ON THE CORRELATION OF PEAK ACCELERATION OF STRONG MOTION WITH EARTHQUAKE MAGNITUDE, EPICENTRAL DISTANCE AND SITE CONDITIONS

by

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ABSTRACT

Statistical analyses and correlations of peak accelerations, with earthquake magnitude, epicentral distance and the geologic conditions of the recording sites have been presented for 188 accelerograms recorded during 57 earthquakes. So far, this is the most complete data set describing strong earthquake ground motion in the Western United States during the period from 1933 to 1971.

It has been shown that the peaks of strong motion acceleration depend on earthquake magnitude only for small shocks. For large magnitude earthquakes this dependence appears to be lost, suggesting that the acceleration maxima recorded close to the source of energy release depend more on the dislocation amplitudes and the stress drop, rather than the overall "size" of an earthquake. It has been shown that the attenuation of strong ground motion with distance can be approximated by the empirical attenuation law developed for the definition of Richter's local magnitude scale. The influence of geologic conditions is shown to be of minor importance for scaling peak accelerations.

INTRODUCTION

In Earthquake Engineering one of the simplest methods of scaling the strong ground motion is to use the peak acceleration recorded in the heavily shaken area. Though one such peak contains only a limited amount of information on the overall spectral and time-dependent properties of ground motion(1), for traditional reasons, as well as simplicity, it appears that such scaling may remain in engineering and seismological practice for some time.

The purpose of this paper therefore is to summarize the existing correlations, to compare them with the trends indicated by the data which are now available for the Western United States, and to present new and more accurate correlations(2, 3) in a form suitable for application in earthquake engineering practice.

SUMMARY OF PREVIOUS CORRELATION WORK

Correlations of peak accelerations with earthquake magnitude and distance were initiated in the early 1940's following the successful recording of the Long Beach 1933 and the Imperial Valley 1940 earthquakes(4) and were supplemented later by the Kern County 1952 earthquake, all in California(5). In 1956 Gutenberg and Richter(6) published the first systematic analysis of peak accelerations. Using an empirical curve for attenuation with distance, they reduced all recorded accelerations to the equivalent "acceleration at epicenter, a_0", and proposed the following correlation with magnitude

$$\log_{10}\left(\frac{a_0}{g}\right) = -2.1 + 0.81\,M - 0.027\,M^2 \tag{1}$$

where a_0 is in cm/sec^2.

In 1965, during the Third World Conference on Earthquake Engineering, Housner(7) and Blume(8) presented revised correlations of peak accelerations with earthquake magnitude and distance. Housner's results have been presented in graphical form and are characterized by accelerations which increase linearly with magnitude, to an absolute maximum acceleration for an M = 8.5 earthquake of about 0.5 g.

The correlation presented by Blume(8) represents an extension of the Gutenberg and Richter(6) correlation in equation (1) and is given by

$$a = \frac{a_0}{1 + (\frac{R}{h})^2} \tag{2}$$

where a is peak acceleration in cm/sec^2, R is epicentral distance, h is the depth of focus and a_0 is the epicentral acceleration given by

$$\log_{10}(\frac{a_0}{g}) = -(\overline{b}+3) + 0.81\ M - 0.027\ M^2\ . \tag{3}$$

It is seen that (3) is a modification of (1), but with a new factor $\overline{b}$, which characterizes the local site conditions. In order that (3) provide the maximum value of a_0, $\overline{b}$ can be approximated by

$$\overline{b} \approx -7.35 + 3.94 \log_{10}(\rho V_s) - 0.41 [\log_{10}(\rho V_s)]^2 \tag{4}$$

where ρ is the average soil density and V_s is the average shear wave velocity for the material underlying the site(8).

Having analyzed the data recorded in California and Japan, Kanai(9) proposed the following correlation

$$a = \frac{5}{\sqrt{T_G}}\ 10^{0.61 M - P \log_{10} R + Q} \tag{5}$$

where a is in cm/sec^2, T_G is the fundamental period of the site, M is the earthquake magnitude, R is the distance to the causative fault in kilometers,

$$P = 1.66 + \frac{3.60}{R} \tag{6a}$$

and

$$Q = 0.167 - \frac{1.83}{R}\ . \tag{6b}$$

Milne and Davenport(10) propose

$$a = \frac{0.69\ e^{1.64 M}}{1.1\ e^{1.1 M} + R^2} \tag{7}$$

where a is the peak acceleration as a percentage of gravity, M is the earthquake magnitude and R is the epicentral distance in kilometers. While this equation fits the data recorded in the Western United States, Milne and Davenport(10) found that it does not apply to Eastern Canada where earthquakes are felt to much greater distances than in California.

Esteva(11) proposed

$$a = 1230\, e^{0.80\,M}\, (R+25)^{-1} \tag{8}$$

where a is in cm/sec², M is the magnitude and R is the hypocentral distance. This curve is derived mainly from the data presented by Gutenberg and Richter(6) and Housner(12).

Donovan(13), using 303 instrumental records, proposed a similar correlation with only slightly different coefficients as follows

$$a = 1300\, e^{0.67\,M}\, (R+25)^{-1.6} \tag{9}$$

where R is the distance to the causative fault in kilometers.

Schnabel and Seed(14) (Figure 6 of their paper) present graphical correlations for average and maximum acceleration on rock. Their correlations have been given for a distance range of 2 to 100 miles from the causative fault.

Figure 1 compares the above-mentioned correlations for a magnitude 6.5 earthquake and presents one additional correlation which has been proposed for that magnitude only(15). The distance and amplitude range for which recorded data are currently available have also been indicated. It is obvious that this type of comparison is a difficult one to present, since various authors use different types and numbers of parameters, or consider a quantity [e.g., T_G, Kanai(9)] which may be quite difficult to interpret.

If we assume that on the logarithmic scale the differences between the epicentral and hypocentral distances and the distance to the fault are not significant, with the exception of very short distances, say less than 10 km, it becomes possible to compare the trend of different correlations and their overall amplitudes. In doing so we find that on the logarithmic scale, the spread of predicted peak accelerations in Figure 1 is almost twice the spread of observed peak accelerations. If one disregards the correlations proposed by Gutenberg and Richter(6) and by Blume(8), the spread of the predicted accelerations reduces and, with the exception of Esteva's curve(11), all correlations appear to be quite consistent with the mean data trend at distances ranging from about 20 to 250 km. For distances less than about 20 kilometers, where only an insignificant number of recorded points are now available, predicted peak accelerations begin to deviate from each other. At small distances from the source, say 1 kilometer, these differences are as large as one order of magnitude.

The estimation of the maximum peak acceleration that could be experienced in the immediate vicinity of a fault has been considered directly or indirectly by most investigators mentioned above. In spite of the fact that it is impossible to compare their estimates under identical conditions, since their results depend on different assumptions, we made an attempt to present a rough comparison in Figure 2(2). By computing the predicted peak at the fault (R = 0), or for R small when a particular correlation is not valid for R = 0, and for the largest recorded magnitude (M = 8.6) or for the largest magnitude for which a given correlation has been presented, we derived the points shown in Figure 2. The maxima of all peaks recorded since 1933 are also shown.

There are some interesting trends shown in Figure 2. With the increase of available records, and as would be expected on a purely statistical basis, the recorded peaks increase in time. This trend, of course, does not reflect any physical change in the earthquake source mechanism in time but merely indicates

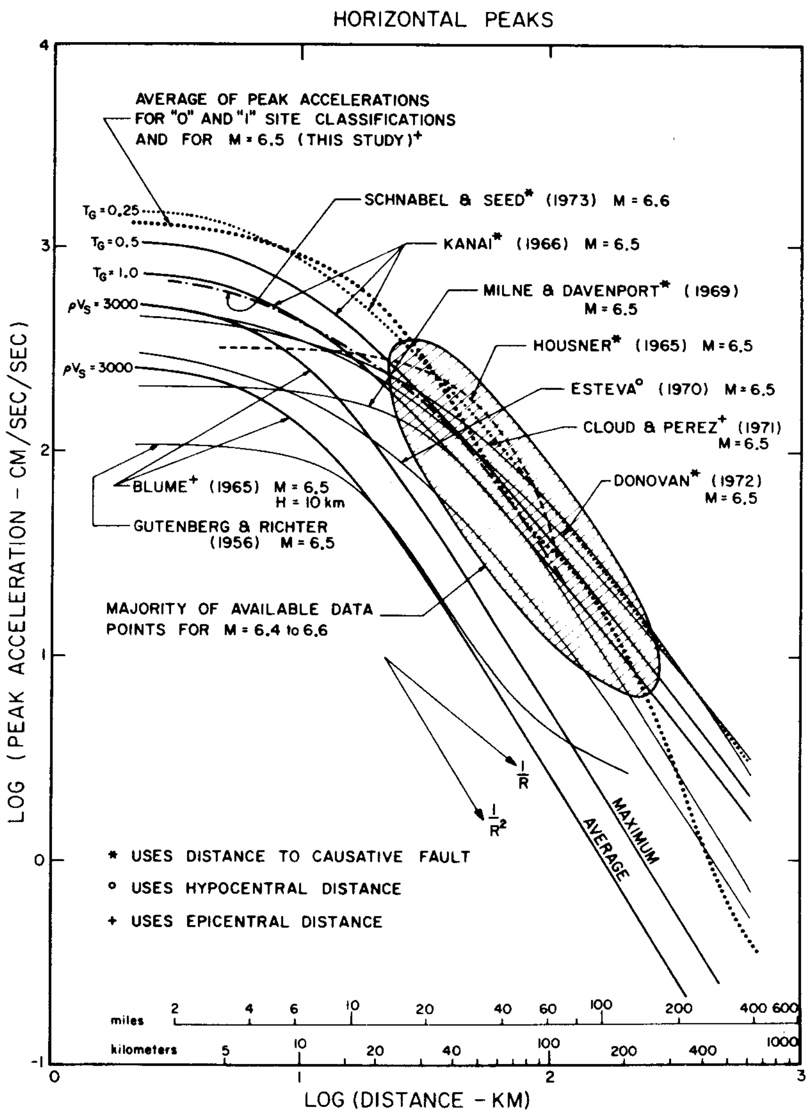

Figure 1. Comparison of the correlations, for a magnitude 6.5 earthquake, of peak acceleration and distance.

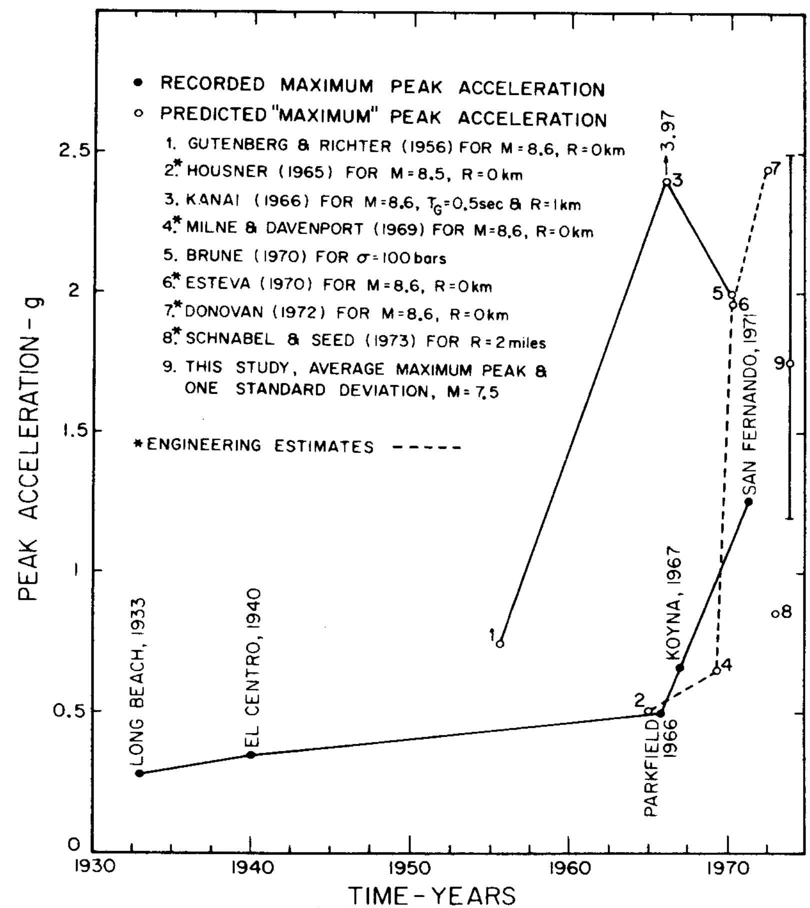

Figure 2. Comparison of recorded and predicted maximum peak acceleration, as functions of time.

that with the increased number of records, the increased number of accelerographs deployed, and the lengthening duration of the strong motion program, we are gradually approaching the minimum number of records offering an adequate and meaningful representation of the infinite set containing all cases that may be encountered.

Predicted maximum peak acceleration also increases with time (Figure 2). As more representative data becomes available (Figure 3), theoretical and empirical peak predictions are updated and, in general, follow the trends indicated by the measurements.

PROPOSED SCALING FOR PEAKS OF STRONG GROUND MOTION

It was recently suggested that the peaks of strong motion acceleration might be scaled by using the following expression(2)

$$\log_{10}[a_{max}] = M + \log_{10} A_0(R) - \log_{10}[a_0(M)] . \qquad (10)$$

In (10), a_{max} represents peak acceleration, M is earthquake magnitude, which in most cases is represented by the local magnitude M_L(16); $\log_{10} A_0(R)$ is the empirically determined function(16) which describes attenuation versus distance (Table I); and $a_0(M)$ represents the magnitude-dependent empirical scaling function for acceleration.

The attenuation function, $\log_{10} A_0(R)$, is employed in equation (10) with its amplitudes as presented by Richter(16). The actual amplitudes of $\log_{10} A_0(R)$ have no significance for our present application, since the function $a_0(M)$ can readily absorb any additional scaling factors needed for the overall amplitude calibration of equation (10). The physical significance of $\log_{10} A_0(R)$ for our present work lies in its relative changes of amplitude with distance.

It is important to note here that $\log_{10} A_0(R)$ is of special value for the correlations presented in this paper because it incorporates empirically the average amplitude attenuation with distance in the Southern California Region and thus experimentally includes the average properties of the earth's crust in this area. Since most strong-motion data have been recorded in the same area, the curve $\log_{10} A_0(R)$ represents the most natural first approximation to be used for scaling the strong-motion data as well.

To calculate the dependence of $\log_{10}[a_0(M)]$ on magnitude and for different site conditions (alluvium, $s = 0$; intermediate, $s = 1$; and basement rock, $s = 2$), the acceleration records were divided into nine groups and the averages and standard deviations computed(2). The results are shown in Table II, where it is seen that the number of data points is barely adequate to suggest the overall amplitudes for the magnitude ranges 4-5 and 7-8. Because of the limited number of data points, and to smooth out the magnitude dependence of the above functions, we calculated the averages and standard deviations for magnitude intervals equal to one magnitude unit, using the ranges 4-4.9, 5-5.9, 6-6.9, and 7-7.9.

The change of averages from those for M = 6-6.9 to those for M = 7-7.9 is close to one magnitude unit (Table II). This means that in this magnitude range, peak acceleration of strong ground motion essentially reaches its magnitude-independent maxima. For low magnitudes the amplitudes of $\log_{10}[a_0(M)]$ tend to level off and seem to reach a constant level for some magnitude less than about 5.

The effects of site classification(17) on the recorded peaks are also seen in Table II. For magnitudes less than 6 it seems that $\log_{10}[a_0(M)]$ may

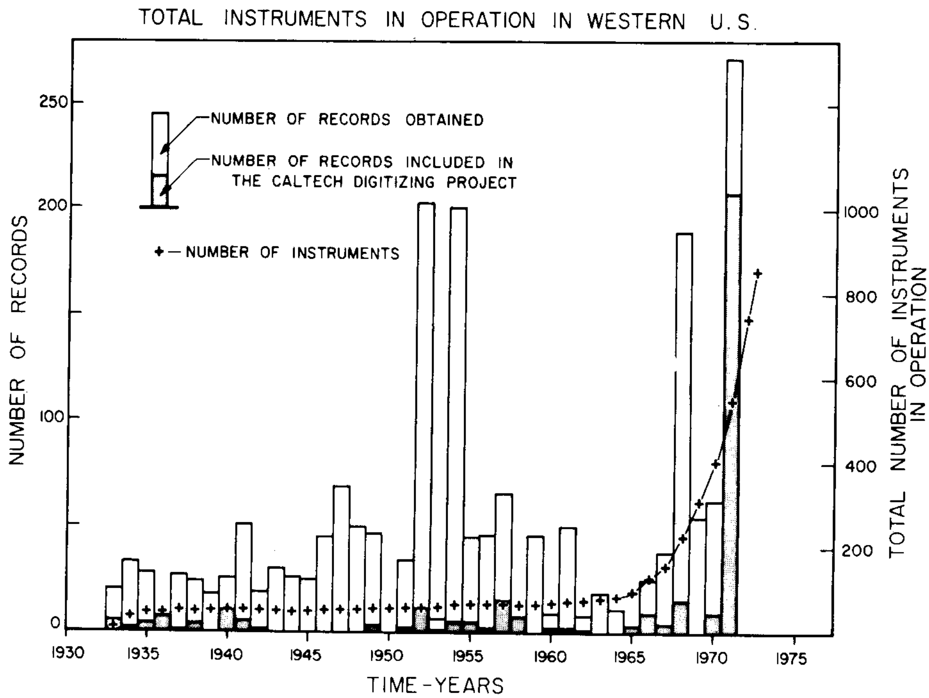

Figure 3. Total number of strong motion accelerographs, and the number of records obtained and analyzed, in the Western United States, 1933 to 1972.

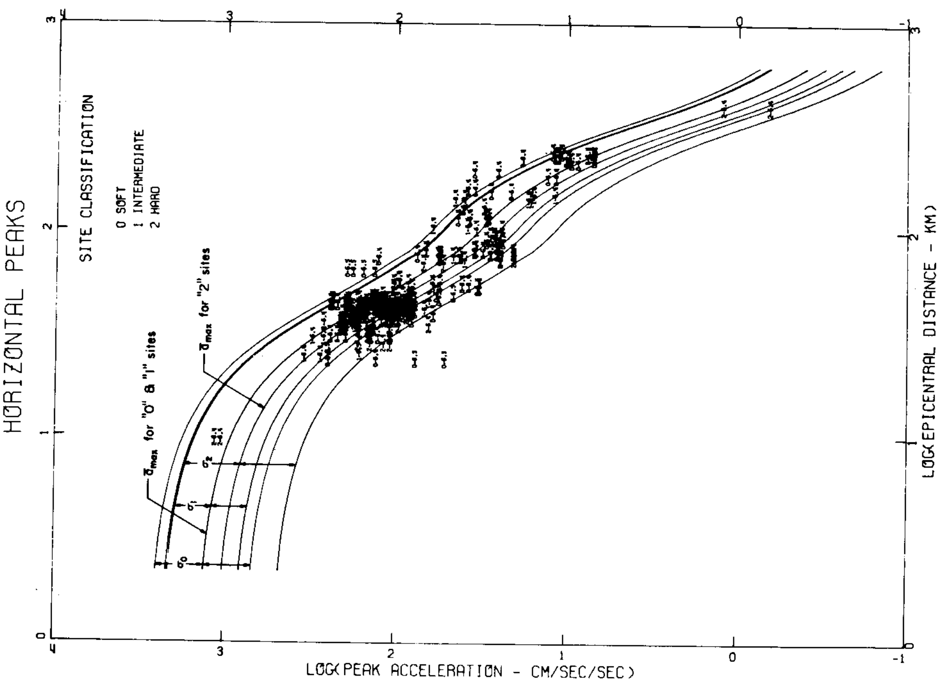

Figure 4. Computed average peak acceleration versus distance and standard deviations, for magnitude M = 6.5 and "0", "1" and "2" site conditions, using equation (10) and Tables I and II. Data points for M = 6.4 to 6.6 are also plotted.

consistently be larger for "0" sites than for "2" sites, which in turn implies that the average peak accelerations for sites on hard basement rocks would be higher than those recorded on top of alluvium ($s = 0$).

The popular earthquake engineering expectations that the peak accelerations are often amplified when seismic waves propagate into soft alluvium layers are not supported by this data set for the Western United States. The extrapolation of the results from simple theoretical computations based on vertically incident S waves to actual earthquake records should therefore be seriously reviewed.

Figure 4 gives an example of computed peak accelerations versus distance for magnitude $M = 6.5$ and "0", "1" and "2" site conditions, by using equation (10) and Tables I and II. Equation (10) shows that the attenuation with distance is governed by the empirically determined attenuation term(16) $\log_{10} A_0(R)$ tabulated in Table I. Transformation of this function to the log-log coordinate axes of Figure 4 results in the shapes of the attenuation curves shown in this figure. All available peak accelerations for magnitudes ranging from 6.4 to 6.6 have also been plotted in this figure to show how the predicted average values and standard deviations compare with the available data.

To extend the applicability of equation (10) so that it can be used for approximate scaling of the peaks of strong motion acceleration when the confidence with which such an estimate is made has been specified, one can write(3)

$$\log_{10}[a_{max,p}] = M + \log_{10} A_0(R) - \log_{10}[a_0(M,p,s,v)] , \tag{11}$$

where M is earthquake magnitude; p is the confidence level associated with the upper bound $a_{max,p}$ for the peak a_{max}; s represents the type of site conditions(17) ($s = 0$ for alluvium deposits; $s = 1$ for "intermediate" rock; $s = 2$ for basement rock); and v is used to describe the component direction ($v = 0$ for horizontal and $v = 1$ for vertical direction). As a first approximation, we assume that within a certain range of M, between M_{min} and M_{max}, the scaling function $a_0(M,p,s,v)$ can be described by

$$\log_{10}[a_0(M,p,s,v)] = ap + bM + c + ds + ev + fM^2 , \tag{12}$$

where a, b, c, d, e and f are coefficients to be determined.

Table III presents the results of least squares fitting of equation (12) to the available strong motion data(3). It presents the estimates of the coefficients a, b, . . . , e and f; the total number of data points which have been used in the fitting; and M_{min} and M_{max}, which are estimates of the lowest and highest M for which equation (12) is assumed to apply. Outside this range, equation (12) takes the form shown in Table III. M_{min} is defined as the point where the calculated parabolic variation with M has zero slope, and for $M \leq M_{min}$ the right hand side of equation (12) takes on the corresponding constant value. M_{max} is defined as the point where the parabola has unit slope, equal to that of M, and for $M \geq M_{max}$ the right hand side of (12) continues linearly with this slope.

CONCLUSIONS

We found the amplitudes of strong ground motion in the near-field of earthquake energy release to be significantly higher than so far predicted by most investigators(4, 7, 8, 9, 10, 11, 13, 14, 18). These differences can be explained by the serious lack of near-field data ($R < 20$ km) and by the use of somewhat arbitrary methods for extrapolation towards the earthquake source in most previous studies.

TABLE I

$\log_{10} A_o(R)$[16] Versus Epicentral Distance R

R (km)	$-\log_{10} A_o(R)$	R (km)	$-\log_{10} A_o(R)$	R (km)	$-\log_{10} A_o(R)$	R (km)	$-\log_{10} A_o(R)$
0	1.400	95	3.020	260	3.827	430	4.549
5	1.500	100	3.044	270	3.877	440	4.579
10	1.605	110	3.089	280	3.926	450	4.607
15	1.716	120	3.135	290	3.975	460	4.634
20	1.833	130	3.182	300	4.024	470	4.660
25	1.955	140	3.230	310	4.072	480	4.685
30	2.078	150	3.279	320	4.119	490	4.709
35	2.199	160	3.328	330	4.164	500	4.732
40	2.314	170	3.378	340	4.209	510	4.755
45	2.421	180	3.429	350	4.253	520	4.776
50	2.517	190	3.480	360	4.295	530	4.797
55	2.603	200	3.530	370	4.336	540	4.817
60	2.679	210	3.581	380	4.376	550	4.835
65	2.746	220	3.631	390	4.414	560	4.853
70	2.805	230	3.680	400	4.451	570	4.869
80	2.920	240	3.729	410	4.485	580	4.885
85	2.958	250	3.779	420	4.518	590	4.900
90	2.989						

TABLE II

Means and Standard Deviations of the Logarithm of the Magnitude-Dependent Scaling Function $a_o(M)$

		Magnitude	4.0-4.9			5.0-5.9			6.0-6.9			7.0-7.9		
		Site Classif.	0	1	2	0	1	2	0	1	2	0	1	2
Acceleration cm/sec^2	Vert.	$\overline{\log a_o}$	1.80	1.39	-	1.83	1.94	1.60	2.21	2.25	2.25	3.21	-	-
		σ	0.036	0.519	-	0.494	0.253	0.213	0.270	0.253	0.332	0.107	-	-
	Horz.	$\overline{\log a_o}$	1.38	1.07	-	1.56	1.54	1.41	1.94	1.94	2.05	2.87	-	-
		σ	0.309	0.368	-	0.503	0.313	0.390	0.278	0.205	0.331	0.163	-	-
No. of Data Used		Vert.	3	2		24	15	2	82	34	12	7		
		Horz.	6	4		47	30	4	164	68	24	14		

TABLE III

Coefficients in the Expression

$$\log_{10}[a_o(M, p, s, v)] = \begin{cases} ap + bM + c + ds + ev + fM^2 - f(M - M_{max})^2 , & M \geq M_{max} \\ ap + bM + c + ds + ev + fM^2 , & M_{max} \geq M \geq M_{min} \\ ap + bM_{min} + c + ds + ev + fM^2_{min} , & M \leq M_{min} \end{cases}$$

a	b	c	d	e	f	Total N Data	M_{min}	M_{max}
-0.898	-1.789	6.217	0.060	0.331	0.186	227	4.80	7.5

For a magnitude 7.5 earthquake, our analysis indicates that the average peak accelerations at the fault should be 1.75 g and that the range corresponding to one standard deviation in the logarithm on either side of this mean is from 1.2 g to 2.5 g (Figure 2). These amplitudes should also be representative of the largest earthquake in the Western United States.

Contrary to the frequently stated opinion that alluvium layers amplify strong-motion acceleration at certain "predominant" frequencies, the data studied in this paper show that on the average peaks recorded on hard rock may be higher, but not significantly, than the peaks recorded on alluvium. This is in accord with our previous study(17) where we demonstrated that for a given Modified Mercalli intensity level peak accelerations recorded on a hard rock site are on the average higher than the same recorded on alluvium.

The approximately parabolic growth of the function $\log_{10}[a_0(M, p, s, v)]$ with magnitude approaches the slope equal to 1 for magnitudes equal to 6 to 6.5. This means that the peak accelerations of strong ground motion do not grow linearly with magnitude and that this rate of growth becomes very small for magnitudes greater than 6.5 to 7.0. For a magnitude 7.5 shock the peaks of the near-field strong-motion acceleration effectively reach the maximum amplitudes. For "soft" site conditions ($s = 0$), magnitude $M = 7.5$, 90 percent confidence level, and epicentral distance $R = 0$, the estimated maximum amplitude of strong-motion acceleration is approximately equal to 4.5 g. According to this analysis, this amplitude would be associated with the largest earthquake in the Western United States.

Finally, it should be pointed out here that, from the practical earthquake engineering point of view, high acceleration amplitudes should not necessarily be associated with a proportionally higher destructive potential. An extended duration(19) of strong ground motion and high acceleration amplitudes characterize destructive earthquake shaking, while one or several high-frequency high-acceleration peaks may, in fact, constitute only minor excitation because of the short duration involved and may lead to only moderate or small impulses when applied to a structural system.

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