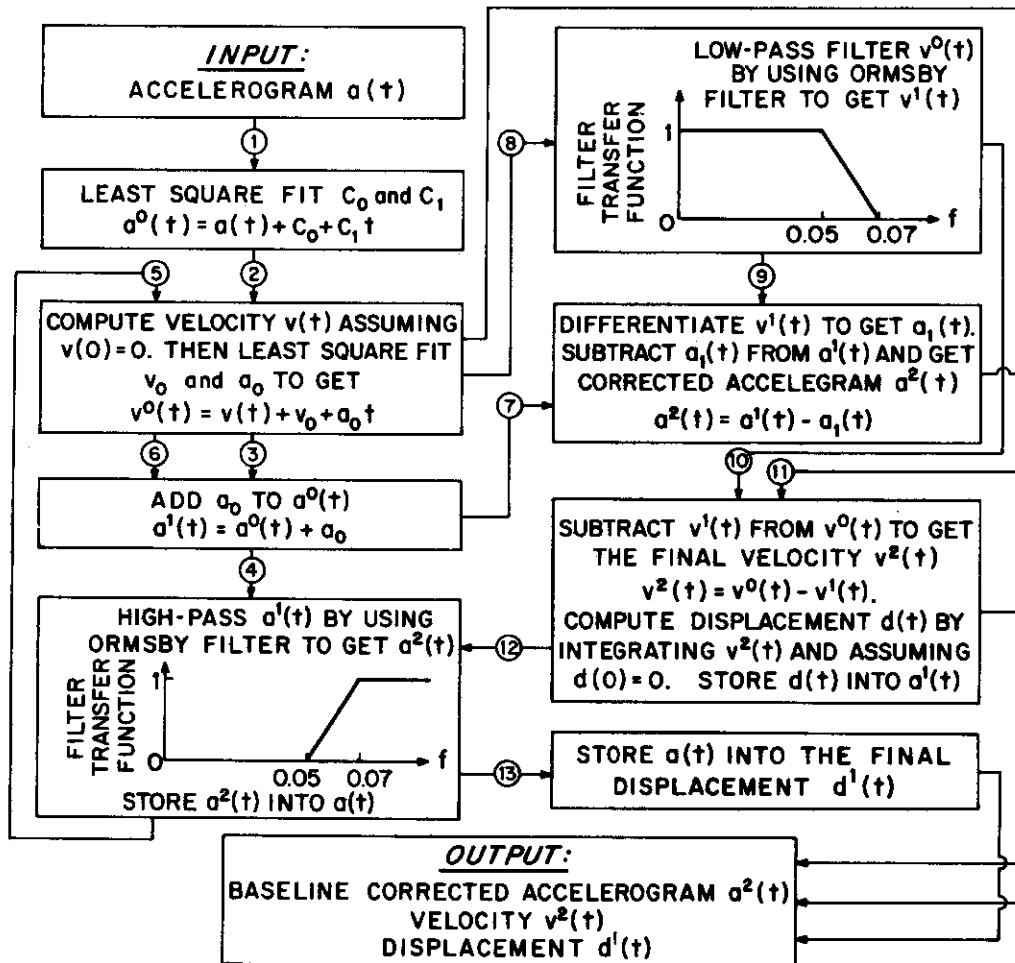


Figure 10

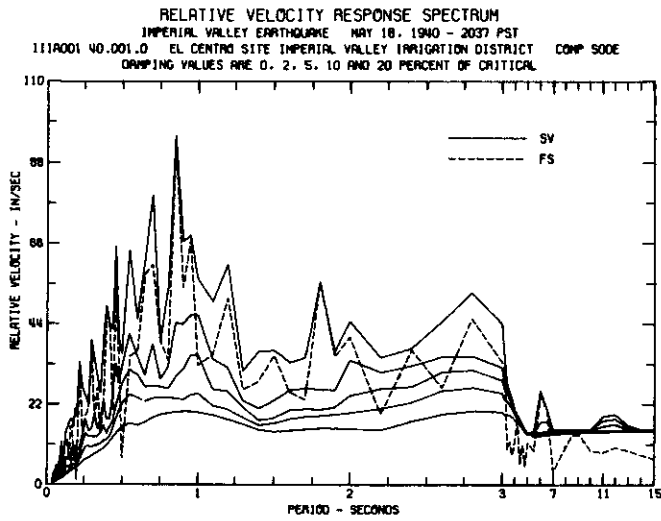


Figure 13

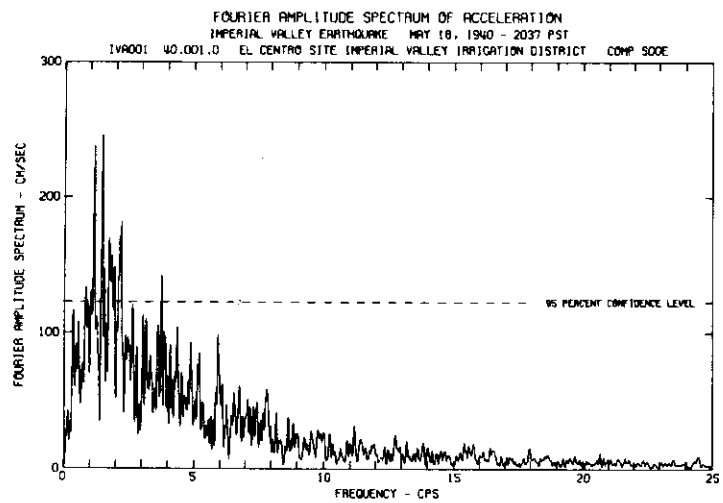


Figure 14

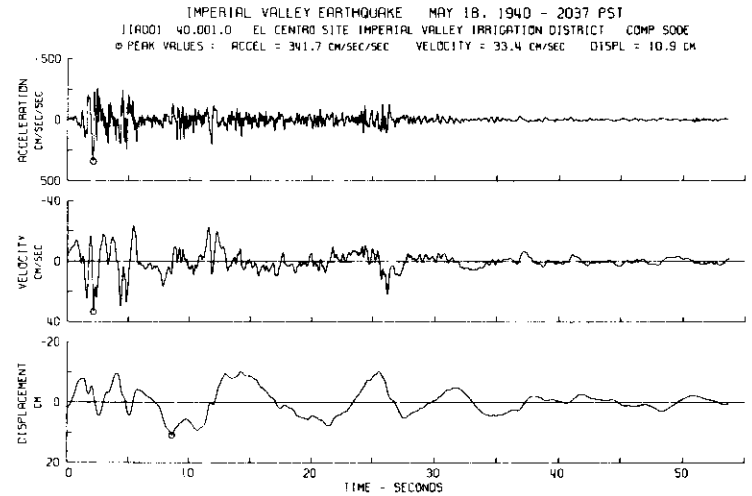


Figure 11

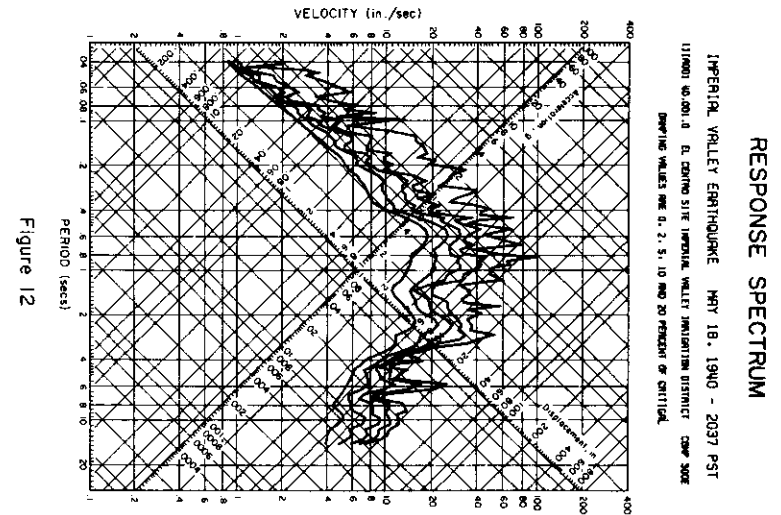


Figure 12