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Transient and permanent shear strains in a building excited by strong earthquake pulses

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ABSTRACT

Nonlinear waves in a building with bi-linear constitutive law are described for excitation by vertically propagating S-wave pulses. Conditions that lead to the nonlinear deformation are described in terms of dimensionless amplitudes and wavelengths of these pulses. It is shown that a building can fail during the first passage of the incident wave (during a time shorter than the travel time from the bottom to the top of the building). Peak amplitudes of (1) transient strains, (2) permanent strains, and (3) the peak ductility in the building are described in terms of (a) the amplitudes of incident pulses and (b) the places where the pulses occur in the building.

1. Introduction

Classical earthquake engineering analyses of response are based on the vibrational formulation of the governing equations [1,15] and employ the concept of the linear- and nonlinear-equivalent oscillators consisting of a mass, a spring, and a dashpot. When this approach is extended to multi-degree-of-freedom systems, represented by lumped-mass models, the nature of modeling requires placement of discrete springs, typically at the top and bottom of the "columns". The nature of such modeling determines *a priori* that the model's nonlinearities can occur only at the model springs. Observations of earthquake damage, however, show complex variations in the location and in the distribution of damage, which are difficult to predict by such simple models. To avoid such spatial constraints, and to enable a more realistic prediction of the locations of early damage, we are led to a different formulation of the solution, which we develop using the wave-propagation method. The vibrational description of linear response mathematically does lead to the unique solution, but when excitation includes strong pulses an accurate solution requires a *superposition of the responses associated with at least several hundred characteristic functions*. Because this is not practical from an engineering standpoint, we are again led to the wave-propagation approach. It is for these reasons that in this paper we examine the response of simple shear beams, with nonlinear material properties, in terms of the wave-propagation method of solution.

Wave-propagation methods in earthquake engineering have been used since the 1930s [7,8,10,11]. In this paper, we study wave propagation through a homogeneous shear layer to describe the elementary relationships among the amplitudes of incident pulses and the building response, with emphasis on transient and permanent strains. One-dimensional (1-D) representation of nonlinear shear waves in a building with constant material properties will be used. This model can describe shear waves in long buildings (when the rocking response associated with soil–structure interaction can be neglected), and it is useful for understanding the elementary aspects of early stages of damage in such buildings [3]. We will consider a building with bi-linear stiffness properties overlying an elastic half-space and excited by S-wave pulses arriving vertically up from the half-space. Our aims are (1) to describe how the amplitudes and duration of incident pulses lead to nonlinear strains (rotations), strain localization, and permanent deformations; and (2) to understand the conditions that determine their locations inside the building.

With minor modifications, the model described in this paper also describes shear waves in a layer of soil overlying an elastic half-space [5]. For linear response, amplification of incident pulse, which depends upon the impedance ratio between the soil and the structure, has been described in Gičev and Trifunac [6].

Throughout this paper, we will use *rotations* and *strains* interchangeably, and will consider only small deformations for which $\varepsilon \sim \tan\varepsilon$.

2. Model and numerical scheme

We analyze horizontal shear deformations, u, in a 1-D building supported by a half-space and excited by a vertically propagating shear wave represented by a half-sine pulse (Fig. 1). The

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Lax–Wendroff [9] finite-difference scheme for solution of this problem with accuracy $O(\Delta t^2, \Delta x^2)$, where Δx and Δt are the space and time increments, leads to the exact solution when $\beta\Delta t/\Delta x = 1$, where β is the velocity of shear waves [4–6]. With a ratio of the spatial intervals $\Delta x_b/\Delta x_s = \beta_b/\beta_s$, this requirement can be satisfied, where x is the coordinate along the wave propagation (Fig. 1). The subscripts b and s designate the values in the building, with height H_b, and in the half-space, respectively. The equation of motion is

$$\nu_t = (\sigma)_x/\rho, \tag{1a}$$

and the relation between the derivative of the strain and the velocity is

$$\varepsilon_t = \nu_x, \tag{1b}$$

where ν, ρ, σ, and ε are particle velocity, density, shear stress, and shear strain, respectively, and the subscripts t and x represent derivatives with respect to time and space.

For shear waves, the boundary conditions (free stress at the top of the building and continuity of stress and displacement at the building–soil interface; point 3 in Fig. 1) and the exact transmitting boundary condition in the soil (at point 1 in Fig. 1) have been discussed in Gičev and Trifunac [5] and will not be repeated here.

For the linear wave motion at the contact (point 3 in Fig. 1), one part of the incoming wave is transmitted into the other medium and one is reflected back into the same medium. The corresponding coefficients are obtained from the boundary conditions of continuity of the displacements and stresses at the contact. For a transmitted wave from medium B to medium A, and for a reflected wave from medium B back into medium B, these coefficients are, respectively,

$$k_{trB\to A} = 2[1 + (\rho_a\beta_a)/(\rho_b\beta_b)]^{-1} \tag{2}$$

and

$$k_{refB\to B} = [1 - (\rho_a\beta_a)/(\rho_b\beta_b)]/[1 + (\rho_a\beta_a)/(\rho_b\beta_b)]. \tag{3}$$

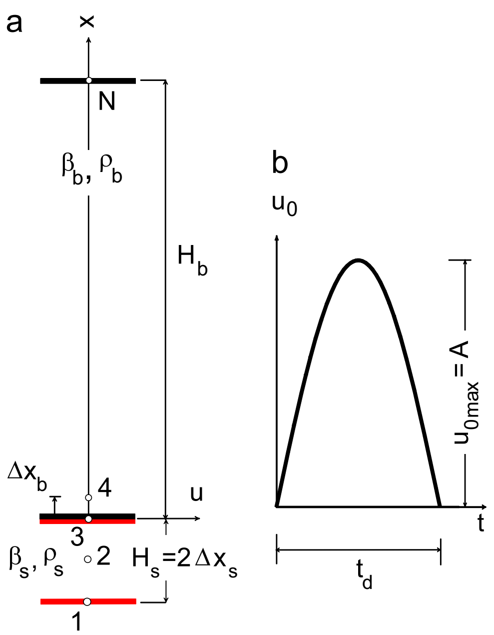

Fig. 1. Building and incoming strong-motion displacement pulse: (a) model of the building, and (b) the pulse in the half-space.

For the opposite direction of propagation, the numerators and denominators in these fractions exchange places.

3. Numerical examples

We consider a building supported by an elastic half-space. The densities of the half-space and the building are assumed to be $\rho_s = 2000\,\text{kg/m}^3$ and $\rho_b = 258\,\text{kg/m}^3$, respectively. The velocity of the shear waves in the half-space is taken as $\beta_s = 250\,\text{m/s}$, and in the building as $\beta_b = 100\,\text{m/s}$. This example will illustrate the nonlinear response of a "typical" building [17,18] supported by a half-space with properties that are representative of many metropolitan areas [16]. In all calculations in this paper we used $H_b = 10\,\text{m}$ and $\varepsilon_{yb} = 0.02$, but we present all results in dimensionless terms.

To describe nonlinear response in the building, we use two dimensionless parameters

$$\text{dimensionless amplitude}: \quad \alpha = A/(H_b\,\varepsilon_{yb}), \tag{4}$$

where A is the amplitude of the incident pulse (Fig. 1), H_b is the height of the building, and ε_{yb} is the yielding strain in the building (Fig. 2); and

$$\text{dimensionless frequency}: \quad \eta = 2H_b/\lambda_b = 2H_b/(\beta_b\,2t_d) = H_b/(\beta_b\,t_d), \tag{5}$$

where λ_b is the wavelength of the wave in the building, β_b is the shear-wave velocity in the building, and t_d is the duration of the incident wave represented by a half-sine pulse (Fig. 1b). The displacement and the strain (rotation) in the linear building are (see [5,6])

$$\begin{aligned} u(x,t) = A\sum_{j=1}^{\infty} k_j \Bigg\{ &\sin\frac{\pi}{t_d}\left(t - t_{j-1} - \frac{x}{\beta_b}\right) \\ &\times \left[H\left(t - t_{j-1} - \frac{x}{\beta_b}\right) - H\left(t - t_{j-1} - \frac{x}{\beta_b} - t_d\right)\right] \\ &+ \sin\frac{\pi}{t_d}\left(t - t_j + \frac{x}{\beta_b}\right)\left[H\left(t - t_j + \frac{x}{\beta_b}\right) - H\left(t - t_j + \frac{x}{\beta_b} - t_d\right)\right]\Bigg\} \end{aligned} \tag{6}$$

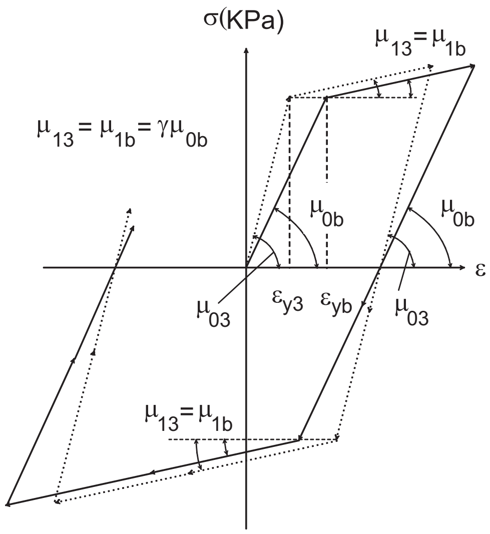

Fig. 2. The constitutive laws, σ–ε, for the building (solid line) and for the interface (dotted line).

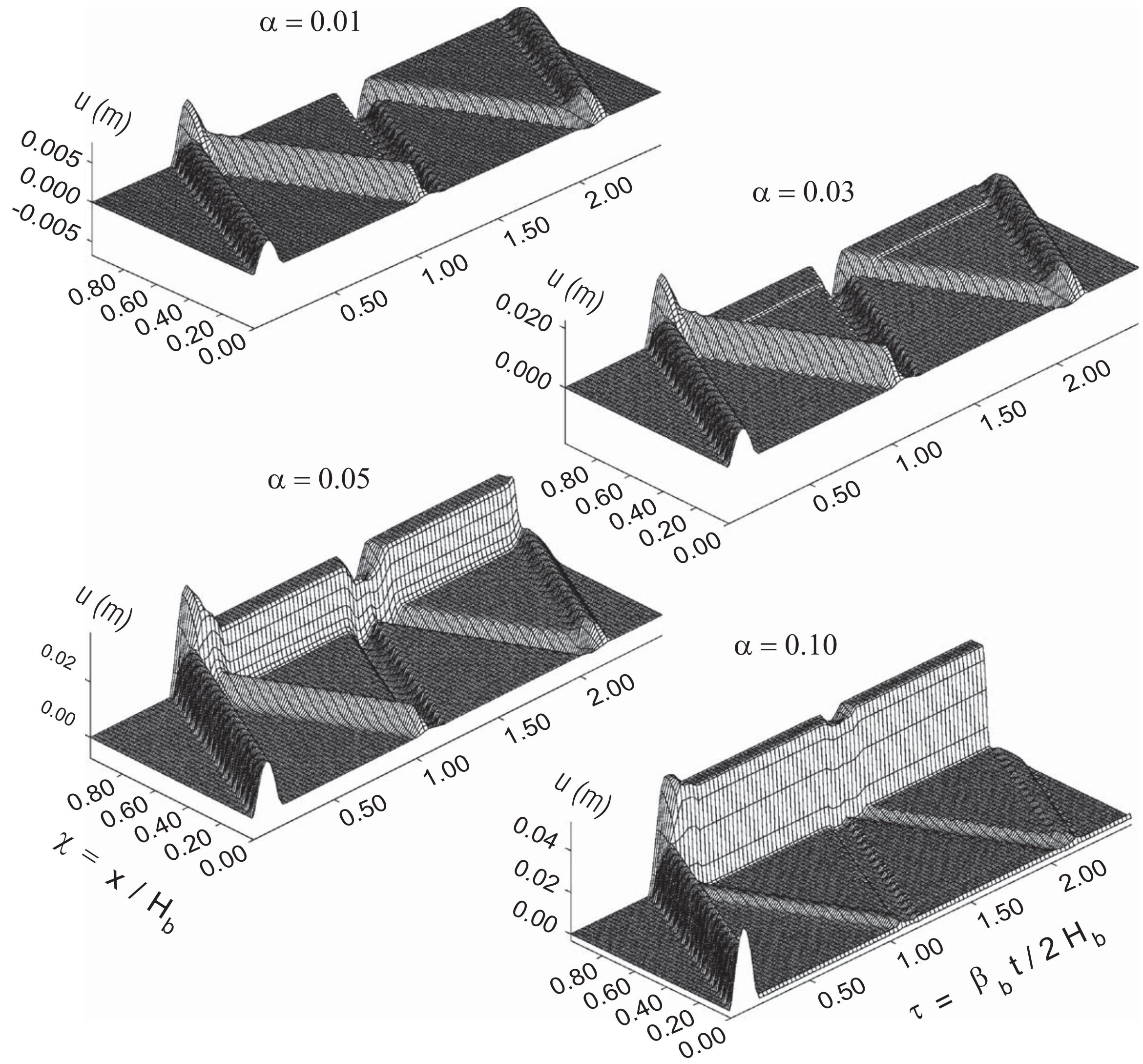

Fig. 3. Linear displacements along the normalized length of the building, $\chi = x/H_b$, versus normalized time $\tau = \beta_b t/2H_b$, for dimensionless pulse amplitude $\alpha = 0.01$ and dimensionless frequency $\eta = 3$ (top), and nonlinear displacements when $\alpha = 0.03$, 0.05, and 0.10.

and

$$\varepsilon(x,t) = A\frac{\pi}{\beta_b t_d}\sum_{j=1}^{\infty} k_j\left\{-\cos\frac{\pi}{t_d}\left(t-t_{j-1}-\frac{x}{\beta_b}\right)\times\left[H\left(t-t_{j-1}-\frac{x}{\beta_b}\right)-H\left(t-t_{j-1}-\frac{x}{\beta_b}-t_d\right)\right]+\cos\frac{\pi}{t_d}\left(t-t_j+\frac{x}{\beta_b}\right)\left[H\left(t-t_j+\frac{x}{\beta_b}\right)-H\left(t-t_j+\frac{x}{\beta_b}-t_d\right)\right]\right\}, \quad (7)$$

where j is the order number of the passage of the wave along the path bottom–top–bottom in the building, $t_j = 2jH_b/\beta_b$ ($j = 0, 1, 2, 3, \ldots$) is the time required to travel the path bottom–top–bottom (two heights) j times, and

$$k_j = k_t k_r^{j-1} \quad (8)$$

is the amplitude factor of the pulse in the half-space during its jth passage along the path bottom–top–bottom through the building, with k_t and k_r defined by Eqs. (2) and (3).

The odd terms in the series of Eqs. (6) and (7) describe the response for the pulse coming from below, while the even terms describe the response for the pulse arriving from above. For t he shear-wave velocities and material densities in our example, ($\beta_s = 250$ m/s, $\rho_s = 2000$ kg/m^3, $\beta_b = 100$ m/s, and $\rho_b = 258$ kg/m^3), $k_t = 1.902$ and $k_r = -0.902$. The displacement and velocity change signs after reflection from the building–half-space interface and do not change signs after reflection from the top of the building. The strain changes sign after reflection from the top of the building and does not change sign after reflection from the interface between the half-space and the building (Fig. 3a). Fig. 3b–d, illustrate the corresponding changes in the nonlinear response when $\alpha = 0.03$, 0.05, and 0.1, respectively.

The constant that multiplies the series in Eq. (7) in terms of dimensionless amplitude and dimensionless frequency is

$$A\pi/(\beta_b t_d) = A_\varepsilon = \pi\alpha\eta\varepsilon_{yb}. \quad (9)$$

To analyze the conditions for occurrence of permanent strain, we consider two characteristic locations in the building: (1) point B ($x = 0$) at the building–half-space interface (point 3 in the finite-difference grid, see Fig. 1), and (2) point T (at $x = H_b - \beta_b t_d/2$), where the amplitudes of the strains with the same sign meet after reflection from the top of the building. The location of this point depends upon the wavelength of the pulse. The first term in Eq. (7) is one if the argument of the cosine function is equal to t_d ($t - t_0 - x/\beta_b = t_d$), and the second term is one if the argument of the second cosine function is equal to 0 ($t - t_1 + x/\beta_b = 0$). We then solve the system of two equations for x (the location of this point) and for t (the time when maximum strain occurs), keeping in mind that $t_0 = 0$ and $t_1 = 2H_b/\beta_b$.

The point T, where the strain (rotation) amplitude is two times larger than the strain entering the building, is at $x = H_b - \beta_b t_d/2$, and the time when this occurs is $t = H_b/\beta_b + t_d/2$. From Eq. (7), during the first passage of the pulse, $t < 2H_b/\beta_b$, and at point B only the first term in the series exists. The strain at point B reaches its absolute maximum at the very beginning, during the entrance of the pulse into the building, and its value is

$$|\varepsilon^1_{B\max}| = \pi\alpha\eta\varepsilon_{yb}k_t. \tag{10}$$

If the strain in Eq. (10) is greater than the yielding strain in the building, a permanent strain at the interface will develop. The condition for this is $|\varepsilon_B{}^1_{\max}| > \varepsilon_{yb}$, or, in dimensionless terms,

$$\alpha\eta > (\pi k_t)^{-1} = (\beta_b\rho_b + \beta_s\rho_s)/(2\pi\beta_s\rho_s) = C_B. \tag{11B}$$

At point T (which does not exist if $t_d > 2H_b/\beta_b$, and which coincides with point B if $t_d = 2H_b/\beta_b$), from Eq. (7) the maximum strain during the first passage is $2A_\varepsilon k_t$, and it occurs at $t = H_b/\beta_b + t_d/2$. The condition for occurrence of the permanent strain is

$$\alpha\eta > (2\pi k_t)^{-1} = (\rho_b\beta_b + \rho_s\beta_s)/(4\pi\rho_s\beta_s) = C_T = C_B/2. \tag{11T}$$

For the material densities and shear-wave velocities in our example, $C_B = 0.16737$ and $C_T = 0.083685$.

When the reflected wave from the top of the building reaches the building–half-space interface ($t = t_1$), the wave begins the second passage. The linear solution for the strain in Eq. (7) at B now involves three terms in the series if the duration of the pulse is longer than $2H_b/\beta_b$, and two terms for shorter pulses. For short pulses ($\eta > 0.5$), if there is no occurrence of permanent strain during the first passage at point T, the response of the building will be linear for all time. For long pulses ($\eta < 0.5$), the strain at B, at the beginning of the second passage, is smaller than the strain at the beginning of the first passage for $\eta < (2\pi)^{-1}\arccos(|k_r|)$, and for $\eta > (2\pi)^{-1}\arccos(|k_r|)$, the former strain is larger than the latter one [4]. For our example, ($k_r = -0.902$), $\eta > 0.0711$ always gives a larger strain at the interface point at the beginning of the second passage than the strain at the beginning of the first passage.

The largest amplification of the strain (rotation) at B is for $\eta = 0.5$ at the beginning of the second passage, when $1 < r < 4.236$, with $r = \beta_s\rho_s/(\beta_b\rho_b)$, and at the beginning of the third passage, when $r > 4.236$ (see [6]). For our example, $r = 19.38$, and amplification at the beginning of the third passage is $[2r/(1+r)]^2 = 3.62$ (see Fig. 7a for small α). Therefore, for long pulses ($\eta \le 0.5$) the first permanent strain can occur later. This means that in addition to conditions (11B) and (11T) there is one further condition for the occurrence of permanent strain, which is valid only for $\eta < 0.5$—namely,

$$\alpha\eta > \frac{1}{\pi k_t(2+|k_r|)} = \frac{\beta_b\rho_b + \beta_s\rho_s}{2\pi\beta_s\rho_s(2+|k_r|)} = C_B/[2-(1-r)/(1+r)], \quad \text{when } 1 < r < 4.236, \tag{11B2a}$$

or

$$\alpha\eta > C_B[(1+r)/(2r)]^2 \ (= 0.04627 \text{ for our example}), \quad \text{when } r > 4.236. \tag{11B2b}$$

In the following, we consider the maximum strain in the building, $\varepsilon_{\max}$. To represent it in dimensionless terms, we consider $v_{lin} = v_{entr}{}^{lin}$, which is the maximum velocity entering the building, supposing that it is linear. The v_{lin} is a linear function of η, which follows from

$$v_{lin} = \pi A k_t/t_d = \pi\alpha\eta\varepsilon_{yb}\beta_b k_t. \tag{12}$$

Then, instead of describing the absolute maximum of the strain, we will consider the normalized maximum strain

$$\varepsilon^{\max}_{norm} = \varepsilon_{\max}\beta_b/v_{lin} = \varepsilon_{\max}/\varepsilon_{lin}. \tag{13}$$

This quantity will show the degree of nonlinearity in the building response and the effects of the interference on the amplification of the linear entry strain. This strain is always larger than one.

We will also describe the normalized strain at the end of the analysis, in terms of the ratio

$$\varepsilon^{end}_{norm} = \varepsilon_{end}\beta_b/v_{lin} = \varepsilon_{end}/\varepsilon_{lin}. \tag{14}$$

This quantity will show the amplitude of the permanent strain (rotation) (after all of the wave energy exits the building) relative to the linear entry strain. This strain can be larger or smaller than one, and for linear waves (when neither condition (11T) nor condition (11B2a) and (11B2b) are satisfied) it is zero.

We will also consider the maximum strain normalized by the yielding strain

$$\varepsilon^{y}_{norm} = \varepsilon_{\max}/\varepsilon_{yb}. \tag{15}$$

If this quantity at some point of the building is larger than the maximum allowed ductility $\mu = \varepsilon_{fail}/\varepsilon_{yb}$, where ε_{fail} is the largest strain that can occur in the system, the building may "collapse". For linear waves, this normalized strain is smaller than one.

In the further discussions, the region (η, x) will be divided into three zones—Zone 1: $Z_1 = \{(\eta, x)|\eta < 0.5, \forall x\}$, Zone 2: $Z_2 = \{(\eta, x)|0.5 \le \eta, x \sim 0\}$, and Zone 3: $Z_3 = \{(\eta, x)|0.5 \le \eta, x > 0\}$.

For $\alpha = 0.01$ in our example, the condition (11B2b) is not satisfied in zone Z_1, ($\eta \le 0.5$), because $\eta > C_B/\alpha(1+r/2r)^2 = 0.04627/0.01 = 4.627 > 0.5$, and so there is no permanent strain in this region. The condition (11T) gives $\eta > C_B/(2\alpha) = 0.16737/(2 \times 0.01) = 8.368 > 5$, which means that in the range $0.06 \le \eta \le 5$

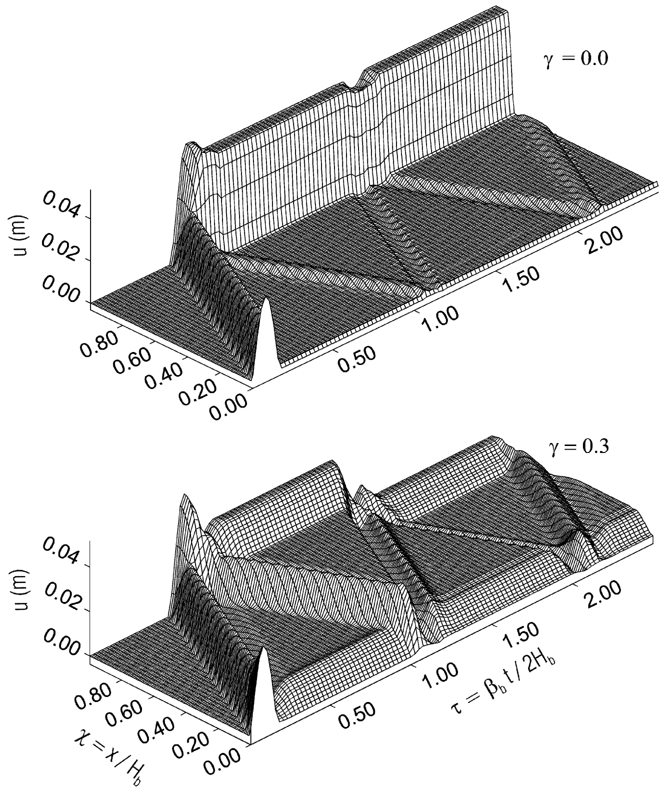

Fig. 4. Displacements in the building along the normalized length of the building χ, versus normalized time τ, for dimensionless amplitude $\alpha = 0.1$, $\eta = 3$, and for $\gamma = 0$ (top), and $\gamma = 0.3$ (bottom).

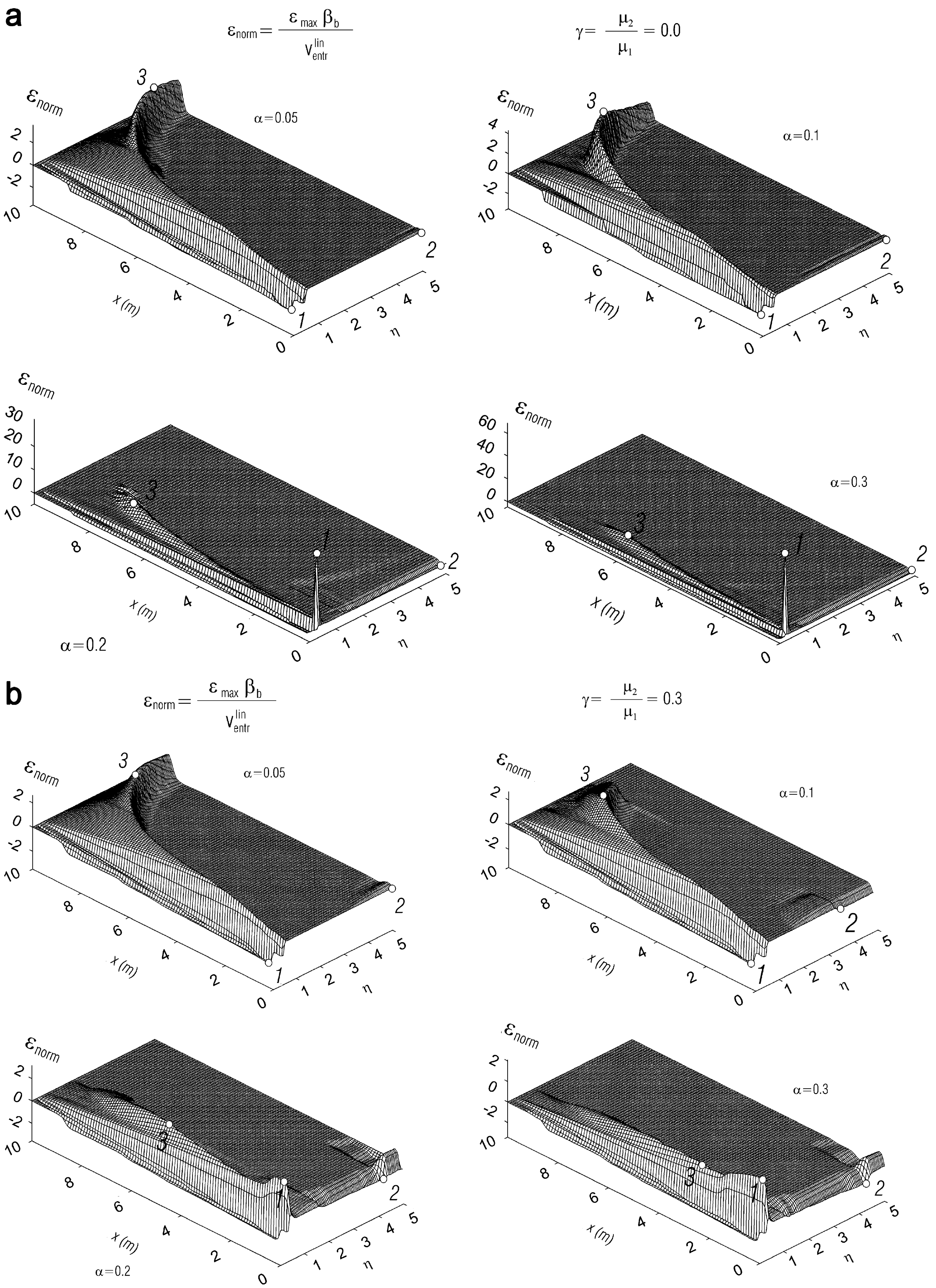

Fig. 5. (a) Normalized strains along the building, when their maxima occur, versus dimensionless frequency η, for $\gamma = 0.0$, for four dimensionless amplitudes $\alpha = 0.05$, 0.10, 0.20, and 0.30. (b) Normalized strains along the building, when their maxima occur, versus dimensionless frequency η, for $\gamma = 0.3$, for four dimensionless amplitudes $\alpha = 0.05$, 0.10, 0.20, and 0.30.

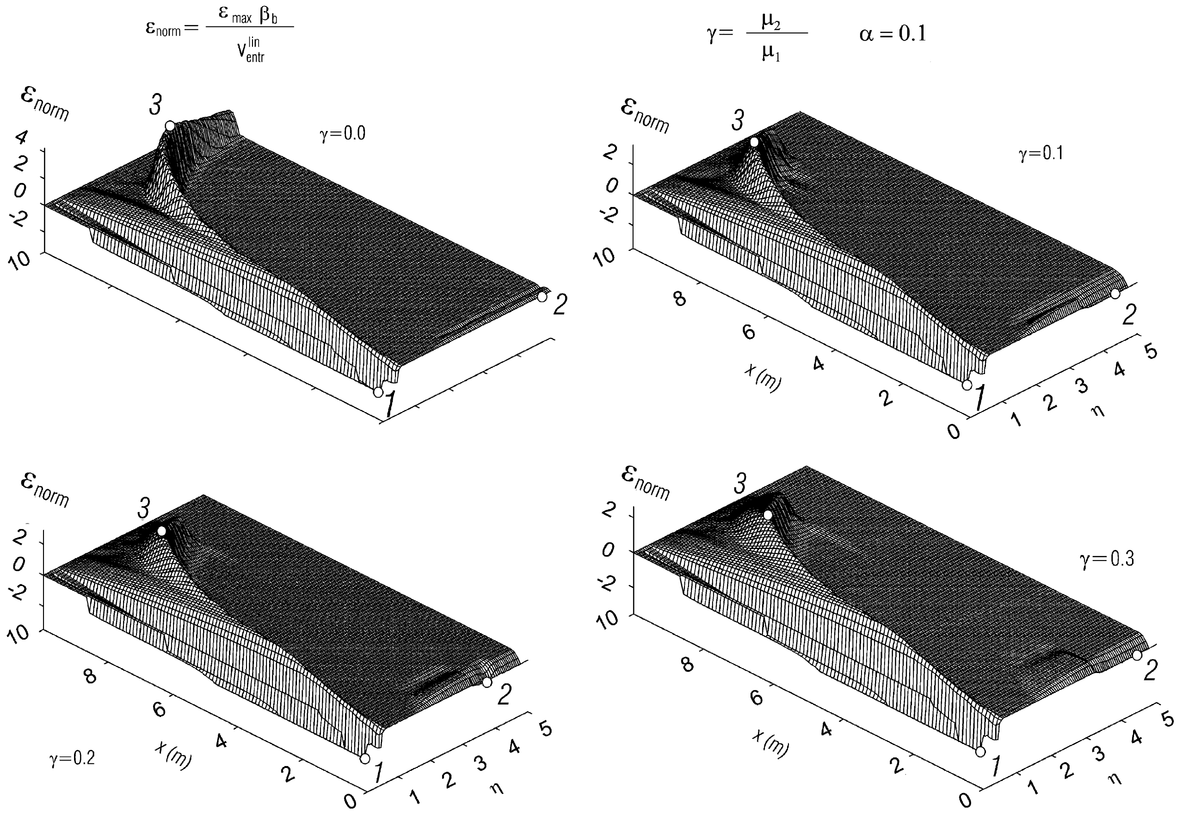

Fig. 6. Normalized peak strain along the building, when its maximum occurs, versus dimensionless frequency η, for $\gamma = 0.0$, 0.1, 0.2, and 0.3 and for dimensionless amplitude $\alpha = 0.1$.

the response of the building for $\alpha = 0.01$ is always linear. The absolute maximum occurs in zone Z_1, for η close to 0.5, at the bottom of the layer. Its value is 3.62. For $\eta > 0.5$, this normalized strain is constant and equal to 2.

To illustrate the dependence of the strain (rotations) on γ, we show in Fig. 4 the propagation of the displacement along the dimensionless length of the building, $\chi = x/H_b$, in dimensionless time, $\tau = \beta_b t/(2H_b)$, and show the first two-and-a-half passages of the wave on the path bottom–top–bottom of the building. The dimensionless amplitude for this example is $\alpha = 0.1$, and the dimensionless frequency is $\eta = 3$. At the top of this figure, the motion is shown for elasto-plastic material, $\gamma = 0.0$, and at the bottom for material with $\gamma = 0.3$. Comparing the top and bottom plots, it is seen that increasing γ reduces the strain. Also, large strains for $\gamma = 0.0$ are localized, while for $\gamma = 0.3$ the nonlinear zone is wide, which results from spreading of the zone with strain localization when γ is greater than zero. This is because the permanent strain for the modulus $\mu_1 = \gamma\mu_0 > 0$ (Fig. 2) can propagate with velocity $\beta_{b1} = \sqrt{\gamma}\,\beta_b$. For elasto-plastic material ($\gamma = 0$), the permanent strain does not propagate but is accumulated in narrow zones of the building, and the value of the strain at the point changes only when the pulse occupies the point. When $\gamma > 0$, with propagation of the permanent strain, there is less accumulation of large strains at a point with each passage of the wave through that point. Thus, as γ increases, the strain ordinates decrease, and the zones of the permanent strain become wider.

In Fig. 4, the nonlinear strain occurs at the beginning of the wave entering the building. After reflecting from the top, almost the whole energy of the input wave is spent for developing permanent strain in narrow zones, and only elastic strain continues to propagate along the building. For the material with $\gamma = 0.3$, with the development of permanent strain at the top, this strain does not remain at the point but propagates with velocity β_{b1}, which causes a substantial amount of energy to continue to spread out, up and down the building.

Fig. 5 illustrates the space–frequency (x–η) dependence of normalized peak strain $\varepsilon_{max}\beta_b/v_{entr}{}^{lin}$ for four normalized excitation amplitudes ($\alpha = 0.05$, 0.1, 0.2, and 0.3), for $\gamma = 0.0$ (Fig. 5a) and 0.3 (Fig. 5b). The normalized strain along the building is plotted at the instant when the absolute maximum of the normalized strain occurs for a given frequency η. This is done for all frequencies considered in this work. Fig. 6 illustrates the dependence of $\varepsilon_{max}\beta_b/v_{entr}^{lin}$ on γ, for $\gamma = 0.0$, 0.1, 0.2, and 0.3, for the normalized amplitude of excitation $\alpha = 0.1$. Figs. 5a and b and 6 also show the peak amplitudes in Zones 1, 2, and 3, respectively.

The normalized strains versus dimensionless amplitude α are illustrated in the subsequent figures for the four values of $\gamma = 0.0$, 0.1, 0.2, and 0.3, in the three zones Z_1, Z_2, and Z_3, using a semi-logarithmic scale. From condition (11B2b), the first nonlinear strain in Zone 1 occurs for

$$\alpha_1 > 0.04627/\eta_{max} = 0.04627/0.5 = 0.09254. \tag{16}$$

As can be seen, in Zone 1 (Fig. 7a), while the strain is linear ($\alpha \le \alpha_1$), there is no dependence upon γ. The normalized maximum strain, ε_{norm}^{max} (see Eq. (13)), is constant, and its value is 3.62, corresponding to the summation of the three strains at the beginning of the third passage. Because the response is linear, in this interval ε_{norm}^{max} shows only the effect of the interference on amplification of the linear entry strain. The normalized end strain, ε_{norm}^{end}, in this interval is zero, showing that the strains are reversible and that the resistance capacity of the building for some future excitation is not diminished. The normalized strain, $\varepsilon_{norm}^{y} = \varepsilon_{max}/\varepsilon_{yb}$, approaches zero as α approaches zero. With

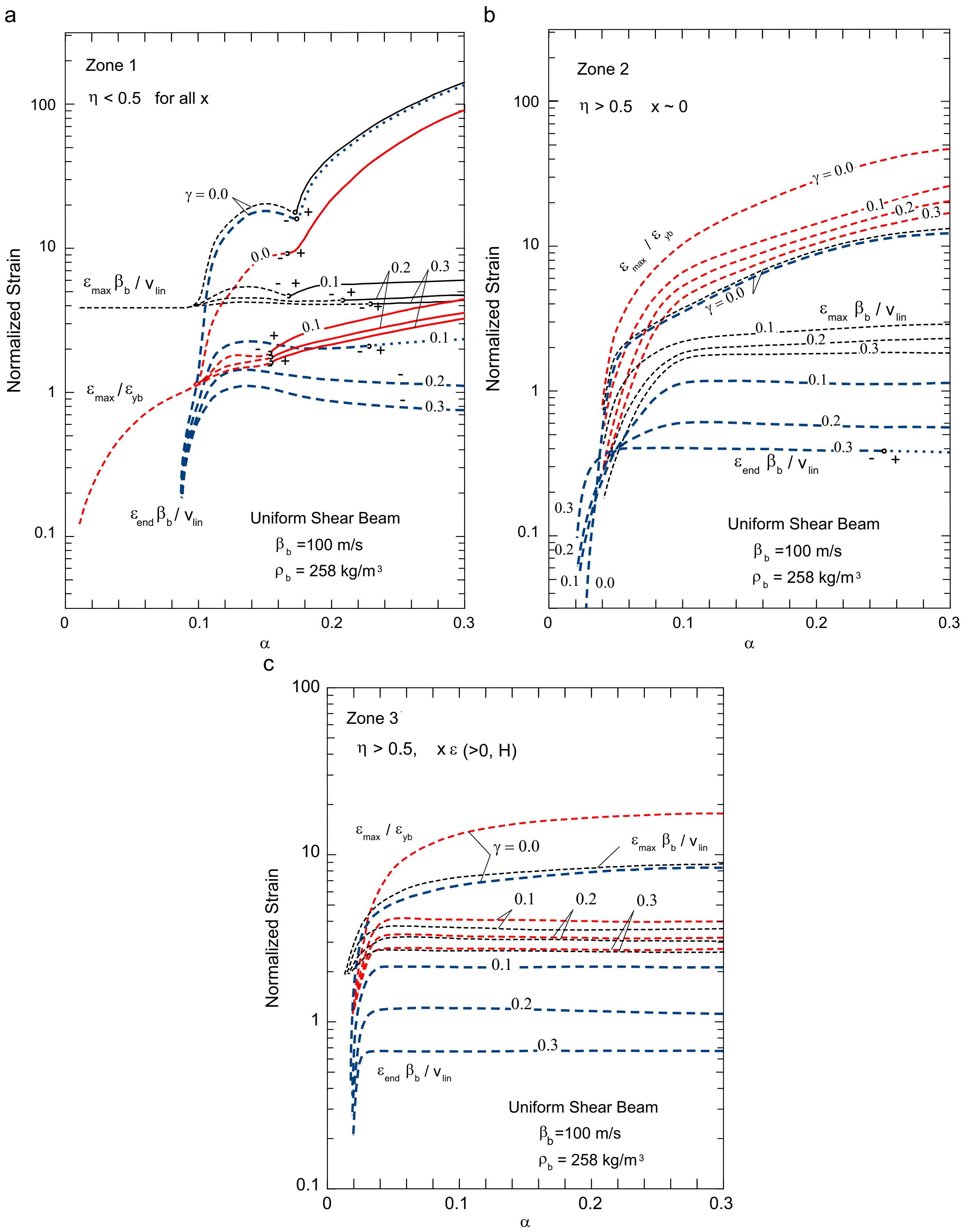

Fig. 7. (a) Dependence of the normalized strains on the dimensionless amplitude, α, in Zone 1, for four different values of $\gamma = 0.0$, 0.1, 0.2, and 0.3. (b) Dependence of the normalized strains on the dimensionless amplitude, α, in Zone 2, for four different values of $\gamma = 0.0$, 0.1, 0.2, and 0.3. (c) Dependence of the normalized strains on the dimensionless amplitude, α, in Zone 3, for four different values of $\gamma = 0.0$, 0.1, 0.2, and 0.3.

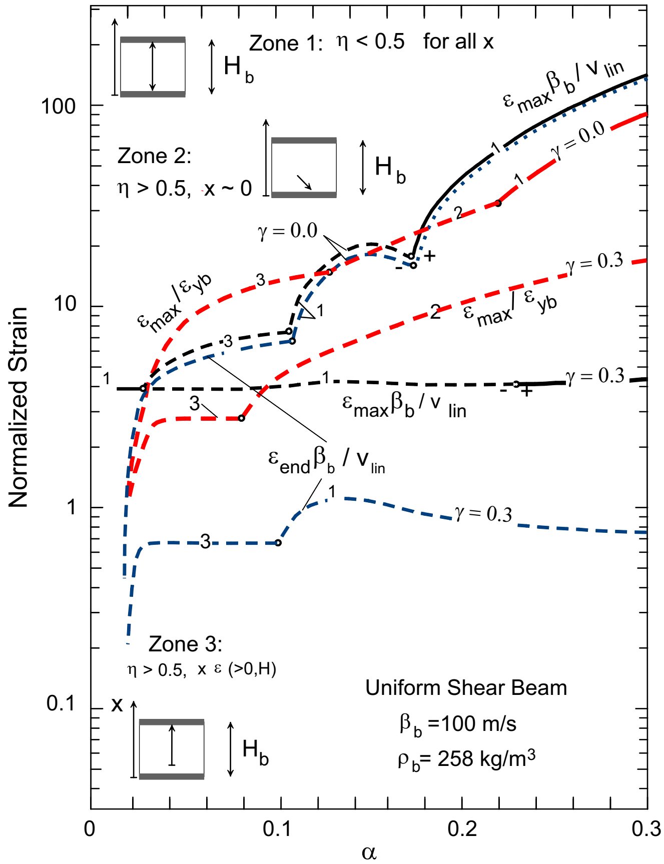

Fig. 8. Dependence of the largest normalized strains versus the dimensionless pulse amplitude, α, showing the zones where the largest peak occurs, for $\gamma = 0.0$ and 0.3.

increasing α, beyond α_1, the response at the bottom of the building becomes nonlinear at the beginning of the third passage, the curves for different values of γ separate, and the normalized strains increase with decreasing γ, being the largest for elasto-plastic material, $\gamma = 0$. The normalized strain ε_{norm}^{max} reaches its maximum values for the largest α shown in the plots, for elasto-plastic material. In this case, the effect of the nonlinearity of the building response dominates the effect of constructive interference of the three strains at the bottom. The remaining normalized permanent strain at the end, after the wave exits the building completely, ε_{norm}^{end}, has almost the same amplitude for the elasto-plastic material, and the normalized strain, ε_{norm}^{y}, is also larger, indicating that the building will fail at large values of α.

In Fig. 7b, the normalized strains versus α in Zone 2 are shown. It is seen that for small amplitudes ($\alpha \le 0.05$) all of the curves converge to zero. As α increases, all normalized strains increase. For α larger than 0.1, the dependence of all of the normalized strains on α for elasto-plastic material, $\gamma = 0$, in this scale resemble a logarithmic function, which means that the normalized strains for elasto-plastic material are approximately linear functions of α. For materials with $\gamma \neq 0$, the normalized strains ε_{norm}^{end} seem to become independent of α, while the normalized strains ε_{norm}^{max} and ε_{norm}^{y} are approximately linear, with the slope $\partial\varepsilon_{norm}^{max}/\partial\alpha$, which is smaller than the slope $\partial\varepsilon_{norm}^{y}/\partial\alpha$.

In Fig. 7c, the normalized strains versus α in Zone 3 are shown. For any γ, the normalized strains ε_{norm}^{max} and ε_{norm}^{y} approach 2 and 0, respectively, as α approaches 0. The normalized strain ε_{norm}^{end} depends only upon γ, and for the range of values of α we considered it is independent of α. The lowest excitation amplitude, when the permanent strain occurs, can be found from condition (11T) at the highest considered η as $\alpha_{min} > 0.083685/5 = 0.016737$. For $\alpha \le \alpha_{min}$, $\varepsilon_{norm}^{end} = 0$.

Fig. 8 shows the zones of the building in which the maximum strains (rotations) occur, versus the dimensionless amplitude of the strong-motion pulse α, in the range from 0.0 to 0.3, and for $\gamma = 0.0$ and 0.3. For all of the normalized strains we examined in this paper ($\varepsilon_{max}\beta_b/v_{lin}$, $\varepsilon_{end}\beta_b/v_{lin}$, and $\varepsilon_{max}/\varepsilon_{yb}$), and for α increasing from 0, essentially all peaks first occur in Zone 3 of the building. There is only one exception to this for $\varepsilon_{max}\beta_b/v_{lin}$, for small values of α, for which the response is linear and for which the peak of this strain occurs in Zone 1. For $\varepsilon_{max}/\varepsilon_{yb}$, and for $\gamma = 0.0$ beyond $\alpha \sim 0.13$, the peak rotations begin to occur in Zone 2. For $\gamma = 0.3$ beyond $\alpha \sim 0.08$, the peak strains $\varepsilon_{max}/\varepsilon_{yb}$ begin to occur in Zone 2.

For the largest normalized pulse amplitudes we considered in this work, $\alpha \sim 0.3$, and for ground motions close to the earthquake fault [13], such that $v_{lin} \sim 100$ cm/s, for example, the results in Fig. 8 imply permanent shearing of the affected parts of the building, which would lead to permanent tilting of the segments along the height of the building, in the range from $\varepsilon_{max} = 4.3\, v_{lin}/\beta_b = 0.043$ rad$\sim 2.5^\circ$ (when $\gamma = 0.3$) to $\sim 60^\circ$ (when $\gamma = 0.0$). The later result for $\gamma = 0.0$ exceeds the range of the response amplitudes ($\varepsilon \sim \tan\varepsilon$) that can be calculated reliably with our model, but it shows nevertheless that the drifts can become very large. The adverse consequences of such large drifts and of the associated shearing deformations in buildings are obvious [2].

4. Discussion and conclusions

We described the energy flow into a shear building with constant material properties, with the objective of contributing to the elementary understanding of the resulting response for excitation by powerful transient pulses. Starting with a linear strong-motion pulse in the half-space and ending with nonlinear waves propagating through a building, we identified the peak transient and permanent strains (rotations) occurring during various stages of response.

For incident short pulses ($\eta > 0.5$), which lead to linear response of the building, the amplification of a pulse, with normalized pulse amplitude α ($= A/(H_b\varepsilon_{yb})$, and represents a ratio of the average drift in the building A/H_b and the yielding strain in the building material ε_{yb}), is equal to 2. It results from interference of the up-propagating wave with the wave reflected from the free top surface and propagating down. For long pulses ($\eta \le 0.5$), the amplification depends upon the impedance ratio between the half-space and the building material, and on the duration of the pulse. It can occur during first-, second-, or higher-order passes of the wave up and down the building. In the example presented in this paper it occurs at the beginning of the third pass, and the amplification is 3.62.

For large values of α, which lead to nonlinear wave motion in the building, amplification of peak strains (rotations) is a strong function of the second slope, γ, in the bi-linear representation of the stress–strain relationship of the building material. For $\gamma = 0.0$, the amplification of all peak rotations and strains grows rapidly with α, while for $\gamma = 0.3$ and larger it is only slightly above the amplification for linear wave motion.

With increasing α and the first appearance of the nonlinear response, the maxima of all normalized strains $\varepsilon_{max}\beta_b/v_{lin}$, $\varepsilon_{end}\beta_b/v_{lin}$, and $\varepsilon_{max}/\varepsilon_{yb}$, first occur in Zone 3 of the building ($\eta > 0.5$, and $0 < x \le H_b$) (Fig. 8). This corresponds to interference of up- and down-propagating short waves after reflection from the top of the building, and it occurs for α less than about 0.08 to 0.12. Beyond $\alpha \sim 0.13$, for $\gamma = 0.0$, and $\alpha \sim 0.08$ for $\gamma = 0.3$, the peaks of $\varepsilon_{max}/\varepsilon_{yb}$ occur just above the interface of the building and the half-space (in Zone 2: $\eta > 0.5$, and $x \sim 0$). The peaks of $\varepsilon_{max}\beta_b/v_{lin}$ occur in Zone

1 ($\eta \leq 0.5$, and $\forall x$) beyond $\alpha \sim 0.11$ for $\gamma = 0.0$, and again in Zone 1 ($\eta \leq 0.5$, and $\forall x$) for $\gamma = 0.3$ and for all α considered in this work.

The above description of the peak rotations and peak strains in nonlinear response is for the example of the shear building in this paper only. For buildings with constant shear-wave velocities, densities, and ε_{yb} different than those considered in our example, the scales of the coordinate axes in Fig. 7a–c and 8 will "shrink" or "extend", but the overall nature of the results will remain similar. For buildings with variable shear-wave velocities and densities, the general appearance of the peaks of $\varepsilon_{\max}\beta_b/v_{lin}$, $\varepsilon_{end}\beta_b/v_{lin}$, and $\varepsilon_{\max}/\varepsilon_{yb}$ will also remain similar, but they will include additional complexities, which will result from reflection and refraction from the impedance jumps caused by changes in the shear-wave velocity along the wave path (building height). Some aspects of those complexities can be seen in the related analyses of the nonlinear waves in a seven-story hotel building in Van Nuys, California, which was damaged by the 1994 Northridge earthquake [3].

The strain localization, which, as the above examples show, can occur almost anywhere in the building, depending upon the α and the η of the pulses, implies that failure can be initiated at any height of the building. Thus, the final outcome will always depend upon the nature of the excitation and upon how many energetic pulses, and of what sizes, are present in the train of strong ground motion [12].

A review of Fig. 7a–c suggests that the normalized peak strains (rotations), $\varepsilon_{\max}\beta_b/v_{lin}$, $\varepsilon_{end}\beta_b/v_{lin}$, and $\varepsilon_{\max}/\varepsilon_{yb}$, respectively, converge to an asymptote or to a monotonically increasing trend when $\alpha \to \infty$. However, this is not so. First, positive and negative peak strains dominate the maxima in a manner that is not simple. In Zones 1 and 3, and for the range of α we considered, most maxima are negative, but in general positive and negative peaks appear and disappear in a manner that is not simple and recognizable. Second, with increasing α, beyond $\sim$0.3 in our examples, what at first appears as a monotonic trend begins to fluctuate, and in some instances the peaks disappear. Our numerical algorithm has been formulated to work with small strains when $\varepsilon \sim \tan \varepsilon$, and so we cannot obtain reliable results for large nonlinear deformations. Furthermore, the differential equations we chose to describe one-dimensional shear waves are also linear and do not include higher-order terms associated with geometric nonlinearities, gravity effects, and dynamic instabilities. Yet, the nature of the problem we study is characterized by large nonlinearities, and therefore for large α it will display the characteristics of chaotic response [14].

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