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ON CORRELATION OF SEISMOSCOPE RESPONSE WITH EARTHQUAKE MAGNITUDE AND MODIFIED MERCALLI INTENSITY

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ABSTRACT

A quantitative measure of the Modified Mercalli Intensity Scale for earthquakes in the western United States has been developed by correlating the peak seismoscope relative displacement response, S_d, with the reported site intensity, I_{MM}. This correlation can be approximated by

$$\bar{S}_d(\mathrm{cm}) \approx \frac{1}{49.2} 10^{0.288 \, I_{MM}}$$

for $I_{MM} \leqq$ VIII and is characterized by one standard deviation of about $0.7 \bar{S}_d$. The data used in this study do not indicate an obvious type of dependence of S_d on local site conditions.

A method for computing the analog of the local earthquake magnitude, $M_{\mathrm{seismoscope}}$, has been presented for possible use in strong-motion seismology and for scaling earthquakes by close-in measurements, when other seismological instruments may go off scale.

INTRODUCTION

Early notable attempts to develop simple strong-motion recording instruments that would provide the structural engineer with information on response spectrum amplitudes have been carried out by Galitzin (1913), Kirkpatrick (1927), Suyehiro (1926), and the U.S. Coast and Geodetic Survey (Ulrich, 1941). Based on these pioneering ideas and motivated by the increasing need for simple and inexpensive strong-motion recorders, the U.S. Coast and Geodetic Survey and the California Institute of Technology developed the modern version of the strong-motion seismoscope (Hudson, 1958). Similar instruments were concurrently developed and deployed in the U.S.S.R. (Medvedev, 1965) and later in India (Krishna and Chandrasekaran, 1965) and several other countries. In 1972, over 1,200 seismoscopes were reported to have been installed in at least 16 countries (Fournier d'Albe, 1973), with the largest concentrations being in the U.S.A. (400), Yugoslavia (320), and the U.S.S.R. (197).

During the last 15 years, numerous seismoscopes registered strong ground motion. These measurements were used to infer the overall response spectrum amplitudes (e.g., Cloud and Hudson, 1961), to study the variability of strong ground motion with distance from an earthquake source and site conditions (e.g., Hudson and Cloud, 1967; Hudson, 1971), and to fill in the detailed information on strong ground motion where accelerographs malfunctioned (Trifunac and Hudson, 1970) or were not available (Scott, 1973). In the U.S.S.R., seismoscope response has also been used for correlations with and determinations of intensity of ground shaking (Medvedev, 1965).

The purpose of this paper is to explore and to suggest the possibility of an extended usage of this simple and rugged instrument. The motivation for our effort results from the fact that the number of installed seismoscopes is already sufficiently large and can provide significant input to the overall problem of scaling and interpreting strong ground motion.

We begin by developing correlations of seismoscope response with earthquake magnitude and local site conditions. We show that on the basis of such correlations seismoscope records might be used to infer the local earthquake magnitude in the epicentral region where other more sensitive seismological instruments normally go off scale. Subsequently, as proposed by Medvedev (1965), we develop correlations between the local estimates of earthquake intensity and seismoscope peak response, and in this way derive the approximate scaling laws for response spectra and Modified Mercalli intensities.

To avoid possible constraints on the above-mentioned correlations that might result from the limited amplitude range that can be recorded on the standard seismoscope recording glass ($S_d \approx 7.5$ cm) and to increase the number of usable correlation points, we derive all correlations from the computed values of 186 seismoscope responses obtained from recorded strong-motion accelerograms.

Computation of Seismoscope Response

The elementary theory of the response and the physical characteristics of the seismoscope (Figure 1) have been presented in previous papers (Hudson, 1958; Cloud and Hudson, 1961; Hudson and Cloud, 1967; Trifunac and Hudson, 1970) and will not be repeated here. Laboratory and field tests have further shown that seismoscope response, for small amplitudes, can be approximated by a system of two uncoupled second-order differential equations, that the effect of vertical accelerations can be neglected, and that the damping can be modeled by a constant equivalent viscous dash-pot (Trifunac and Hudson, 1970).

For large deflections of the seismoscope pendulum, however, more refined theory is required, and the Coulomb-type friction between the smoked glass and the recording needle and the effects of vertical accelerations may be considered. If one neglects the contributions of torsional and higher modes to the calculated response, then the nonlinear, coupled differential equations for the deflection angles φ and ψ (Figure 2) become

$$\ddot{\varphi}+2\omega_n\zeta\dot{\varphi}+\omega_n^2 \sin\varphi \cos\psi = -(\omega_n^2/g)(\cos\varphi\ddot{x}+\sin\varphi\ddot{z}) \tag{1a}$$

$$\ddot{\psi}+2\omega_n\zeta\dot{\psi}+\omega_n^2 \sin\psi \cos\varphi = -(\omega_n^2/g)(\cos\psi\ddot{y}+\sin\psi\,\ddot{z}). \tag{1b}$$

In these equations ω_n is the natural frequency of the seismoscope pendulum normally equal to 8.38 rad/sec, ζ is the fraction of critical damping for equivalent viscously damped systems, and g is the acceleration of gravity. $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ represent the absolute components of acceleration of the pendulum support. For accurate response calculations the ζ dependence on the recorded amplitude is as shown in Figure 3. This functional form has been confirmed by many laboratory experiments (Hudson, 1958). It represents a superposition of the viscous amplitude-independent (5 to 7 per cent of critical) damping resulting from the pendulum mass moving in the magnetic field (Figure 1) and the equivalent viscous damping coefficient, which results from Coulomb friction between the glass and the needle, and is inversely proportional to the response amplitude.

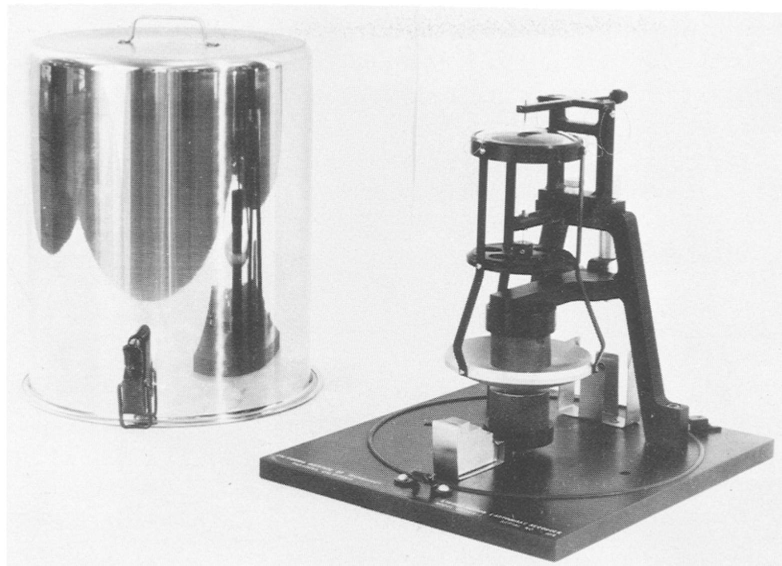

FIG. 1. Photograph of the Wilmot seismoscope with protective cover removed.

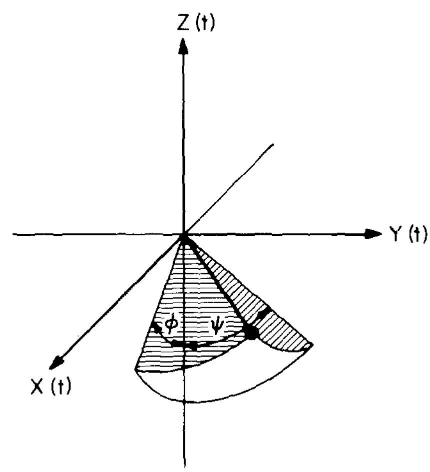

FIG. 2. Coordinates φ and ψ used in describing the relative seismoscope motion. The general motion of the seismoscope support is described by the general displacement $x(t), y(t), z(t)$.

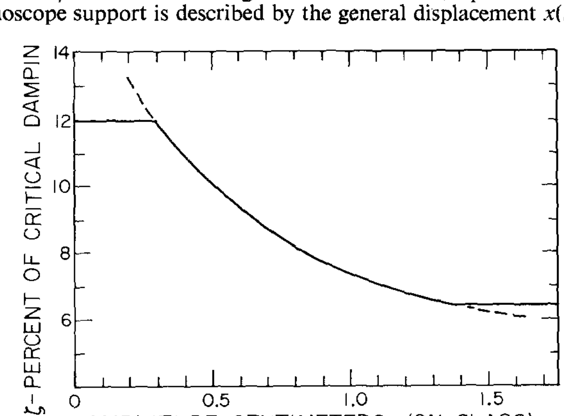

FIG. 3. Functional dependence of the fraction of critical damping on the amplitude of the seismoscope response measured on the recording glass.

The distance between the midpoint of the pivot spring suspension and the top surface of the spherical standard watch glass is about 6.00 cm. This distance is required in converting the angular deflections of the pendulum into the recorded amplitude on the spherical glass surface.

From the analogy of the differential equations describing the relative motion of a single degree-of-freedom viscously damped oscillator which is used in computations of relative displacement response spectra, S_d, and the equations for small amplitude seismoscope response, it can be shown that (Hudson and Cloud, 1967)

$$S_d = \frac{gT^2}{4\pi^2}\alpha_{\max}, \tag{2}$$

where T is the natural period of the seismoscope pendulum, normally 0.75 sec, and where $\alpha_{\max}$, measured in radians, is the largest angular deflection of the seismoscope pendulum relative to the z axis (Figure 2) during a particular excitation. The value of $\alpha_{\max}$ can be computed from the time histories $\varphi(t)$ and $\psi(t)$. For large φ and ψ when nonlinear terms in equations (1a) and (1b) become significant, equation (2), of course, ceases to correspond to its linear displacement spectrum analog.

Although valid for large pendulum deflections φ and ψ, equations (1a) and (1b) neglect the contributions to the seismoscope response that may result from torsional, second (Scott, 1973), and higher modes of vibration of the seismoscope pendulum. However, in computing the synthetic seismoscope responses in this paper, we shall neglect those torsional and higher mode contributions because the comparison between the calculated and actually recorded seismoscope responses appears to be quite good. This has been demonstrated on several occasions (e.g., Figures 9 and 10 of Hudson and Cloud, 1967) when both a seismoscope and an accelerograph recorded strong ground motion at the same point permitting direct comparison of the recorded and computed seismoscope responses.

Correlation of Seismoscope Response and Modified Mercalli Intensity

Earthquake intensity scales are of necessity only descriptive and qualitative measures of vibrational effects caused by earthquakes. These scales depend on numerous factors that can change from one area to another. For example, the material and the type of construction used for residential, industrial and public buildings, the type of geological environment, the time of day when the earthquake occurs, public awareness of an existing earthquake hazard, the methods used in collecting and interpreting intensity data (e.g., Lomnitz, 1970), all can result in significant differences of intensity determination in various parts of the world. One possible remedy for these variations would be the establishment of some instrumental basis for intensity determinations.

One instrumental basis for the MKS intensity scale, now used in the U.S.S.R., has been proposed by Medvedev (1965). To this end a conical pendulum, with static magnification of 1.1, has been developed. This instrument, called an SBM seismometer, is very similar in construction to the Wilmot type seismoscope used in the U.S. (Hudson, 1958). It records on a spherical smoked glass, has a natural period of $T = 0.25$ sec, and the fraction of critical damping $\zeta = 0.08$. By recording ground motions caused by earthquakes, explosions and machine vibrations, the empirical correlation between the MKS intensity scale (Medvedev and Sponheuer, 1969) and the maximum recorded amplitude on the SBM seismometer has been established (Medvedev, 1965).

To develop similar correlations between the Modified Mercalli intensity and the maximum amplitude on the Wilmot Seismoscope ($T = 0.75$ sec, $\zeta \approx 0.10$), we calculated synthetic seismoscope responses for 186 strong-motion accelerograph records (Trifunac and Brady, 1975) now available in digital form. These accelerograms represent the best strong-motion data recorded between 1933 and 1971 in the western United States and are representative of earthquakes with magnitudes ranging from $M = 4$ to about $M = 7.5$. The Modified Mercalli intensity levels at the recording sites were estimated from the isoseismal maps produced in the annual publication "United States Earthquakes" by the U.S. Department of Commerce. Where detailed intensity levels were indicated in these maps close to a recording site, these levels took precedence over the isoseismals. The resulting intensity levels range from III to X, although the 186 accelerograph records provide adequate data only for the intensities V, VI and VII.

TABLE 1

AVERAGE SEISMOSCOPE RESPONSE COMPUTED FROM 186 ACCELERATION RECORDS

Modified Mercalli Intensity	$\bar{S}_d$ (cm)	σ One Standard Deviation of S_d (cm)	No. of Data
III	0.30	—	1*
IV	0.43	0.19	3
V	0.52	0.40	34*
VI	1.14	1.03	66*
VII	2.55	1.52	75
VIII	3.63	2.06	6
IX	—	—	—
X	23.5	—	1
		Total	186

*In Table 3 of Trifunac and Brady (1975) the number of data listed for intensities III, V and VI are 2, 33 and 67, respectively, rather than those listed above. These changes reflect the latest improvements in the classification of the basic data-set and in no way affect the results and inferences presented in this or our previous paper.

Most of the above 186 accelerograph records have been registered on alluvium or in a "soft" geological environment (117 of 186 or 63 per cent) or on "intermediate" sedimentary type rocks (54 of 186 or 29 per cent). Only a few records are available from strong-motion stations located on "hard" rock (15 of 186 or 8 per cent). For simplicity of notation in this paper, the data or the derived results that correspond to the stations recording on "soft," "intermediate," or "hard" rocks will be labeled by the arbitrarily chosen symbols 0, 1, and 2, respectively. The simplified geological data used to select these 0, 1, and 2 labels and the methods employed in their selection have been presented in our previous paper (Trifunac and Brady, 1975).

Table 1 and Figure 4 give the average maxima of the seismoscope responses and their standard deviations for different Modified Mercalli intensities. It is seen that both $\bar{S}_d$ and its standard deviation rapidly increase for higher intensities. As already mentioned, the number of data points is adequate to define $\bar{S}_d$ for the Modified Mercalli intensities V, VI and VII only. However, to present all available information, we included even the single point measurements in Figure 4.

It should be noted here that Table 1 and Figure 4 present the S_d data versus the Modified Mercalli intensity, which is normally reported in terms of integer values, only for convenience in computations. Since the seismoscope response S_d is an instrumental measurement, it represents a more accurate quantity than a Modified Mercalli intensity level and the scatter of S_d versus the Modified Mercalli intensity in Table 1 and Figure 4 most probably is due to the poor accuracy of the intensity scale levels.

The functional relationship between the average S_d seismoscope reading and the Modified Mercalli intensity cannot be derived from basic physical principles because the earthquake intensity scale has no physical units. It has been derived by adopting a set of conventions and definitions which give a qualitative but not quantitative description of the level of shaking. By developing the empirical correlations of the type proposed by Medvedev (1965), it may eventually become possible to "calibrate" various intensity

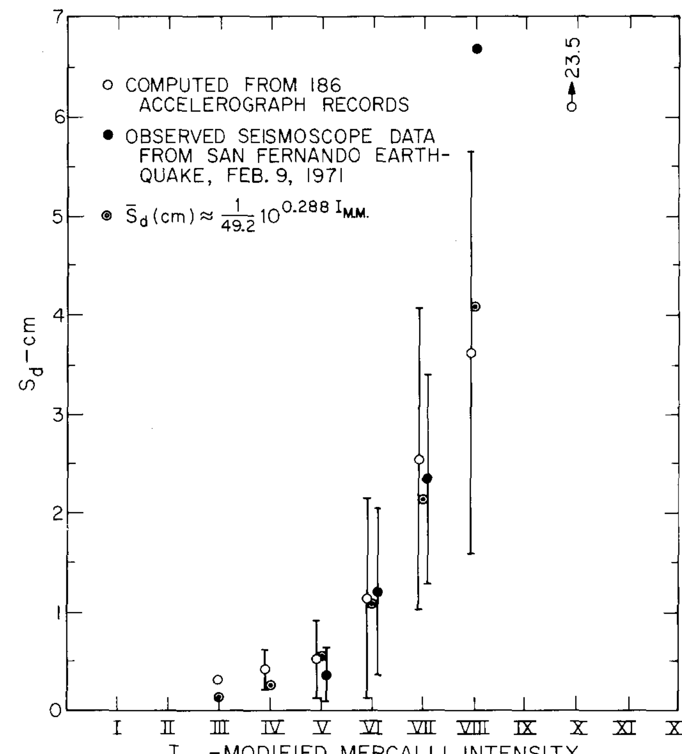

FIG. 4. Average maxima of both the computed and observed seismoscope responses and their standard deviations for different Modified Mercalli intensities (see Tables 1 and 2).

scales by assigning to each intensity level a certain range of possible amplitudes recorded on seismoscopes, accelerographs, seismometers or other related instruments. The data we have at our disposal to date are certainly not adequate for this purpose but can be used to derive the preliminary and approximate correlations. One such correlation that can be used for preliminary prediction of S_d could be of the following form

$$\bar{S}_d(\text{cm}) \approx \frac{1}{49.2}\, 10^{0.288 I_{MM}}; \quad I_{MM} \leqq \text{VIII}, \tag{3}$$

where I_{MM} represents the Modified Mercalli intensity. The data of Table 1 suggest that the average standard deviation of S_d would be of the order of $0.7\, \bar{S}_d$.

To test the validity of the simplified seismoscope model (Figure 2) and the methods of integrating the equations (1a) and (1b) and thus the quality of the derived correlations in Figure 4, Table 1 and our subsequent analysis, we correlated 116 seismoscope readings

(Hudson, 1971), which were recorded in the greater Los Angeles area during the San Fernando, California, earthquake of February 9, 1971, with the corresponding Modified Mercalli intensity. The results of this correlation are given in Table 2 and Figure 4. As may be seen from Figure 4, the agreement between the trends indicated by the measured and calculated seismoscope responses is very good. We interpret this to mean that our method of computing the seismoscope response can be used as a substitute for actual seismoscope recordings. This is clearly a useful result, since it allows us to derive numerous correlations involving seismoscopes in the absence of actual seismoscope recordings.

In developing the above correlations (Tables 1, 2, Figure 4), we tabulated the S_d readings irrespective of the local site and geological conditions at the recording stations. Since the peaks of strong ground motion appear to be influenced by the type of site conditions (Trifunac and Brady, 1975), it is appropriate to examine to what extent those conditions may also be reflected in the seismoscope records computed from the same list of 186 accelerograph records. The results of such analyses have been presented in Table 3 and Figure 5, where S_d and its standard deviation have been tabulated versus Modified Mercalli intensity and for different site classifications. Perusal of Table 3 shows that the number of data used in this analysis is barely adequate to suggest the possible trends of $\bar{S}_d$ and its standard deviation for intensities V, VI and VII and shows that many more recordings will be required before we can develop reliable inferences on what effects the site conditions may have on the recorded seismoscope response. Thus, the data plotted in Figure 5 may eventually merge into a well-defined trend that might result from the effects of geological conditions at the recording site, but does not, at this time, indicate any obvious trends. In fact, some variations of $\bar{S}_d$ and its standard deviation in Figure 5 and Table 3 may be accidental and result from averaging over a small number of points which are not representative of the whole population.

Correlations of Seismoscope Response with Magnitude and Site Conditions

Typically, an earthquake magnitude is proportional to the logarithm of the peak amplitude of a seismogram recorded by a standard instrument. Starting from this elementary definition, numerous magnitude scales have been developed for different source-to-station distances, different types of recorded waves and for a variety of instruments employed (e.g., Richter, 1958).

At close distances, typically less than 50 km, the amplitudes of ground motion caused by even moderate earthquakes drive the conventional seismic instruments off scale and the computation of magnitude must depend on more distant recordings. This leads to several problems, the most important one being that in some cases no quick and local information on the size of an earthquake may be available when the rescuing authorities need it most. Therefore, it is especially fortunate when a strong-motion accelerograph is located so as to record near-field strong ground motions and thus provide statistics on the local magnitude characteristics.

One simple and inexpensive solution to this problem seems to be the use of the seismoscope. Although only sensitive to a narrow band of input frequencies centered about its natural frequency of 1.33 Hz, the seismoscope offers additional advantages in that it represents a simple building model and thus also provides direct information on the amplitude of response spectrum curves. With this in mind, we develop the correlations between the peak seismoscope relative displacement response S_d and the magnitude scale by using 182 accelerograph recordings obtained from 57 earthquakes in the Western United States (see Table 2 of Trifunac and Brady, 1975).

TABLE 2

AVERAGE OBSERVED SEISMOSCOPE DATA FROM S.F. EARTHQUAKE

Modified Mercalli Intensity	$\bar{S}_d$ (cm)	σ_{sd} One Standard of Deviation S_d (cm)	No. of Data
VIII	6.70	—	1
VII	2.34	1.07	40
VI	1.20	0.84	52
V	0.34	0.28	23
		Total	116

TABLE 3

AVERAGE SEISMOSCOPE RESPONSE COMPUTED FROM 186 ACCELERATION RECORDS AND FOR DIFFERENT SITE CLASSIFICATIONS*

Modified Mercalli Intensity	$\bar{S}_d$ (cm)	σ One Standard Deviation of S_d (cm)	No. of Data
III–0	0.30	—	1
III–1			
III–2			
IV–0	0.70	—	1
IV–1	0.30	—	2
IV–2			
V–0	0.55	0.45	17
V–1	0.47	0.31	15†
V–2	0.60	0.50	2
VI–0	1.02	0.80	43
VI–1	1.35	1.30	16†
VI–2	1.41	1.35	7
VII–0	2.40	0.98	49†
VII–1	3.01	2.31	21†
VII–2	2.02	1.14	5
VIII–0	3.63	2.06	6
VIII–1			
VIII–2			
X–0			
X–1			
X–2	23.5	—	1
		Total	186

*See Trifunac and Brady (1975).

†In Table 5 of Trifunac and Brady (1975) the "number of data" listed for intensities V–1, VI–1, VII–0 and VII–1 are 14, 17, 50 and 20 rather than those shown above. These changes reflect the latest improvements in the classification of the basic data-set and in no way affect the results and inferences presented in this or our previous paper.

For most earthquakes studied in this paper, the magnitude determination has been arrived at by using the standard definition of the local Richter magnitude M_L (Richter, 1958)

$$M_L = \log_{10} A - \log_{10} A_0(\Delta), \tag{4}$$

where A is the amplitude in millimeters recorded on the standard Wood-Anderson seismograph whose natural period is 0.8 sec, static magnification $V_s = 2800$, and the fraction of critical damping is 0.8 to 1.0. The empirically determined $A_0(\Delta)$ represents the amplitude in millimeters with which a standard seismometer would register an earthquake of magnitude zero. Table 4, reproduced from Richter (1958), gives $\log_{10} A_0(\Delta)$ versus

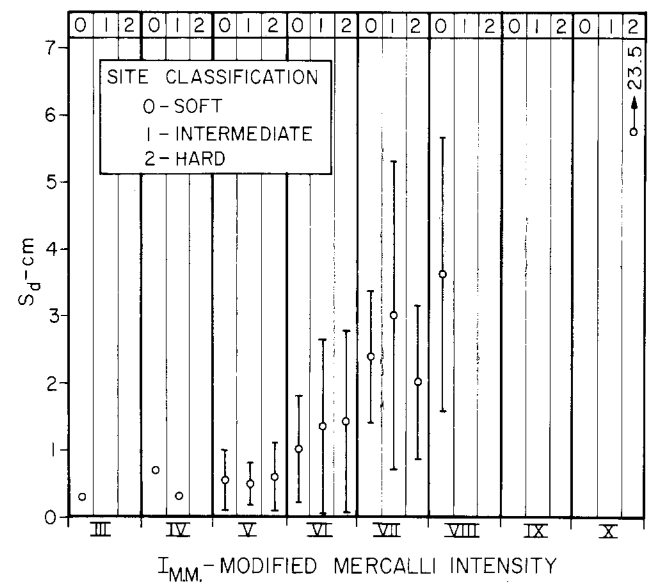

FIG. 5. Average maxima of seismoscope response computed from 186 acceleration records and their standard deviations, for different Modified Mercalli intensities and for different site classifications (see Table 3).

distance. In analogy with the definition (4), we can propose the magnitude derived from seismoscope record, $M_{\text{seismoscope}}$, to be

$$M_{\text{seismoscope}} = \log_{10} S_d(\text{cm}) + 1 - \log_{10} A_0(\Delta) + \log_{10} 2800 - $$
$$- \log_{10} (\text{seismoscope transfer function}). \tag{5}$$

It is convenient to incorporate the effects of scaling and the transfer function into one seismoscope scaling factor defined by

$$\log_{10} S_{d_0} = \log_{10} (\text{seismoscope transfer function}) - 1 - \log_{10} 2800.$$

For a lightly damped seismoscope with the fraction of critical damping equal to about 0.10 and for a steady state sinusoidal excitation, the amplitude of the seismoscope transfer function at its natural frequency would be $1/2\zeta \approx 5$. This would then give

$$\log_{10} S_{d_0} \equiv \log_{10} S_d(\text{cm}) - M_{\text{seismoscope}} - \log_{10} A_0(\Delta) \approx -3.75. \tag{6}$$

To calculate the $\log_{10} S_{d_0}$ for different magnitudes and recording site classifications, we substituted the reported earthquake magnitudes for $M_{\text{seismoscope}}$ in equation (6), computed $\log_{10} S_d(\text{cm})$ from the maximum of the synthetic seismoscope response, and computed $\log_{10} A_0(\Delta)$ from Table 4 by using the known epicenter to station distance. For the four magnitude intervals $M = 4.0$ to 5.0, 5.0 to 6.0, 6.0 to 7.0, and 7.0 to 8.0 and three site classifications 0, 1, and 2, we then least-squares fitted the best estimate of $\log_{10} S_{d_0}$. Table 5 and Figure 6 summarize the results obtained in this way. As may be seen from Figure 6, the scaling factor $\log_{10} S_{d_0}$ decreases with magnitude in a way that emphasizes the progressively smaller influence of larger reported magnitudes on the amplitude of seismoscope response. This is in agreement with our previous work (Trifunac, 1973) which suggested that close to the source of energy release the stress drop and the final DC component of ground displacement should govern the overall

TABLE 4*

LOGARITHMS† OF THE AMPLITUDES A_0 IN MILLIMETERS, WITH WHICH A STANDARD TORSION SEISMOMETER ($T_0 = 0.8$, $V = 2800$, $h = 0.8$) SHOULD REGISTER AN EARTHQUAKE OF MAGNITUDE ZERO

Δ (km)	$-\log A_0$	Δ (km)	$-\log A_0$	Δ (km)	$-\log A_0$
0	1.4	150	3.3	390	4.4
5	1.4	160	3.3	400	4.5
10	1.5	170	3.4	410	4.5
15	1.6	180	3.4	420	4.5
20	1.7	190	3.5	430	4.6
25	1.9	200	3.5	440	4.6
30	2.1	210	3.6	450	4.6
35	2.3	220	3.65	460	4.6
40	2.4	230	3.7	470	4.7
45	2.5	240	3.7	480	4.7
50	2.6	250	3.8	490	4.7
55	2.7	260	3.8	500	4.7
60	2.8	270	3.9	510	4.8
65	2.8	280	3.9	520	4.8
70	2.8	290	4.0	530	4.8
80	2.9	300	4.0	540	4.8
85	2.9	310	4.1	550	4.8
90	3.0	320	4.1	560	4.9
95	3.0	330	4.2	570	4.9
100	3.0	340	4.2	580	4.9
110	3.1	350	4.3	590	4.9
120	3.1	360	4.3	600	4.9
130	3.2	370	4.3		
140	3.2	380	4.4		

*Reproduced from Richter (1958).

†Since A_0 is less than 1, its logarithm is negative, and the table shows values for $-\log A_0$.

shape of the response spectra, while the influence of magnitude progressively diminishes. As seen in Figure 6, the slope defined by the two averages of $\log_{10} S_{d_0}$ for 0 site classification and for magnitude intervals 6.0 to 7.0 and 7.0 to 8.0 is almost equal to -1, indicating that the peak seismoscope response and therefore the response spectrum amplitudes are practically independent of magnitude in the magnitude range larger than about 6. A more precise description of the dependence of $\log_{10} S_{d_0}$ on the magnitude scale and the determination of the magnitude for which the slope of $\log_{10} S_{d_0}$ ceases to be magnitude-dependent will, of course, have to await a more abundant set of accelerograph data, covering a much larger magnitude range and recorded on a more complete sample of different recording sites than the data we have at our disposal now. In the meantime, we hope that the trends indicated in Figure 6 will prove useful for the preliminary correlations of seismoscope data and reported earthquake magnitudes, as well as for the approximate predictions of earthquake magnitude where only seismoscope records are available.

Recent source mechanism studies of strong ground motion close to the earthquake energy release (e.g., Trifunac, 1972a; 1972b), as well as numerous teleseismic observations of body-wave spectra (e.g., Hanks and Wyss, 1972), have demonstrated that large

TABLE 5

DEPENDENCE OF $\log_{10}(S_{d_0})$ ON MAGNITUDE AND SITE CONDITIONS

Magnitude Range	Site Classification*	$\log_{10}(S_{d_0})$ (cm)	σ One Standard Deviation $\log_{10}(S_{d_0})$	No. of Data
4–5	0	−3.028	0.242	4
4–5	1	−3.339	0.296	3
5–6	0	−3.353	0.460	23
5–6	1	−3.470	0.328	15
5–6	2	−3.739	0.408	3
6–7	0	−3.651	0.308	83
6–7	1	−3.699	0.291	33
6–7	2	−3.938	0.469	11
7–8†	0	−4.455	0.248	7
			Total	182‡

*See Trifunac and Brady (1975).

†For most earthquakes used in this work, the magnitude has been calculated from the Wood-Anderson seismograph records and represents, by definition, the Local Richter magnitude M_L. For earthquakes larger than about 6.5 to 7.0, local stations usually go off scale and the "magnitude" has to be determined from other teleseismic seismograms. For this reason, there may be systematic deviations between the magnitude estimates for shocks less than and greater than 6.5 to 7.0.

‡The four calculated seismoscope records not included in this table either had the assigned magnitude less than 4.0 or no magnitude information was available.

earthquakes lead to progressively greater energy content in the long-period waves, while the high-frequency end of the displacement spectrum falls off like ω^{-2} to ω^{-3}. The intuitive physical basis for such scaling has been given by Brune (1970) who proposed an approximate shape of earthquake displacement spectra based on the ω^{-2} falloff at high frequencies and with a flat DC displacement spectrum amplitude proportional to the seismic moment. In this theory the corner frequency, where the flat DC spectrum amplitude begins to fall off like ω^{-2}, decreases with increasing seismic moment and/or the size of an earthquake. Thus, when the predominent frequency band of the recording instrument is at frequencies smaller than the corner frequency, the magnitude derived from the peak of a seismogram may be expected to be representative of the earthquake "magnitude"; while if the frequency band is higher than the corner frequency, the peak on a seismogram becomes more representative of the amplitude of the ω^{-2} part of the displacement spectrum.

The Wood-Anderson seismometer, which represents the standard instrument for published magnitudes for the majority of earthquakes studied in this paper, has a transfer function proportional to f^2 for frequencies shorter than 0.8 Hz and equal to unity for frequencies higher than 0.8 Hz. Thus, for small earthquakes having high corner frequency (>1 Hz), the Wood-Anderson instrument can sample the amplitudes of the flat portion of the displacement spectrum quite well. For large earthquakes, as the corner frequency becomes smaller than about 1 Hz, the Wood-Anderson seismometer samples the amplitudes of the high-frequency spectra and ceases to grow with magnitude (Brune, 1970).

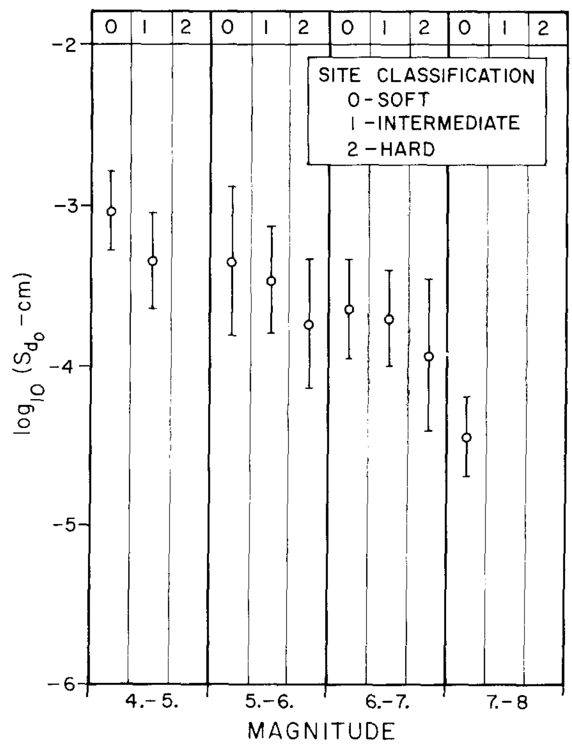

FIG. 6. Dependence of $\log_{10}(S_{d_0})$ on magnitude and site classification (see Table 5).

When a seismoscope is used for magnitude determinations, the above qualitative discussion should become even more applicable since a seismoscope samples only a narrow part of the input displacement spectrum close to 1.33 Hz. Therefore, it might be expected that the scaling given by $\log_{10} S_{d_0}$ should be nearly constant for small magnitude earthquakes, whereas for large magnitudes it would decrease by one for each unit of increasing magnitude, if one were to assume that the amplitude of the ω^{-2} portion of the displacement spectrum has some absolute maximum which is independent of earthquake magnitude (Brune, 1970; Thatcher and Hanks, 1973).

The effect of site classification on $\log_{10} S_{d_0}$ is consistent for all magnitude ranges covered by the available data. Although not significantly different in all cases, the mean values of $\log_{10} S_{d_0}$ are consistently smaller for "intermediate" and "hard" sites than for the "soft" sites. These differences show that the seismoscope response recorded on "soft" alluvium can be up to ~ 2.5 times larger than the corresponding response on "hard" rock formations.

Rewriting equation (6)

$$M_{\text{seismoscope}} = \log_{10} S_d(\text{cm}) - \log_{10} A_0(\Delta) - \log_{10} S_{d_0} \tag{6}$$

and using Tables 4 and 5, one can compute the $M_{\text{seismoscope}}$ magnitude by observing the maximum deflection on seismoscope records, $\alpha_{\max}$, and by computing S_d from equation (2). If many seismoscope records are available, then one standard deviation of such an estimate, obtained by averaging over all seismoscope records, may be on the average 0.35 magnitude units (Table 5), which is close to the accuracy with which the local Richter magnitude, M_L, can be determined (Richter, 1958).

The difficulty in this approach, however, arises from the fact that the $\log_{10} S_{d_0}$ is magnitude-dependent and that several iterations may be required before one arrives at an estimate of the $M_{\text{seismoscope}}$ magnitude. For large earthquakes where $\log_{10} S_{d_0}$ versus magnitude attains the slope of -1, i.e., the seismoscope response becomes insensitive to earthquake magnitude, the above method of computing $M_{\text{seismoscope}}$ breaks down and, of course, should not be used. Precisely at what magnitude level this takes place cannot be determined from the present set of data, and one can only guess that this is probably in the range of magnitude 6 or 7.

If one does know the approximate magnitude range for the earthquake studied, the standard error of each individual estimate of magnitude determined from a seismoscope record would be about 0.35 magnitude units. If one does not know the range of possible magnitude of an earthquake and one works only with Tables 4 and 5 using the trial and error procedure, the results may not be unique, i.e., there may be two magnitude estimates for the two adjacent magnitude intervals given in Table 5 and Figure 6. In that case, the error of an estimate can be as large as one magnitude unit. In normal circumstances, however, at least several seismoscope records will be available, and by averaging the results from all seismoscopes the accuracy of the determined magnitude may be significantly improved.

The important assumption in deriving equation (6) and subsequently the average values of $\log_{10} S_{d_0}$ (Table 5) has been that the apparent attenuation with distance for the Western United States can be adequately described by the empirical data in Table 4. These data have been tabulated by Richter (1958) and result from numerous calculations of the local magnitude scale. Therefore, our results in Table 5 merely represent an extension of the methods used by Richter in deriving the definition of the local magnitude scale. The magnitude-dependent changes of $\log_{10} S_{d_0}$ (Table 5 and Figure 6) then reflect the inferred variations between our estimate of the local magnitude in terms of $M_{\text{seismoscope}}$ and the "magnitude" reported in the literature for the 57 earthquakes used in our analysis. For the majority of shocks used in our work, the reported magnitude does represent the local Richter magnitude. For larger earthquakes, however, when the local Wood-Anderson instruments go off scale, the magnitude reported in the literature and used in our correlations represents the surface-wave magnitude. Table 2 of our paper dealing with the correlation of seismic intensity scales with the peaks of recorded ground motion (Trifunac and Brady, 1975) gives all pertinent data for the 57 earthquakes studied in this paper and lists their assigned magnitudes.

Conclusions

In this paper, we presented correlations of peak seismoscope response S_d with the Modified Mercalli intensity. Although the range of the seismoscope response has been found to vary appreciably for a given intensity level, I_{MM}, illustrating the imprecise nature of scaling the level of strong shaking with the Modified Mercalli intensity, there is a definite trend of the mean seismoscope response which can be approximated by

$$\bar{S}_d \approx \frac{1}{49.2}\, 10^{0.288 I_{MM}},$$

for $I_{MM} \leqq$ VIII. In the absence of reliable estimates of intensity of strong ground shaking, when seismoscope records are available, this correlation can prove useful in supplying additional intensity information. Since the seismoscope represents a typical building vibrating at its first mode with the period of 0.75 sec and fraction of critical damping equal to 10 per cent, the above correlation also provides means for predicting the expected amplitude of the relative displacement response and the pseudo-relative velocity spectrum.

By subdividing all available data into three groups, corresponding to the type of geological formations underlying the recording stations used in this study, we found no systematic trends in the above correlation that would indicate strong dependence on site conditions.

We extended the standard methods used for calculation of the local earthquake magnitude, M_L, and applied them to the calculation of the local magnitude, $M_{\text{seismoscope}}$, using the recorded peak of the seismoscope relative displacement response S_d. We found that the scaling function that relates the magnitude computed from the seismoscope reading, S_d, with the magnitude reported in the literature is strongly magnitude-dependent. We found that the data used in this study is not adequate to describe this magnitude-dependent scaling factor over a sufficiently broad magnitude range of practical interest. The available data indicate, however, that there exists an upper bound for S_d, and therefore the relative displacement spectra, which is practically attained for an earthquake with magnitude somewhere between 6 and 7.

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