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# A note on strength-reduction factors for design of structures near earthquake faults

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## Abstract

Strength-reduction factors are analyzed for simplified near-fault, fault-normal and fault-parallel strong-motion displacements. It is shown that the common design rules for selection of the strength-reduction factors are conservative and properly describe the reduction amplitudes near faults of strong earthquakes, for fault-normal pulses. However, for fault-parallel displacements, the same reduction factors are not conservative and should be changed. It is recommended that for design close to active faults, the strength-reduction factors for all components of motion should be constant for all periods and equal to  $(2\mu-1)^{1/2}$ , where  $\mu$  is ductility. © 2007 Elsevier Ltd. All rights reserved.

**Keywords:** Strength-reduction factors; Non-linear response; Earthquake motion near faults; Design of structures

## 1. Introduction

Engineering design of earthquake-resistant structures usually begins with pushover analyses (FEMA [1,2]; ATC [3]), which are scaled by target displacements determined from the inelastic response of the corresponding single-degree-of-freedom system (SDOF). Since the design ground motions are usually specified in terms of the response spectra of linear oscillators, this requires that the relationship between the peak relative displacement response,  $u_0$ , of the linear SDOF system and of its inelastic counterpart,  $u_m$ , be properly specified for the required design parameters and environment. By defining the yield strength-reduction factor as  $R_y = u_0/u_y$ , where  $u_y$  is the yielding displacement of the SDOF system equivalent spring, and ductility as  $\mu = u_m/u_y$ , for the same ground motion, the ratio  $u_m/u_0$  is then equal to  $\mu/R_y$ .

The relationship between the maxima of non-linear and linear responses of SDOF systems,  $u_m$  and  $u_0$  (Fig. 1a), for the same excitation, has been studied extensively during the past 40 years. In one of the first studies by Veletsos and Newmark

[4] it was shown that for a long-period SDOF system, when its natural period  $T_n \rightarrow \infty$ ,  $u_m/u_0 \rightarrow 1$  (equal deformation rule), and that for the response amplitudes governed mainly by the peak excitation velocities,  $u_m/u_0 \rightarrow \mu/\sqrt{2\mu-1}$  (equal strain energy rule). With the gradual increase in the number of the recorded strong-motion accelerograms [5] these rules were further refined to reflect the trends observed in the responses to the recorded data [6–9] for different site conditions and ductility factors [10,11], for rupture distance and the earthquake magnitude [12], and for fault-normal near-field records in the zone affected by directivity [13,14]. For reinforced concrete structures, the range of  $T_n$ , where equal deformation rule applies was described by Shimazaki and Sozen [15], and it was suggested by Ruiz-Garcia and Miranda [16] that the average value of the ratio  $u_m/u_0$  is not much influenced by the recording site classification, by earthquake magnitude, when  $u_0/u_y < 4$  ( $u_y$  is yield deformation), or by rupture distance. Other studies have shown that  $u_m/u_0$  for the high frequencies of the response (acceleration-sensitive region) are reduced by the postyield stiffness [17,18] and increased by stiffness degradation [18–20] and pinching of the hysteresis loop [21,22].

The weakness of many of the above-mentioned studies is that in a given paper only one or two governing parameters

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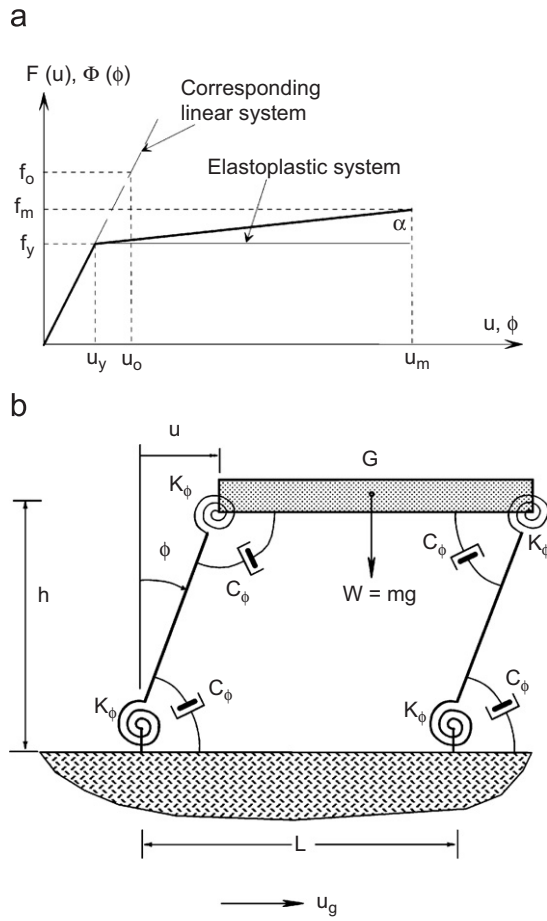


Fig. 1. (top) Force–displacement (moment–rotation) relationship for bilinear spring. (bottom) Relative response,  $u$ , of the single-degree-of-freedom system excited by motion of rigid base,  $u_g$ .

are considered, with de facto averaging over all other parameters. Furthermore, the data set considered in most studies is not large enough to detect all significant dependencies with confidence. The exceptions to this trend can be found in two papers, one by Tiwari and Gupta [23] and the other by Chakraborti and Gupta [24]. Both papers present comprehensive regression models based on large data sets, and show the clear dependence of the strength-reduction factors on magnitude predominant period, duration of strong motion, and geologic site conditions.

In the near field of large earthquakes, and especially close to surface rupturing faults, the strong-ground motion can be dominated by the permanent displacements (typically parallel to the fault surface) and by large pulses (often perpendicular to the fault). Traces of these large displacements and pulses may not always be obvious in the processed records of the recorded motions because of the band-pass filtering, designed to eliminate digitization and processing noise [25–28]. This processing filters out the long-period ground displacements, and thus eliminates all permanent displacement components.

The purpose of this paper is to show how the strength-reduction factors depend on the amplitudes and duration of the near-fault pulses and displacements, in the condi-

tions when these dominate the near-fault ground motion. The results should help in the selection of the correct design criteria for structures located in the vicinity of active faults and in the development of the regression equations for the strength-reduction factors when the distance between the site and the fault surface approaches zero. In general, the strength-reduction factors depend on the time history and the transient nature of the excitation, on the linear and non-linear characteristics of the SDOF system adopted for the analysis, and on the initial conditions. Since the recorded strong-motion data at or very close to the causative fault continue to be limited, the results of this study should help us understand how the known results for intermediate and far-field records of strong motion would change when extrapolated toward the fault rupture.

## 2. Near-source ground motion

Strong-ground motion near faults can be complicated due to irregular distribution of fault slip [25,29,30] because of non-uniform distribution of geologic rigidities surrounding the fault, non-uniform distribution of stress on the fault, and complex non-linear processes that accompany the faulting. Thus, in general, it is not possible to predict the detailed nature of the near-fault ground motion and of the associated pulses. In this paper we adopt a simplified approach and model these motions by smooth pulses, which have correct average amplitudes and duration, and which have been compared to and calibrated against the observed fault slip and the recorded strong motions in terms of their peak amplitudes in time and their spectral content [31,32].

Fig. 2 shows schematically a fault and two characteristic simple motions,  $d_N$  (Brune's near-field displacement) and  $d_F$  (Brune's far-field pulse), which describe monotonic growth of the displacement toward the permanent static offset, and a pulse, which near faults is usually perpendicular to the fault and associated with failure of a nearby asperity or passage of dislocation under or past the observation point [33].

For excitation by a pulse, we chose the Brune's "far-field" pulse (Fig. 2-center)

$$d_F(t) = A_F t e^{-\alpha_F t}, \quad (1)$$

where the values of  $A_F$  and  $\alpha_F$ , for different earthquake magnitudes, are shown in Table 1 [31]. Since the strong-motion data are abundant only up to about  $M = 6.5$ , we place the values of  $\alpha_F$  and  $A_F$ , for  $M = 7$  and 8, in Tables 1 and 2 in parentheses to emphasize that those are based on extrapolation. For the near-field permanent displacement, we consider the Brune's displacement (Fig. 2-bottom)

$$d_N(t) = \frac{A_N}{2} (1 - e^{-(t/\tau_N)}), \quad (2)$$

where the values of  $A_N$  and  $\tau_N$ , for different earthquake magnitudes, are shown in Table 2 [31].

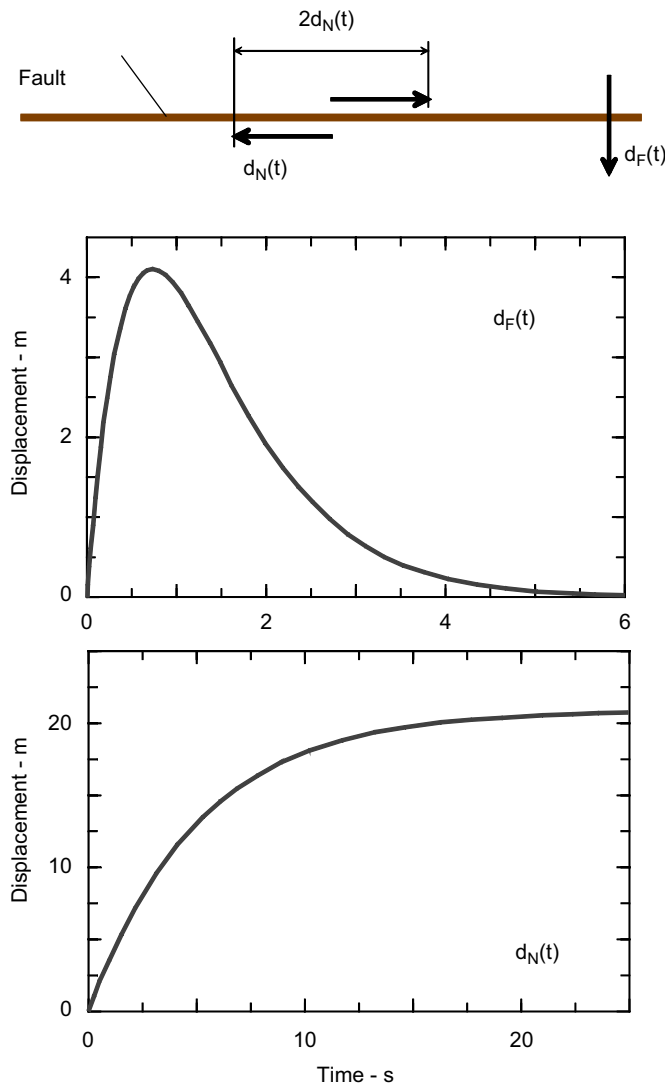


Fig. 2. Fault-parallel,  $d_N(t)$ , and fault-normal,  $d_F(t)$ , displacements.

Table 1  
Characteristics of Brune's pulse displacement (Trifunac [31])

M (magnitude)	$\alpha_F$ (1/s)	$A_F$ (cm/s)	$d_{F,max}$ (cm)	$\dot{d}_{F,max}$ (cm/s)
4	14.04	56.48	1.48	56.48
5	7.90	151.61	7.06	151.61
6	4.44	546.97	45.32	546.97
7	(2.50)	(860.34)	(126.6)	(860.34)
8	(1.40)	(1560.29)	(410.0)	(1560.29)

Table 2  
Characteristics of Brune's near-field displacement (Trifunac [31])

M (magnitude)	$\tau_N$ (s)	$A_N$ (cm)	$d_{N,max}$ (cm)	$\dot{d}_{N,max}$ (cm/s)
4	0.55	4.9	2.45	4.45
5	1.2	29.2	14.6	12.17
6	1.8	245.5	122.75	68.19
7	(3.0)	(1288.0)	(644.0)	(214.7)
8	(5.0)	(4169.0)	(2084.5)	(416.9)

The amplitudes of  $d_F$  and  $d_N$  have been studied in numerous regression analyses of recorded peak displacements at various distances from the fault and in terms of the observed surface expressions of fault slip. The latter are traditionally presented as average dislocation amplitudes,  $\bar{u}$ , and are related to  $d_N$ , as  $\bar{u} = 2d_N$  (see Fig. 2 top). Fig. 3 (bottom shaded zone) shows, for example, the range of average ( $p = 0.5$ , where  $p$  is the probability of no exceedance) peak ground displacements,  $d_{max}$ , versus magnitude, calculated at epicentral distance  $R = 0$  km for the recording site conditions in the range from sediments ( $s = 0$ ), to the basement rock ( $s = 2$ ) [34]. The top gray zone shows the corresponding range for the 10% probability of being exceeded. The same figure also shows  $d_{F,max}$  (see Table 1), the peak amplitudes of  $d_F$  used in this study. It is seen that the amplitudes of  $d_{F,max}$  shown in Table 1 are consistent with extrapolations from the recorded strong-ground motion in the far field, and are close to  $d_{max}$  for  $p = 0.5$  and  $s = 0$ , and  $p = 0.9$  and  $s = 2$ .

Fig. 4 summarizes the trends of average dislocation amplitudes,  $\bar{u} = 2d_N$ , versus magnitude  $M$ . Average dislocation is the value of dislocation amplitudes averaged over the fault surface and is the quantity used in spectral interpretations of near-fault and near-field motions and of the body wave amplitudes in the far field. Various symbols show the results extracted from the studies of selected earthquakes [35,36,59], while the two gray zones outline the

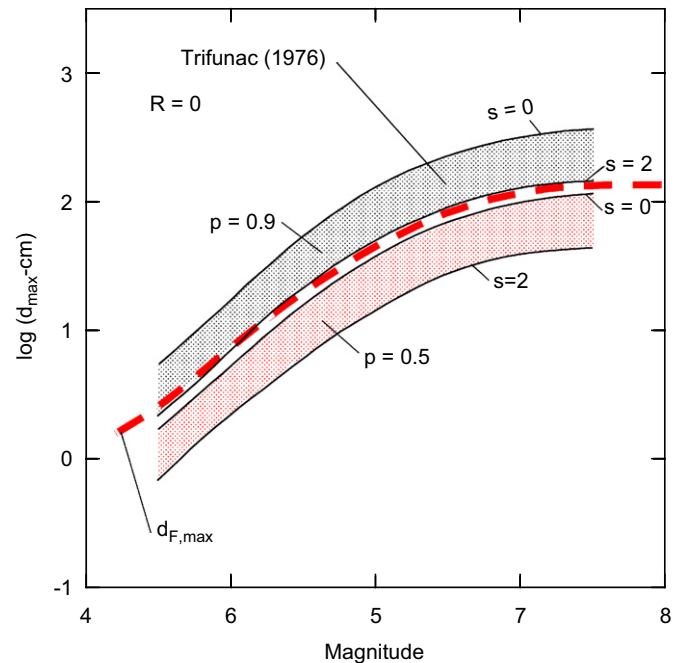


Fig. 3. Range of peak ground displacements,  $d_{max}$ , determined by regression analysis of recorded strong-ground motion. The dashed line shows  $d_{F,max}$  (see Table 1). The bottom shaded zone gives the average ( $p = 0.5$ ), while the top gray zone gives  $d_{F,max}$  with amplitudes having 10% probability of being exceeded ( $p = 0.9$ ). The width of the shaded zones describes variations associated with geological site conditions. The bottom of the zones is for  $s = 2$  (basement rock), while the top is for  $s = 0$  (sediments).

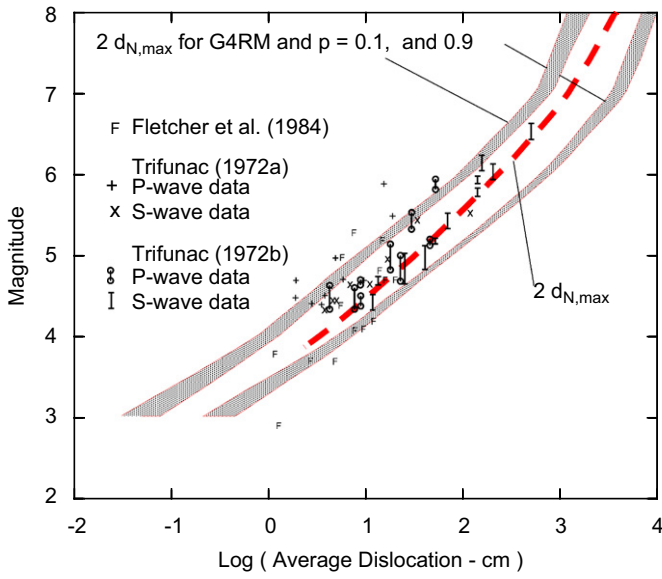


Fig. 4. Comparison of the distribution of the average dislocation amplitudes on the fault,  $\bar{u} = 2d_{N,max}$ , computed for the group of four regression models (G4RM), determined from the recorded strong-motion amplitudes (gray zones for  $p = 0.1$  and  $0.9$ ), with the amplitudes of  $d_{N,max}$  (Table 2), adopted for scaling  $d_N(t)$  in this paper (dashed line).

80% confidence interval (bounded by  $p = 0.1$  and  $0.9$ ) for the amplitudes of  $\bar{u} = 2d_N$  based on four regression models (G4RM), which describe attenuation of strong-motion amplitudes [31,37–40]. The dashed line shows the amplitudes of  $2d_{N,max}$  as used in this paper (see Table 2). It is seen that the agreement is excellent.

An important physical property of the  $d_F$  and  $d_N$  functions, as used in this paper, is their initial velocity. It can be shown that  $\dot{d}_F \sim \sigma\beta/\mu_s$ , where  $\sigma$  is the effective stress ( $\sim$  stress drop) on the fault surface [31,39],  $\beta$  is the velocity of shear waves in the fault zone, and  $\mu_s$  is the rigidity of rocks surrounding the fault. For  $\dot{d}_N$  it can be shown that  $\dot{d}_F = 0.5C_0\sigma\beta/\mu_s$ , at  $t = 0$ , where typical values of  $C_0$  are 0.6, 0.65, 1.00, 1.52, and 1.52 for  $M = 4, 5, 6, 7$ , and  $8$  [31,39]. The largest peak velocities observed so far, are in the range of and exceed 200 cm/s (170 cm/s, 5–10 km above the fault of Northridge, California earthquake of 1994 [41], and 229 cm/s at station TCU068, near the end of surface expression of Che-lungpu fault, during Chi-Chi, Taiwan earthquake of 1999 [42]). Because there are no strong-motion measurements of peak ground velocity at the fault surface, the peak velocities  $\dot{d}_F$  and  $\dot{d}_N$  can be evaluated only indirectly in terms of  $\sigma$ . The accuracy of the stress estimates depends upon the assumptions and methods used in interpretation of recorded strong-motion records and is typically about one order of magnitude. Therefore, solving the above equations for  $\sigma$  we can use  $\sigma \sim 2\mu_s\dot{d}_N/(\beta C_0)$  (dotted lines in Fig. 5) and  $\sigma \sim \mu_s\dot{d}_F/\beta$  (continuous lines in Fig. 5), to check their consistency with other published estimates of  $\sigma$  [35,36,59]. Fig. 5 show this comparison for typical values of  $\mu_s$  and  $\beta$ .

Since the dependence of both  $u_m/u_0$  and the yield strength-reduction factor  $R_y = u_0/u_y$ , versus period  $T_n$ ,

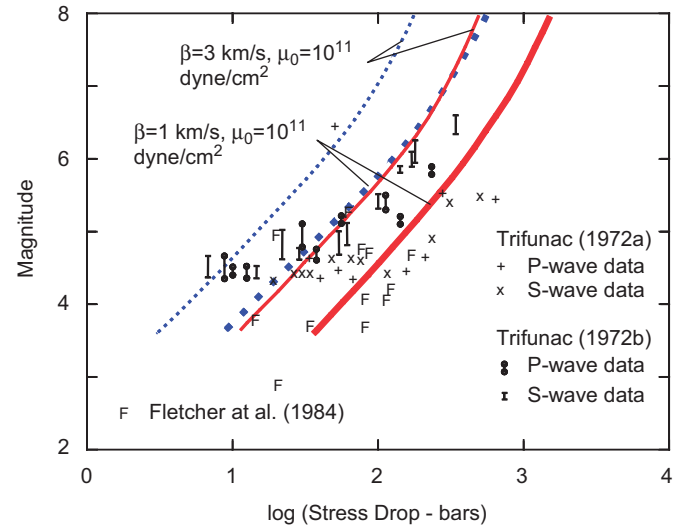


Fig. 5. Estimates of stress drop, from near-source recordings of strong motion, with the stress drop associated with  $\dot{d}_F$  (solid lines, Table 1) and  $\dot{d}_N$  (dotted lines, Table 2) used in this paper.

are both functions of excitation and of the corresponding spectral amplitudes for the linear response, we next describe the PSV spectral amplitudes for  $d_F(t)$  and  $d_N(t)$  motions at the fault. Figs. 6a and b show the linear pseudo relative velocity (PSV) spectra computed for ground motions represented by  $d_F$  (Fig. 6a) and  $d_N$  (Fig. 6b), for magnitudes  $M = 4, 5, 6, 7$ , and  $8$ , and for the fraction of critical damping  $\zeta = 0.05$ . In both figures, the role of large, non-zero initial velocity is evident. It dominates the amplitudes and the shape of PSV spectra, for both  $d_F$  and  $d_N$  excitations, with spectral amplitudes approaching constant asymptotes as  $T_n \rightarrow 0$ , whose amplitudes are equal to  $\dot{d}_F$  and  $\dot{d}_N$ , respectively. At long periods, the corresponding relative displacement spectra,  $SD = PSV/\omega_n$ , approach the asymptotes whose amplitudes are equal to the peaks of  $d_F$  and  $d_N$ ,  $d_{F,max}$ , and  $d_{N,max}$ , respectively. With increasing magnitude, the corner frequencies, which are direct functions of  $\alpha_F$  and  $\tau_N$  [31] progressively move toward longer periods. To enable a direct relative comparison of spectral amplitudes, and to emphasize how different the PSV spectral shapes are for  $d_F$  and  $d_N$  excitations, we superimpose in Figs. 6a and b the average PSV spectra estimated by regression analysis of more than 550 PSV spectral amplitudes computed from recorded accelerograms in the western US; they were recorded on sediments ( $s = 0$ ) or on geological basement rock ( $s = 2$ ), for a fraction of critical damping  $\zeta = 0.05$ , at “epicentral” distance  $R = 0$  km, and for magnitudes  $M = 4.5, 5.5, 6.5, 7.5$ , and  $8.5$  [43].

Amplitudes of PSV spectra of the linear response of SDOF systems can be viewed, and scaled, in three period ranges, where the PSV amplitudes are proportional to (1) peak ground acceleration (short periods), (2) peak ground



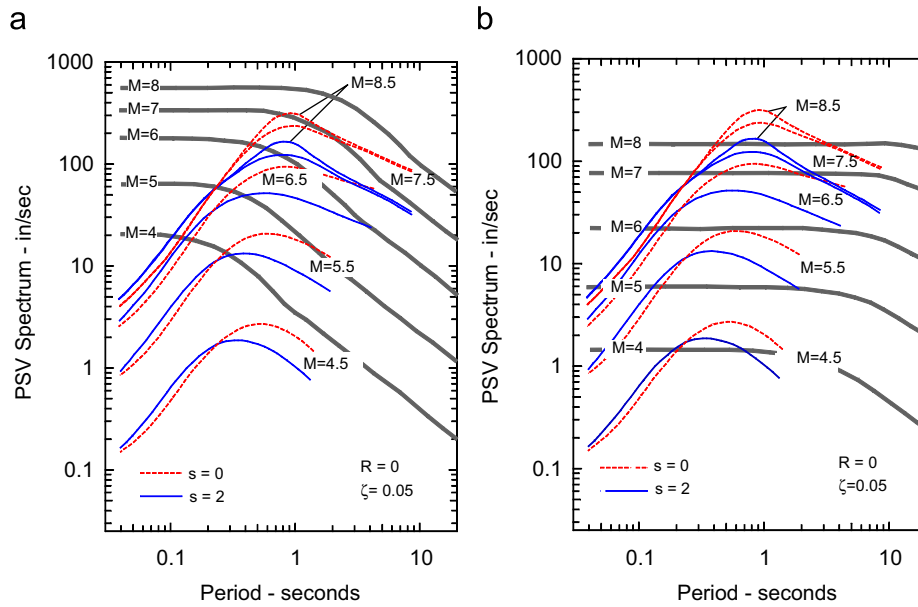


Fig. 6. (a) Comparison of PSV spectra for the pulse at the fault,  $d_F(t)$  (wide lines), with average ( $p = 0.5$ ) spectra of recorded strong-ground motion extrapolated to  $R = 0$  km, for  $\zeta = 0.05$ , and for sites on basement rock ( $s = 2$ , solid lines), and on sediments ( $s = 0$ , dashed lines) (redrawn from Trifunac, [43]). (b) Same as Fig. 6a, but for excitation by the near-field displacement  $d_N(t)$  (wide lines).

velocity (intermediate periods), and (3) peak ground displacement (long periods) [6]. As will be seen throughout this paper, when the motion begins with a sudden jump in ground velocity (caused by a sudden stress drop on the fault surface), this large initial velocity also dominates the spectral amplitudes for the short periods of the oscillator and the “acceleration-dominated” zone of PSV amplitudes disappears. This results in essentially constant PSV amplitudes in the short-period range.

We note that the effect of large initial velocities,  $\dot{d}_F$  and  $\dot{d}_N$ , on the PSV spectral amplitudes in the short-period range is reminiscent of the effects of differential motions, particularly for stiff structures, at soft soil sites, and for large plan dimensions. There, the peak strains in the soil (proportional to peak ground velocity) lead to constant PSV spectral amplitudes at short periods [44,45].

The presence of the motions resembling  $d_N$  in the recorded velocities and displacements filtered by data processing will be noticed by a trained eye in numerous plots of processed strong-motion records [46]. The frequency of the occurrence and the amplitudes of such pulses are larger for the motions recorded closer to the causative faults. For the examples of  $d_F$  and  $d_N$  in this paper there is a Dirac delta function for accelerations at time zero. In the observed motions, because of wave propagation through sediments and soil, this will correspond to large but not infinite accelerations.

Fig. 7 (top) shows one of the early examples of the ground displacement, perpendicular to the fault, recorded during the Parkfield, California earthquake of 1966 [30]. This displacement, computed by double integration from the recorded accelerogram [47] is used here to illustrate the actual near-fault “pulse-like” ground motion, which in this

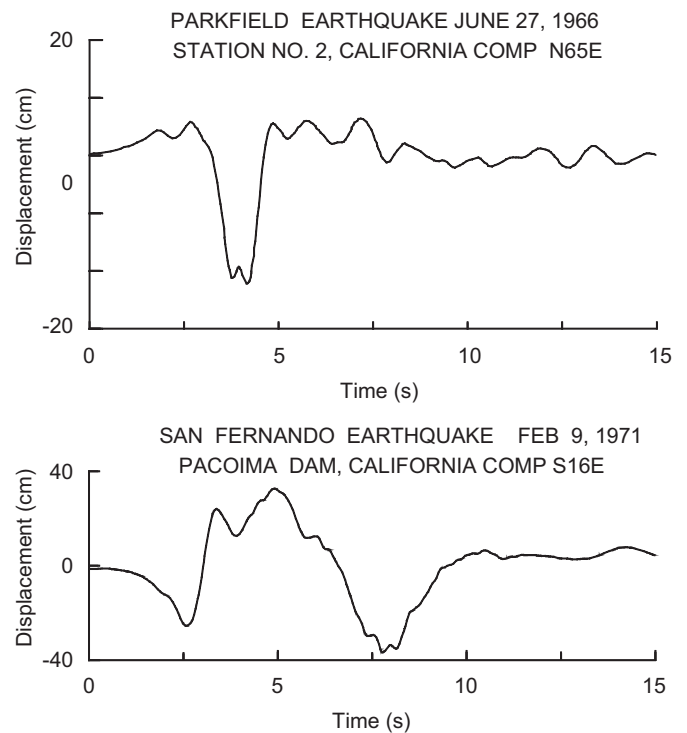


Fig. 7. (top) Ground displacement, perpendicular to the fault surface, near the end of the surface expression of a vertical strike-slip fault (redrawn from Housner and Trifunac, [47]). (bottom) Ground displacement recorded near center and above a thrust fault (Trifunac, [29]).

paper is modeled by  $d_F$  (shown in Fig. 2, middle). Fig. 7(bottom) shows the ground displacement computed during the San Fernando, California earthquake of 1971 [48]. This displacement has been high-pass filtered by the routine data processing methods [26], and therefore does

not contain periods of motion longer than 15 s. However in spite of the high-pass filtering it shows clearly two episodes of permanent ground displacements, starting near 2.5 and 7 s. Further examples of how  $d_N$  for this earthquake appeared near fault can be found in Figs. 6 and 10 of Trifunac [29], which are based on synthetic computation of the fault slip during the San Fernando earthquake of 1971.

### 3. Inelastic deformation ratio

The yield strength-reduction factor is  $R_y = f_0/f_y = u_0/u_y$ , where all the quantities are defined as in Fig. 1a. For ductility  $\mu = u_m/u_y$ , the inelastic deformation ratio  $u_m/u_0$  is then equal to  $\mu/R_y$ . The inelastic deformation ratio, for known yield strength, and same excitation  $C_{R_y} = u_m/u_0 = \mu/R_y$ , can be computed from calculated peak responses  $u_m$  and  $u_0$ . For given  $R_y$ , the associated ductility demand is then determined from  $\mu/R_y$ . Iterations are required to compute the inelastic deformation ratio,  $C_\mu = u_m/u_0$ , for specified ductility factor because different values of  $f_y$  may lead to the same  $\mu$ . Because  $C_\mu$  is not unique, the convention is to choose the largest  $f_y$  corresponding to the largest  $C_\mu$  [49].

For stiff, high-frequency oscillators, when  $T_n \rightarrow 0$ , the relative displacement of the mass,  $u$ , and its relative velocity,  $\dot{u}$ , are essentially zero. Then the absolute acceleration of the mass  $m$  is the same as the absolute acceleration of the ground  $\ddot{u}_g$ , and  $f = -m\ddot{u}_g$ , independent of the force-displacement relationship, and therefore  $f_m \approx f_0$ . From Fig. 1a it is seen that  $f_y + (\mu - 1)\alpha f_y = f_m$ , and when  $f_m \approx f_0$ ,  $1 + (\mu - 1)\alpha = R_y$ . Then, as  $T_n \rightarrow 0$ , for  $\alpha = 0$ ,  $R_y \rightarrow 1$ , and  $u_m/u_0 \rightarrow \mu$ . For  $\alpha > 0$ ,  $R_y$  is larger than one, and  $u_m/u_0 < \mu$ .

For large initial velocity of ground motion, and for the oscillator periods, for which the response amplitudes are dominated by the peak amplitudes of ground velocity, equating the kinetic and potential energies of response leads to the requirement that the linear and non-linear oscillators, at the peaks of their responses, should have the same maximum strain energy. This leads to  $f_0 u_0 / 2 = f_y u_y / 2 + (u_m - u_y) f_y + \alpha (u_m - u_y)^2 / 2$  (see Fig. 1a). For  $\alpha = 0$ , this simplifies to  $R_y = (2\mu - 1)^{1/2}$ , and for  $\mu = 1.5, 2, 4$ , and eight  $R_y = 1.41, 1.73, 2.65$ , and  $3.87$ , respectively. Then  $u_m/u_0 = \mu(2\mu - 1)^{-1/2}$ .

For long periods of the oscillator, when  $T_n \rightarrow \infty$  and its stiffness approaches zero, both  $u_m$  and  $u_0$  approach the negative of the peak ground displacement  $u_g$ , and then  $u_m/u_0 \rightarrow 1$ , and  $R_y \rightarrow \mu$ . This is known as the equal displacement rule [4].

### 4. Strength-reduction factor

Since the inelastic deformation ratio  $u_m/u_0$  is equal to  $\mu/R_y$ , for estimation of the non-linear response of an SDOF system, it suffices to describe the dependence of  $R_y$  versus  $T_n$  in terms of the parameters that characterize strong-ground motion and the physical characteristics of the structure (SDOF). The characteristics that will determine the nature of the strong-ground motion may include; for example, the size of the earthquake (usually in terms of a magnitude scale); geometry and the slip on the causative fault; the geometrical relationship between the site and the fault plane, usually expressed in terms of epicentral or some other characteristic distance; directivity; the nature of the medium between the source and the site; and the nature of the geological and soil conditions at the site. The characteristics of the SDOF system may include the nature

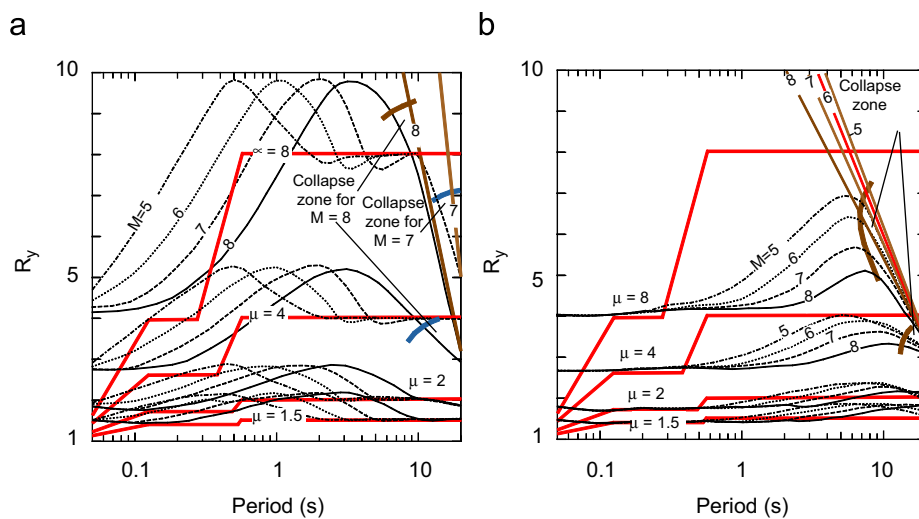


Fig. 8. (a) Comparison of strength-reduction factors computed for near-field pulse,  $d_F(t)$ , for  $M = 5, 6, 7$ , and  $8$ ,  $\zeta = 0.05$ , and  $\mu = 1.5, 2, 4$ , and  $8$  (continuous, dashed, dotted, and dash-dot lines), with piecewise straight representation of  $R_y$  based on the studies of Biggs, Elghadamsi, Hall, Lai, Mohraz, Riddell, and Hewmark (redrawn from Chopra, [50]). Above the wide lines, with negative slopes, for periods longer than about 10 s, the oscillators collapse, for  $M = 7$  and  $8$ . For long periods as  $R_y$  asymptotically move towards those lines from below, their amplitudes decrease, approaching the collapse zones. (b) Same as Fig. 8a, but for near-field displacement  $d_N(t)$  (continuous, dashed, dotted, and dash-dot lines). In this figure, the collapse boundaries are shown for  $M = 5, 6, 7$ , and  $8$ , and the collapse zones begin for periods longer than about 7 s and  $\mu = 8$ .

of its force displacement properties like degradation of stiffness in terms of the number of cycles and amplitudes of the response, pinching of the hysteresis loop, consequences of geometric non-linearities, the effects of gravity, dynamic instability, coupling of the response to multi-component excitation, and so on.

With the gradual increase in the number of the recorded and processed strong-motion accelerograms, the dependence of  $R_y$  versus  $T_n$  could be investigated using the computed non-linear response first from several and then from several hundred strong-motion records (e.g.), [4,12,14]. Piecewise straight segments of thick lines in Figs. 8 show a typical example of  $R_y$  versus period  $T_n$ , for  $\mu = 1.5, 2, 4$ , and  $8$ .  $R_y$  is equal to  $1$  for  $T_n < T_a$ , to  $(2\mu - 1)^{1/2}$  for  $T_b < T_n < T_c$ , and to  $\mu$  for  $T_n > T_c$ , where on firm ground  $T_a = 1/33$ ,  $T_b = 1/8$ , and  $T_c = 0.55$  s in this example, but in general depends on damping. This example, taken from Chopra [50], is based on and is representative of the studies by Biggs, Elghadamsi, Hall, Lai, Mohraz, Riddell, and Newmark [51,52,53]. The sloping straight lines provide transition between the two constant segments and  $R_y = 1$  for  $T_n \rightarrow 0$ . The trends of  $R_y$  versus  $T_n$ , shown in Figs. 8, are average trends based on the computation of non-linear response to a small number of recorded strong-motion records and ignore any dependence on earthquake magnitude, source to site distance, or local site conditions.

Irregular thick lines, for  $\mu = 2$  and  $4$ , in Figs. 9, redrawn from MacRae et al. [14], show  $R_y$  versus period  $T_n$ , for average strike-normal records at distances of 20–30 km from epicenter, in the direction of rupture propagation, for

closest distance to rupture 0–10 km, and for fraction of critical damping equal to 2%. Those are compared with  $R_y$  computed for the near-fault pulse  $d_F$  (Fig. 9a) and for the near-fault displacement  $d_N$  (Fig. 9b). MacRae et al. [14] analyzed 154 earthquake records and reported that the directivity effects cause a more severe response only for periods  $T_n$  between 1 and 3 s. Their results showed no significant dependence of  $R_y$  versus period  $T_n$  on earthquake magnitude.

Smooth wide lines in Figs. 10, for  $\mu = 1.5, 2$ , and  $4$ , redrawn from Krawinkler and Nassar [54] show  $R_y$  versus period  $T_n$  curves, which have been used in several other studies [55–58]. As in many other previous studies, the dependence of  $R_y$  on earthquake magnitude is assumed to be small and hence is not considered.

The purpose of Figs. 8–10 here is to illustrate the range of the variations in the shape of  $R_y$  versus period  $T_n$  functions, based on different subsets of the recorded strong-motion data, and to contrast those shapes with the ones computed in this paper for near-fault pulse and near-fault displacement.

## 5. Strength-reduction factors for near-fault motions $d_F$ and $d_N$

We computed the relative response,  $u = h \sin \phi$ , of an SDOF system (Fig. 1b) consisting of a rigid mass of length  $L$  supported by two rigid mass-less columns, which are connected at the top to the mass by circular springs and at the bottom also by circular springs to the ground. The stiffness of the springs is assumed to be elastic–plastic, as shown in Fig. 1a, with  $\alpha = 0$ . The massless columns are

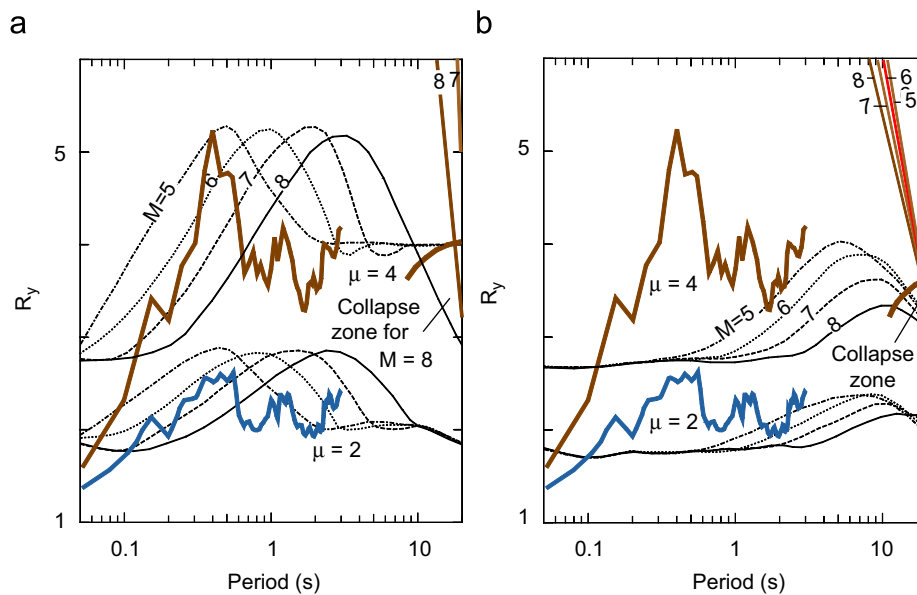


Fig. 9. (a) Comparison of strength-reduction factors computed for near-field pulse,  $d_F(t)$ , for  $M = 5, 6, 7$ , and  $8$ , and  $\mu = 2$  and  $4$  (continuous, dashed, dotted, and dash-dot lines), with  $R_y$  computed by MacRae et al. [14] for 154 earthquake records, for epicentral distances less than 30 km, in the direction of rupture propagation, and for  $\zeta = 0.02$  (irregular wide lines). The collapse boundaries for  $M = 7$  and  $8$ , and the collapse zone for  $M = 8$  appear for periods longer than about 10 s, and for  $\mu = 4$ . (b) Same as Fig. 9a, but for  $R_y$  computed for near-field displacement  $d_N(t)$  (continuous, dashed, dotted, and dash-dot lines). The collapse boundaries for  $M = 5, 6, 7$  and  $8$ , and the corresponding collapse zones appear for periods longer than about 10 s, and for  $\mu = 4$ .

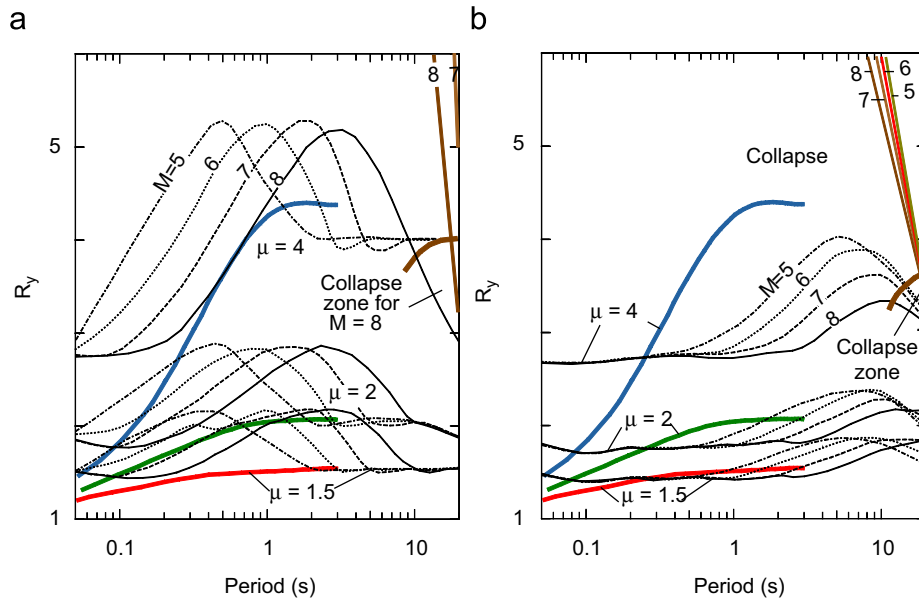


Fig. 10. (a) Comparison of strength-reduction factors computed for near-field pulse,  $d_F(t)$ , for  $M = 5, 6, 7$ , and  $8$ ,  $\zeta = 0.05$ , and  $\mu = 1.5, 2, 4$ , and  $8$  (continuous, dashed, dotted, and dash-dot lines), with  $R_y$  (wide lines) computed by Krawinkler and Nassar [54]. The collapse boundaries for  $M = 7$  and  $8$ , and the collapse zone for  $M = 8$  appear for periods longer than about 10s, and for  $\mu = 4$ . (b) Same as Fig. 10a, but for  $R_y$  computed for near-field displacement  $d_N(t)$  (continuous, dashed, dotted, and dash-dot lines). The collapse boundaries for  $M = 5, 6, 7$  and  $8$ , and the corresponding collapse zones appear for periods longer than about 10s, and for  $\mu = 4$ .

connected to ground and to the rigid mass by circular dashpots providing the fraction of critical damping equal to 5%. Rotation of the columns is assumed not to be small, which leads us to consider the geometric non-linearities. The mass is acted upon by the acceleration of gravity,  $g$ , and is excited by the horizontal ground acceleration,  $\ddot{u}_g$ , at its base. The functional form of  $\ddot{u}_g$  is defined by the second derivative with respect to time of the near-fault displacements  $d_F$  and  $d_N$ , as in Eqs. (1) and (2), respectively. The governing differential equation for the SDOF system in Fig. 1b is then

$$\ddot{\phi} + 2\omega_n\zeta\dot{\phi} + \Phi(\phi)/mh^2 - (g/h)\sin\phi = -(\ddot{u}_g/h)\cos\phi, \quad (3)$$

where when  $\alpha = 0$  (Fig. 1a),

$$\Phi(\phi) = \begin{cases} k\phi & \text{when } |\phi| < \phi_y \\ \pm k\phi_y & \text{when } \phi > \phi_y \text{ or } -\phi < -\phi_y \end{cases} \quad (4)$$

and  $u_y = h \sin \phi_y$ , as in Fig. 1a.

When  $\phi$  is small, Eq. (3) reduces to

$$\ddot{\phi} + 2\omega_n\zeta\dot{\phi} + \omega_n^2(1 - gmh/k)\phi = -\ddot{u}_g/h, \quad (5)$$

where  $\omega_n^2 = k/(mh^2)$ , and  $g$  is acceleration of gravity.

In the presence of gravity, the SDOF system is stable when  $\phi < \phi_s$ , and  $\phi_s$  is determined from

$$\Phi(\phi_s) = hmg \sin \phi_s. \quad (6)$$

At  $\phi = \phi_s$ , the overturning moment by gravity force  $hmg \sin \phi_s$  is equal to the restoring moment in the helical springs  $\Phi(\phi_s)$ . For simplicity, and to save computer time, we assume that the system will collapse when  $\phi > \phi_s$ .

It is seen that for large excitation amplitudes, this structure will experience permanent distortions and may collapse. In all calculations we consider simultaneous action of the effects of gravity forces, dynamic instability, and geometric non-linearities. For this structure, we calculate  $u_m$  and  $u_0$ , for  $d_F$  and  $d_N$  corresponding to earthquake magnitudes  $M = 5, 6, 7$ , and  $8$ , for ductilities  $\mu = 1.5, 2, 4$ , and  $8$ , and then plot  $R_y$  versus  $T_n$ . We overlay the results in Figs. 8 through 10 for comparison with previous studies on how  $R_y$  depends upon  $T_n$ . In Figs. 8a, 9a, and 10a we show our results for  $R_y$  versus  $T_n$  for excitation by the pulse  $d_F$  (Fig. 2 center), and in Figs. 8b, 9b, and 10b for excitation by the near-fault displacement  $d_N$  (Fig. 2 bottom).

In Figs. 8a through 10b, for periods  $T_n$  in the vicinity and longer than about 10s, the amplitudes of  $R_y$  become sensitive to action of the gravity load. There, for large magnitudes and ductilities,  $R_y$  dips to smaller amplitudes, below the constant asymptotes equal to  $\mu$ . There, the destabilizing effect of gravity and of horizontal excitation lead to conditions that are close to collapse ( $\phi > \phi_s$ ). In Figs. 8a through 10b, for  $\mu > 4$  the values of  $R_y$  for which collapse occurs are shown by almost straight lines with negative slopes, asymptotically approaching  $R_y$  curves from above. In the zones labeled ‘‘Collapse zone,’’ only minor increases in  $R_y$  will lead to collapse.

The classical strength-reduction factors equal to  $\mu$  for  $T_n$  longer than about 0.5s, and to  $(2\mu-1)^{1/2}$  for periods between about 0.1 and 0.4s (Fig. 8a), compare favorably with our calculations and would be conservative if used with near-fault excitation  $d_F$ , except for short intervals in  $T_n$  and for  $M = 8$ . For all other magnitudes our results are



above  $\mu$  or  $(2\mu-1)^{1/2}$ . For short periods, our results for  $R_y$  tend to an asymptote with amplitude equal to  $(2\mu-1)^{1/2}$ , which is a consequence of strong initial velocities associated with a sudden onset of  $d_F$ . Such behavior is observed even when non-linearity is not present, as can be seen in Fig. 6 for PSV spectra of  $d_F$  and  $d_N$ .

Fig. 9a shows similarity in the trends of our results for  $M = 5$  with average trends presented by MacRae et al. [14]. Because their results show average trends, and because most of their strong-motion data are for magnitudes less than 7, it is possible to argue that their results reflect a presence of strong-motion pulses corresponding to the failure of single asperities for earthquakes less than about 5, and of multiple asperities for larger-magnitude events. The physics behind this reasoning is described in two papers by Trifunac [31,39]. His studies of spectral characteristics of the recorded strong motion in California, as viewed in the frequency range of interest for engineering (say 0.1–25 Hz), suggest that most earthquakes smaller or equal to about  $M = 5$ , could be viewed as failures of one asperity, while the larger events can be viewed as failures involving multiple asperities (with indicative numbers of 3, 15, and 90 asperities, for  $M = 6, 7$ , and 8).

Our results for  $R_y$  versus  $T_n$  also agree qualitatively with those of Krawinkler and Nassar [54], shown in Fig. 10a, except that their results would not be conservative if used for near-fault pulses when  $T_n$  is longer than about 1 s. In Figs. 8b, 9b and 10b, for periods  $T_n$  longer than about 0.2–0.4 s, and for ductilities 4 and 8, the traditional design curves illustrated here [14,50,54] are not conservative.

## 6. Discussion and conclusions

We have described the variations in the strength-reduction factor,  $R_y$  versus  $T_n$ , for different earthquake magnitudes,  $M$ , for typical motions on the surface fault, modeled using the near-fault displacement pulse,  $d_F$ , characteristic of the fault-normal displacements, and the permanent displacement,  $d_N$ , which describes fault-parallel motions. We found strong dependence of  $R_y$  versus  $T_n$  on magnitude. These results are different and contradict the conclusions of many previous studies, which typically found little or no dependence in  $R_y$  versus  $T_n$  on magnitude. Our results are in qualitative agreement with those presented by Tiwari and Gupta [23] and Chakraborti and Gupta [24]. These observed differences can be explained by the nature of the physical process in question. The large static and dynamic parts of the near-fault ground motions, which occur near the moving faults, can be shown to decay rapidly with distance from the fault,  $\Delta$ , as  $\Delta^{-4}$  and  $\Delta^{-2}$  [33] and therefore become small relative to the body and surface waves for  $\Delta$  larger than 1–2 source dimensions [29,31]. The study by Mac Rae et al. [14], however, was able to detect the dependence on directivity of fault-normal versus fault-parallel strong motion for a selected group of strong-motion records for “small” fault-to-station distances. Their average trend for  $R_y$  versus  $T_n$  curves is also

consistent with the results of this study, if we assume that typical multiple asperities broken by the larger magnitude events (e.g. larger than 6) are representative of approximately magnitude 5 events [31,39].

Our results show that the average trends of most  $R_y$  versus  $T_n$  curves used in the design are conservative for pulse-like fault-normal motions near faults, which we modeled in this paper by  $d_F$ . However the same design curves are not conservative for fault-parallel motions, modeled in this paper by  $d_N$ , and for periods  $T_n$  longer than about 0.2–0.4 s. The simple remedy for this problem is to replace the strength-reduction factors by  $(2\mu-1)^{1/2}$  for all periods  $T_n$ , for all designs near faults.

Most previous studies of the strength-reduction factors show  $R_y = 1$  when  $T_n \rightarrow 0$ . This results from the common assumption that for stiff, high-frequency oscillators, when  $T_n \rightarrow 0$ , the relative displacement of the mass,  $u$ , and its relative velocity,  $\dot{u}$ , are essentially zero. Also, in the response analyses of linear and non-linear response of SDOF systems excited by the recorded strong-motion accelerograms it is assumed that the initial conditions for  $u$  and  $\dot{u}$  are sufficiently small, so that those can be approximated by zero. Our selection of the functional forms for  $d_F$  and  $d_N$  leads to large initial strong-motion velocities at  $t = 0$ , which result from the physical nature of ground motion caused by sudden stress drop on the fault surface [31,39]. As can be seen from the results presented in Figs. 8 through 10, this causes our  $R_y$  versus  $T_n$  curves to not converge to  $R_y = 1$  when  $T_n \rightarrow 0$ , but to an asymptote with an amplitude of  $(2\mu-1)^{1/2}$ . For the near-fault fault-parallel displacement,  $d_N$ , this initial velocity dominates the relative response for virtually all periods.

In summary, the results of this study suggest that for conservative design of structures near faults the simple and effective modification of the strength-reduction curves would be to replace the current values for  $R_y$  versus  $T_n$  by a constant equal to  $(2\mu-1)^{1/2}$ , for both fault-normal and fault-parallel motions. This is because in nature the faults are rarely straight and regular surfaces, and so it is difficult to predict when the near-fault motions will follow closely the examples of  $d_F$  and  $d_N$  employed in our study. Thus, it would be conservative to use the strength-reduction factor  $(2\mu-1)^{1/2}$  for all near-fault motions, irrespective of their orientation.

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