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# Biot response spectrum ☆

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**Abstract**

The response spectrum method (RSM) in earthquake engineering was conceived and then fully developed by M.A. Biot (1905–1985) during the period of about 10 yr, between 1932 and 1942. On the occasion of the 100th anniversary of Biot's birth, this paper reviews his contributions to earthquake engineering, and then briefly outlines the milestones in the later evolution of the RSM, and the eventual introduction and acceptance of the method by the engineering disciplines in the early 1970s. The role of the Biot spectrum in the formulation of design codes is illustrated briefly, using examples from code development in California. Finally, the limitations of the linear response superposition method are discussed, and future directions for the development of earthquake-resistant design tools are suggested.


## 1. Introduction

The year 2005 marks the 100th anniversary of the birth of the inventor of the response spectrum method (RSM)—Maurice A. Biot. It is also approximately the 30th anniversary of the general acceptance of the RSM by the engineering profession, following the 1971 San Fernando, California earthquake. On the occasion of these anniversaries, this paper reviews the ideas that led to the formulation of the RSM in 1942, its evolution during the following 40 yr, its use in design during the past 30 yr, its limitations and the prospects for its continued use in the future.

The response spectrum is a plot of the maximum displacements or velocities of the relative response of a family of damped single-degree-of-freedom oscillators to strong earthquake ground motion, specified in terms of absolute ground acceleration. The maximum relative displacement can be used to compute maximum drifts and maximum dynamic forces acting in the structure, and thus it serves as a basis for earthquake-resistant design in terms of linear representation of response. The method is formulated in a manner that permits separation of the characteristics of particular structures from those of the earthquake, the latter being given by the "response spectra." This approach is used for the design of many earthquake-resistant structures, and it is also the main tool for preliminary design of important structures, before the final design is further refined via the dynamic response using numerical integration in time for the response of the detailed mathematical model of the structure.

Extension of the RSM to the nonlinear response has been studied extensively, with varying degrees of success. In the following, we will cite only a few examples, leaving the complete review of such analyses for a future paper. This paper will focus on (1) the development of the RSM for the "linear" response of structures, (2) its role in influencing the development of design codes, and (3) its suitability for analysis of transient response to impulsive loading.

## 2. Response spectrum

### 2.1. Historical notes

In the early 1930s, Professor Theodore von Kármán[2] and his student Maurice Biot were studying the theoretical

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[1]Professor.

[2]Theodore von Kármán, born in Budapest, Hungary (1881–1963), engineer, applied scientist, teacher, and visionary, was the first director of the Daniel Guggenheim Graduate School of Aeronautics at the California

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dynamics problem of how to estimate the maximum response of oscillators to transient excitation, and proposed what would later become known as the RSM in earthquake engineering. These ideas were first described in 1932, and further developed during the following 10 yr. Today, almost three quarters of a century later, the theory remains essentially intact and still forms the core of the linear mechanics framework of earthquake engineering.

The origin of RSM can be traced to the second chapter of Biot's Ph.D. dissertation in the Aeronautics Department at Caltech, entitled "Vibration of Buildings During Earthquakes" [3]. Biot's ideas, and further studies undertaken at the suggestion of Professor von Kármán, were developed further while Biot was a Research Fellow at Caltech in 1932 [4,5]. In his 1933 paper, communicated on 19 January 1933, 50 days before the first strong earthquake ground motion was recorded on 10 March 1933 in Long Beach, California, Biot stated: "The study of seismogram spectral distributions has not yet been made; it is however, the author's opinion that this study would be of great importance for two reasons: (1) The peaks of spectral curves will reveal the presence of certain characteristic frequencies of the soil at a given location. (2) By applying the preceding theorem, the maximum effect of earthquakes on buildings will be easily evaluated…."

Biot's interest in the peak response of solids and fluids subjected to transient impulses first became evident in his papers written in 1932. It was this general interest that led to his study of the response of buildings during earthquakes, in the second chapter of his Ph.D. dissertation, and to the first phase of his formulation of RSM [3–5].

From the very beginning of his career, Biot displayed many varied interests, covering a broad spectrum of subjects in mechanics and engineering. In 1930 and 1931, he had already studied such different subjects as induced electrical currents, hydrodynamics and the guidance of airplanes. While at Caltech in 1933, he wrote about photoelasticity, analysis of bending moments in airplanes using tests in a wind tunnel, and beams on elastic foundations. Back at Louvain, during 1933, Biot wrote about photoelastic analyses of thermal stresses, and studied stresses in dams and the vibration of buildings. At Harvard, in 1935 and 1937, Biot wrote about stress distribution in sediments during consolidation, about beams on elastic foundations, and about flood waves. During the 3 yr period from 1938 to 1941 while at Columbia, he further expanded the horizon of his research, and started to work on the theory of elasticity with large displacements and rotations, nonlinear elasticity in a body under initial stress, the influence of initial stress on stability, on wave propagation, finite difference calculations, use of analogies in differential equations to investigate stress distribution in bending and the calculation of response of simple oscillators to strong-motion acceleration (torsional pendulum). It was also during this time that he started to write the first papers in his now famous series of 21 papers on poromechanics [6].

At Columbia, Biot briefly returned to the subject of earthquake engineering, describing the computation of response spectra by means of a mechanical analyzer [7], and then he finally formalized the general theory and principles of response analysis and response spectrum superposition at Caltech in 1942 [8,9].

Between 1942 and 1985, Biot published 134 journal papers and articles and wrote two books. Further references about Biot and about the impact of his work on modern engineering can be found in Trifunac [10].

## 3. Shape of "standard" elastic earthquake response spectrum

### 3.1. Fixed-shape response spectra

In his 1934 paper, Biot stated: "If we possessed a great number of seismogram spectra we could use their envelope as a standard spectral curve for the evaluations of the probable maximum effect on buildings." In Biot [7], he continued: "These standard curves…could be made to depend on the nature and magnitude of the damping and on the location. Although the previously analyzed data do not lead to final results, we…conclude that the spectrum will generally be a function decreasing with the period for values of the latter greater than about 0.2 s. A standard curve for earthquakes of the Helena and Ferndale…for values $T>0.2$ s, could very well be the simple hyperbola $A = 0.2g/T$ and for $T<0.2$ s, $A = g(4T + 0.2)$, where $T$ is the period in seconds and $g$ the acceleration of gravity. This standard spectrum is plotted in Figs. 1–4. Whether this function would fit other earthquakes can only be decided by further investigations."

Housner, 15 yr later, averaged and smoothed the response spectra of three strong-motion records from California (El Centro, 1934, $M = 6.5$; El Centro, 1940, $M = 6.7$ and Tehachapi, 1952, $M = 7.7$) and one from Washington (Olympia, 1949, $M = 7.1$). He advocated the use of this average spectrum shape in earthquake engineering design (Fig. 1, [11,12]).

Newmark and co-workers [13,14] noted that the shape of response spectra can be determined by specifying peak acceleration, peak velocity and peak displacement of strong ground motion. Spectrum shape was further studied by Mohraz et al. [15] using 14 strong-motion records, and by Blume et al. [16] who analyzed 33 records. The joint recommendations of the Newmark and Blume studies of the shape of the response spectra [17] were later adopted by the US Atomic Energy Commission (now the US Nuclear Regulatory Commission, USAEC, 1973 [18]) for use in the design of nuclear power plants (Fig. 2).

(*footnote continued*)
Institute of Technology, where he arrived in 1930 from Aachen, Germany. Von Kármán had foresight, creativity, and a remarkable talent for getting people together across professional, national, and language barriers. He was one of the foremost leaders in the world of aviation and space technology (see, e.g., Ref.for example [2]).

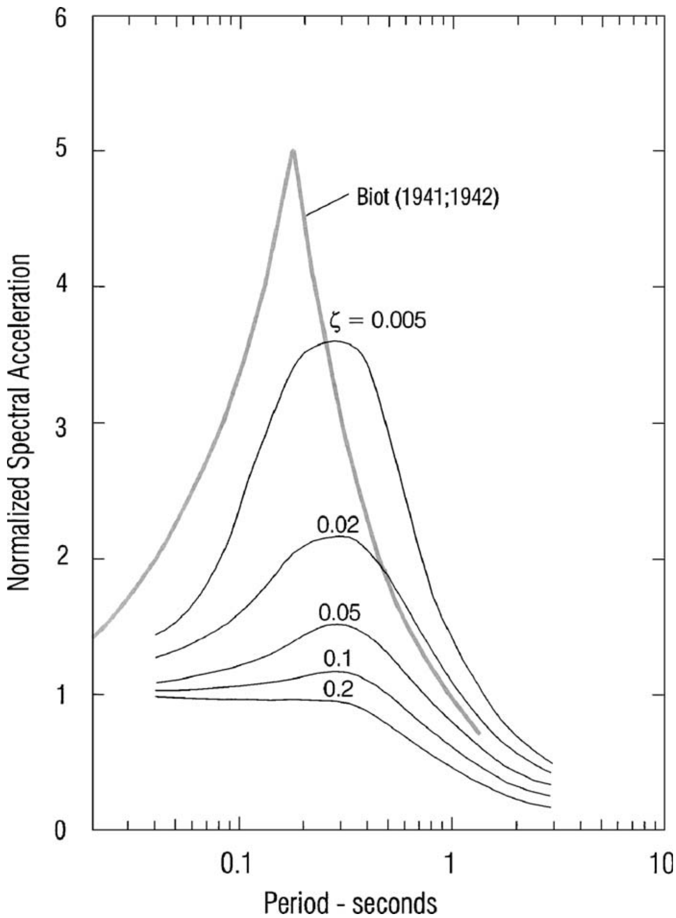



Fig. 1. Comparison of Biot [7,8] "standard spectrum" (heavy line) with average spectrum of Housner [11,12].

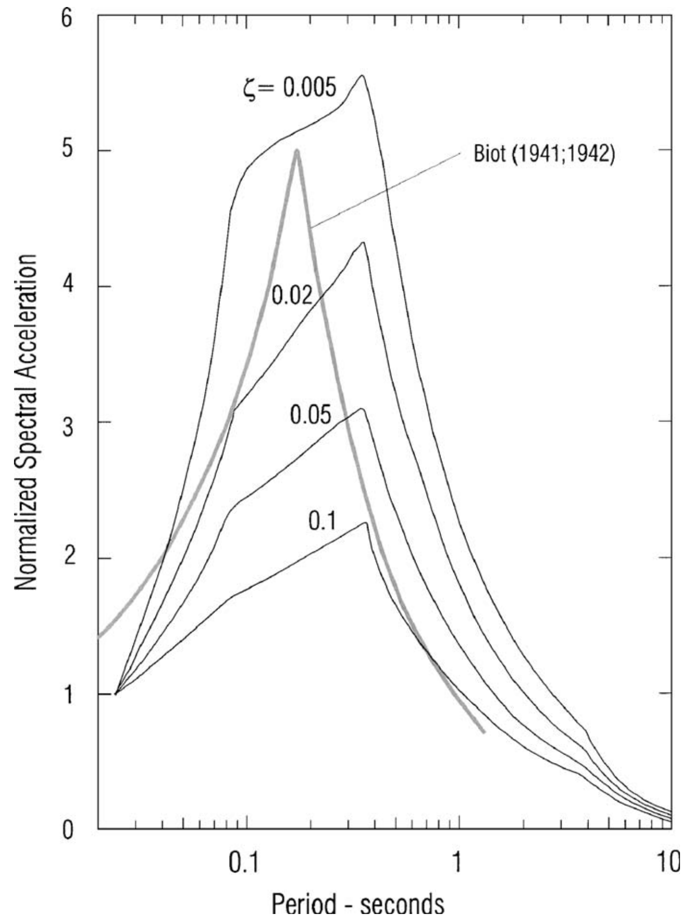



Fig. 2. Comparison of Biot [7,8] "standard spectrum" (heavy line) with regulatory guide 1.60 spectrum [18].

In engineering design work, the fixed shapes of Housner and Newmark spectra, normalized to unit peak acceleration, were scaled by selecting the "design" peak acceleration. This procedure, which was first systematically used in the design of nuclear power plants, emerged as the "standard" scaling procedure for determination of design spectra in the late 1960s and early 1970s.

### 3.2. Site-dependent spectral shapes

In one of the first studies to consider the site-dependent shape of earthquake response spectra, Hayashi et al. [19] averaged spectra from 61 accelerograms in three groups, according to the recording site conditions (A—very dense sands and gravels; B—soils with intermediate characteristics and C—very loose soils), and showed that the soil site condition has an effect on the shape of average response spectra. This result was later confirmed by Seed et al. [20], who considered 104 records and four site conditions (rock, stiff soil, deep cohesionless soil and soft-to-medium clay and sand; Fig. 3).

Mohraz et al. [15] suggested that the peak ground displacement, $d$, and peak ground velocity, $v$, were $d = 36$ in. and $v = 48$ in./s for "alluvium" sites and $d = 12$ in. and 28 in./s for "rock" sites, both corresponding to a $1g$ peak ground acceleration. However, because of the small number of recorded accelerograms on rock in 1972, conclusive recommendations on how to describe the dependence of spectra on site conditions did not appear possible at that time.

A major and persistent problem in the evaluation of site-dependent spectra of strong earthquake motion is the lack of generally accepted procedures on how to characterize a site. Gutenberg [21] studied the amplification of weak earthquake motions in the Los Angeles area and published the results on average trends and amplification of peak wave motions in sedimentary basins for periods of motion longer than about 0.5 s. His site characterization could be termed "geological," because he considered the "site" on the scale of kilometers, and used the term "rock" to represent geological basement rock. Gutenberg's results were shown to be in excellent agreement with the empirical scaling of Fourier amplitude spectra of strong-motion accelerograms of 186 records [22], 20 yr later. While it is clear today that both geotechnical and geological site characterizations must be considered simultaneously [23,24], there is so far no general consensus on how to do this.

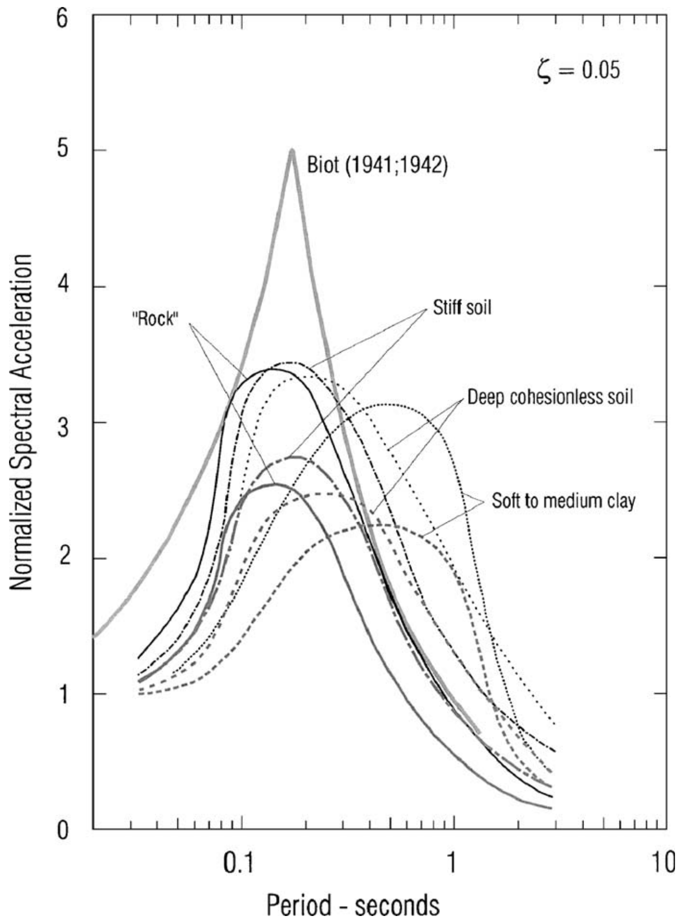



Fig. 3. Comparison of Biot [7,8] "standard spectrum" (heavy line) with average (heavy lines) and average plus standard deviation spectra (light lines) of Seed et al. [20] for four soil site conditions.

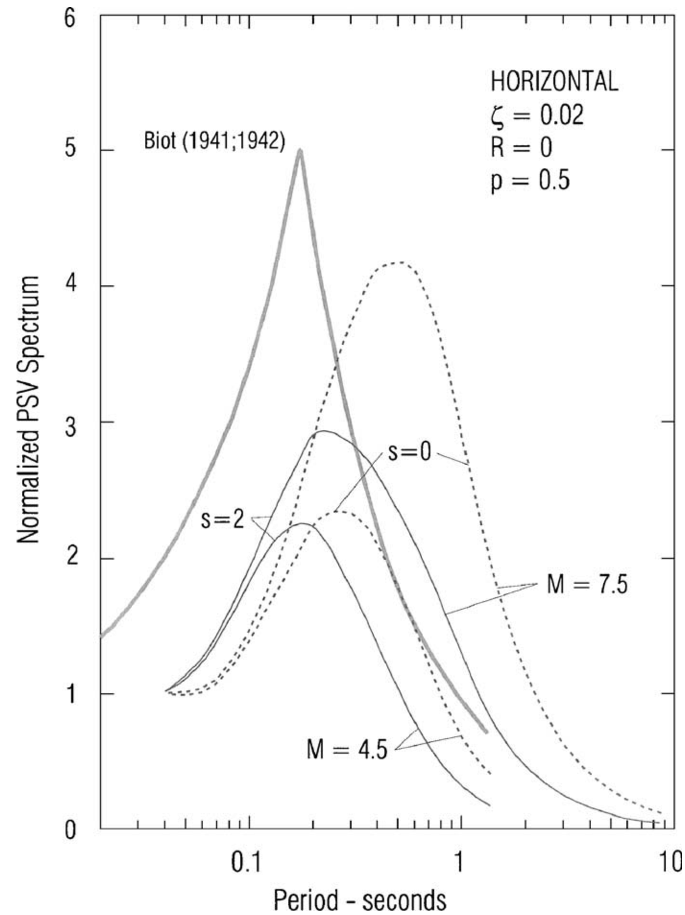



Fig. 4. Comparison of Biot [7,8] "standard spectrum" (heavy line) with spectral shapes, which depend upon magnitude ($M = 4.5$ and 7.5) and geological site conditions ($s = 2$ for basement rock and $s = 0$ for sediments), for average spectral amplitudes ($p = 0.5$), at zero epicentral distance ($R = 0$) and for 2% of critical damping ($\zeta = 0.02$; [26]).

### 3.3. Site-, magnitude- and distance-dependent spectra

The occurrence of the San Fernando, California earthquake in 1971, and the large number of new recordings it contributed to the strong-motion database [25] opened a new chapter in the empirical studies of earthquake response spectra. For the first time, it became possible to consider multi-parametric regressions and to search for the trends in recorded strong-motion data. It became possible to show how spectral amplitudes and spectrum shape change, not only with local soil and geologic site conditions, but also with earthquake magnitude and source-to-station distance (Fig. 4; [26]). During the following 20 yr, the subsequent regression studies evolved into advanced empirical scaling equations, contributing numerous detailed improvements and producing a family of advanced, direct-scaling equations for spectral amplitudes in terms of almost every practical combination of scaling parameters. The literature on this subject is voluminous, and its review is beyond the scope of this paper. The readers can find many examples and a review of this subject in Lee [27].

Figs. 1–4 compare Biot's "standard" spectrum shape with other examples of fixed (Figs. 1, 2, and 3) and variable (Fig. 4) spectral shapes. These comparisons are only qualitative, because the methods used in their development and the intended use of the spectral shapes differ. Biot's spectrum was originally thought to correspond to zero damping, but it was later discovered that it has small variable damping, probably less than 3% of critical. It was based on the spectra of two earthquakes only (Helena, Montana, 1935, $M = 6.0$ and Ferndale, California, 1934, $M = 6.4$). Housner (Fig. 1), NRC (Fig. 2) and Seed et al. (Fig. 3) spectra were based on progressively larger numbers of recorded accelerograms (4, 33 and 104, respectively) and on recordings during large earthquakes. Therefore, they have broader spectral shapes. The variable shape spectrum in Fig. 4 shows only the dependence of spectral shape (normalized to $1-g$ acceleration) on magnitude and geological site conditions, but it does show clearly how the spectra broaden with increasing magnitude, and how larger magnitudes contribute larger long-period spectral amplitudes.

## 4. Response spectrum superposition

In can be demonstrated that for a linear multiple-degrees-of-freedom system the dynamic response of the $r$-th mode alone may be described by an equation that corresponds to an equivalent single-degree-of-freedom system, e.g. Ref. [28], and then that the total displacement response may be computed by adding contributions of all individual modes. For example, the contribution of the $r$-th mode,$\{x^{(r)}(t)\}$, to the total displacement of a multiple-degree-of-freedom system is

$$\{x^{(r)}(t)\} = \frac{\{A^{(r)}\}\alpha_r}{\omega_r\sqrt{1-\zeta_r^2}}\int_0^t \ddot{z}(\tau)\,\mathrm{e}^{-\zeta_r\,\omega_r(t-\tau)}\sin\omega_r\sqrt{1-\zeta_r^2}\,(t-\tau)\,\mathrm{d}\tau, \tag{1}$$

where $\{A^{(r)}\}$is the $r$-th mode shape, $\alpha_r$ its participation factor, $\omega_r$ its circular frequency and $\zeta_r$ its fraction of critical damping. This contribution is seen to be directly proportional to $1/\omega_r\sqrt{1-\zeta_r^2}$ times the integral term and will depend upon the integral's maximum absolute value. In terms of the displacement, $\mathrm{SD}^{(r)}$, and velocity, $\mathrm{SV}^{(r)}$, spectra, Eq. (1) can be written as

$$\{|x^{(r)}(t)|\}_{\max} = \{|A^{(r)}|\}\alpha_r\,\mathrm{SD}^{(r)} = \{|A^{(r)}|\}\alpha_r\,\frac{T_r}{2\pi}\,\mathrm{SV}^{(r)}, \tag{2}$$

where the superscripts ($r$) on SD and SV indicate that these spectral values are computed for damping $\zeta_r$ and frequency $\omega_r = 2\pi/T_r$, corresponding to the $r$-th mode of vibration. However, the different modal maxima do not occur at the same time and, therefore, the individual modes do not simultaneously contribute their peak values to the maximum total response.

The sum of maximum modal responses [9]

$$\sum_{r=1}^{n}\{|x^{(r)}(t)|\}_{\max} = [|A|]\begin{Bmatrix}\mathrm{SD}^{(1)}\\ \mathrm{SD}^{(2)}\\ \vdots \\ \mathrm{SD}^{(n)}\end{Bmatrix} \tag{3}$$

gives an upper bound to the total system response, but may be too conservative. An alternative approach, based on statistical considerations [29], is to take the square root of the sum of the squares of the individual modal maxima. This method (RMS) has been shown to give reasonable results [30] for structures in which the main contributions come from the lowest few modes.

The subject of mode superposition has been studied extensively [31–39], and the conditions under which a meaningful degree of conservatism can be achieved have been determined. When the maxima of each response quantity have been determined from equations analogous to Eq. (2), the RMS approximation is given by

$$\{x\}_{\max} \approx \begin{Bmatrix} \left[\sum_{i=1}^{n}\left(x^i_{1_{\max}}\right)^2\right]^{1/2} \\ \left[\sum_{i=1}^{n}\left(x^i_{2_{\max}}\right)^2\right]^{1/2} \\ \vdots \\ \left[\sum_{i=1}^{n}\left(x^i_{n_{\max}}\right)^2\right]^{1/2} \end{Bmatrix}. \tag{4}$$

The expressions for maximum shears $\{V\}_{\max}$ and moments $\{M\}_{\max}$ follow from Eq. (4) [28].

The above-described response spectrum superposition method provides only an approximate indication of the maximum response in the multiple-degree-of-freedom systems. The advantage of this method is that it avoids lengthy computations associated with the exact method and, at the same time, takes into account the dynamic nature of the problem. It can often provide reasonable results for design purposes.

## 5. Biot spectrum and evolution of earthquake design codes

Work on developing building design codes appears to have begun in Italy in 1908, following the Massina disaster in which more than 100,000 persons were killed; in Japan following the 1923 Tokyo disaster, in which more than 150,000 perished and in California after the Santa Barbara earthquake of 1925 [40,41]. In 1927, the "Palo Alto Code," developed with the advice of Professors Willis and Marx of Stanford University, was adopted in Palo Alto, San Bernardino, Sacramento, Santa Barbara, Klamath, and Alhambra, all in California. It specified the use of a horizontal force equivalent to 0.1, 0.15 and 0.2$g$ acceleration on hard, intermediate and soft ground, respectively.

"Provisions Against Earthquake Stresses," contained in the proposed US Pacific Coast Uniform Building Code was prepared by the Pacific Coast Building Officials Conference and adopted at its Sixth Annual Meeting, in October 1927, but these provisions were not generally incorporated into municipal building laws [40]. The code recommended the use of horizontal force equivalent to 0.075, 0.075 and 0.10$g$ acceleration on hard, intermediate and soft ground, respectively. Following the 1933 Long Beach earthquake, the Field Act was implemented. Los Angeles and many other cites adopted an 8% $g$ base shear coefficient for buildings, and a 10% $g$ for school buildings. In 1943, the Los Angeles Code was changed to indirectly take into account the natural period of vibration of the structure.

San Francisco's first seismic code (the Henry "Vensano Code") was adopted in 1948, with lateral force values in the range from 3.7% to 8.0% of $g$, depending upon the building height ([42,43]). The Vensano Code called for higher earthquake coefficients than were then common in Northern California, and higher than those prescribed by the Los Angeles 1943 code. Continued opposition by San Francisco area engineers led to a general consensus-building effort, which resulted in the "Separate 66" report in 1951. "Separate 66" was based on Maurice Biot's "standard" fixed shape response spectrum [44].

In Los Angeles, until 1957 (for reasons associated with urban planning rather than earthquake safety, and to prevent development of downtown "canyons") no buildings higher than 150 ft (13 story height limit) could be built. In 1957, the fixed height limit was replaced by the limit on the amount of floor area that could be built on a lot. After the San Fernando, California earthquake of 1971, Los Angeles modified the city code in 1973 by requiring dynamic analysis for buildings over 16 stories high (160 ft).

In 1978, the Applied Technology Council (ATC) issued its ATC-3 report on the model seismic code for use in all parts of the United States. This report, written by 110 volunteers working in 22 committees, incorporated many new concepts, including "more realistic ground motion" intensities. Much of the current Uniform Building Code was derived from the ATC-3 report.

## 6. Limitations of RSM

Biot's mathematical formulation of the response of structures uses the vibrational approach, in which the solution of the governing differential equations is represented by superposition of characteristic functions (mode shapes) of the problem. Physically, characteristic functions (mode shapes) represent standing waves that have been created by constructive interference of the waves incident through and reflecting from the boundaries of the model. All other wave energy does enter the structure but, after some time, it dies out due to destructive interference, scattering transmission and refraction, and propagation out of the structure.

### *6.1. Low-pass filtering effects*

In practical applications, and for most structures, the mode participation factors for the lowest frequencies are usually the largest. In applications using detailed models (lumped mass, finite elements, finite differences, etc.), the contributions of high-frequency modes are routinely neglected, because these contributions to the response can be shown to be "small." However, this practice is equivalent to low-pass filtering of the actual motions, and it results in reduction of the estimated transient peak response amplitudes. In applications that consider only the fundamental mode of vibration, this low-pass filtering effect is the largest.

### *6.2. Short, impulsive excitation*

It can be shown that the modal approach is not appropriate to represent "early" transient response, particularly for excitation consisting of strong motion pulses with duration shorter than the travel time, $t$, of an incident wave to reach the top of the building ($t<H/\beta$; $H$ and $\beta$ are the building height and the velocity of shear waves in the building). As the modes of vibration result from constructive interference of the incoming wave and the wave reflected from the top of the building, the building "starts vibrating" in the first mode only after time $t=2H/\beta$ has elapsed from the time the shaking started. Although, in principle, the representation of the response as a linear combination of the modal responses is mathematically complete and, therefore, can be used to represent any response, a short, impulsive excitation would require the consideration of many modes (infinitely many for a continuous model), which is impractical. Thus, the wave propagation methods are more natural for representation of the "early" transient response.

Wave propagation models of buildings have been used for many years [45–47], but they have only recently begun to be verified against actual observations. Continuous, 2D wave propagation models (homogeneous, horizontally layered and vertically layered shear plates) have been used to study the effects of traveling waves in the response of long buildings [48–55]. Discrete-time, 1 and 2D wave propagation models have been used to study the seismic response of tall buildings [56,57].

### *6.3. Soil–structure interaction*

In general, RSM cannot be used for evaluation of the relative response of structures supported by flexible or multiple foundations, and in the presence of nonlinear deformations in the soil. The complex role that flexible soil plays in the response of structures to incident wave excitation has been recognized and studied since the 1930s [41,46,47]. In an unpublished note, Biot [8] states that "the problem is extremely complex because it involves a complete knowledge of the propagation and properties of the seismic waves in the strongly heterogeneous surface layers of the earth, as well as their diffraction and reflection by objects built on the surface.... In the present investigation, we have attempted to answer the following question: What is the influence of the elasticity of the ground on the rocking motion of a building? How resistant is the surrounding soil to the rocking displacement of a foundation; what are the factors influencing this rigidity, and can we expect this effect to have a practical influence in the action of earthquakes on buildings? The problem is simplified by neglecting the radiation of elastic waves due to the rocking." The ideas and equations from this unpublished note appear in abridged from in Section 5, entitled "Influence of Foundation on Motion of Blocks" in Biot's paper (Ref. [9]).

Between 1970 and 1980, the research on soil–structure interaction grew steadily. Important theoretical problems were solved, and key full-scale experiments were conducted [58]. However, soil–structure interaction is rarely considered in the routine design of engineered structures and, when it is considered, it is based on the most elementary models.

A common assumption in many models that consider the soil–structure interaction effects is that the foundation is rigid. This reduces the number of degrees of freedom of the model and gives good approximations of response for ground motions composed of long wavelengths relative to the foundation dimensions [59]. For short wavelengths, this assumption can result in non-conservative estimates of the relative deformations in the structure [60–62] and, in general, such an assumption can be expected to result in excessive estimates of scattering of the incident wave energy and in excessive radiation damping [50,63,64]. The extent to which this simplifying assumption is valid depends upon the stiffness of the foundation system relative to that of the soil and on the overall rigidity of the structure [56,65–67].

Rigid foundation models are usually combined with lumped-mass, discrete representations of the structure. The entire system is then described by a system of differential equations, and the solution is given in terms of the motion of different building floors. A soil–rigid foundation–lumped-mass structural model is usually limited to representation of 1D models and offers useful approximation for the lower-frequency modes of relative response. The response spectrum superposition method can be used in deterministic or in probabilistic form [32,36,37] with such models.

The other extreme is to neglect the stiffness of the foundation system, ignore the soil–structure interaction and assume that the wave energy in the soil drives the building according to the principles of wave propagation. This approximate approach underestimates the scattering of the incident wave energy by the foundation and overestimates the energy entering the structure [68].

As the soft soil surrounding the foundation begins to experience nonlinear deformations for much smaller levels of shaking than the structure, the soil–structure system experiences significant shifting of system frequencies typical of nonlinear softening spring behavior [69–71]. Because this occurs most of the time, ignoring soil–structure interaction and interpreting response solely through the RSM can result in gross misrepresentation of the response within the structure.

### *6.4. Nonlinear systems*

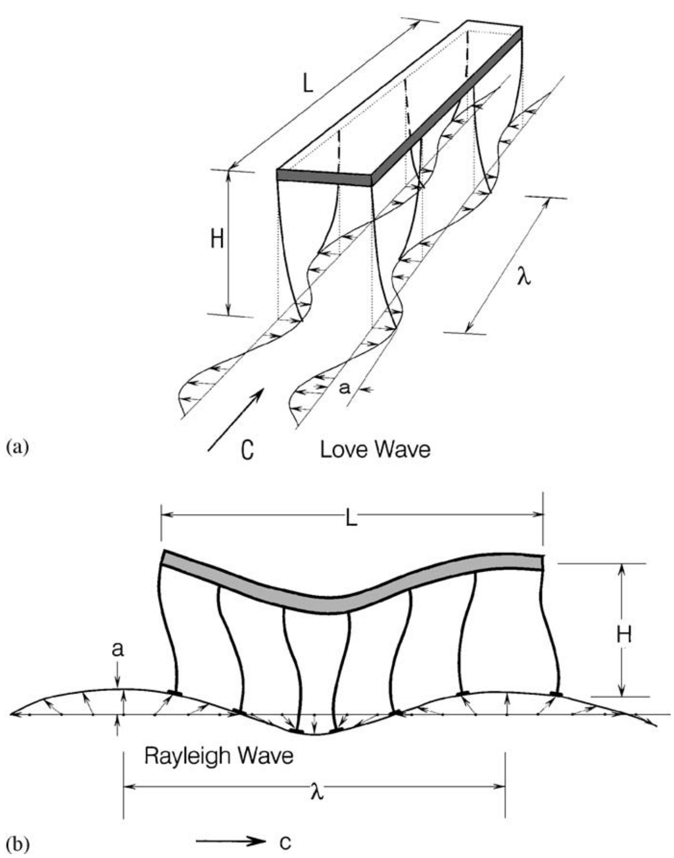
By definition, response spectrum amplitude corresponds to the peak response of the single-degree-of-freedom system (SDOF), irrespective of the length of the excitation and the number and sign of the other peaks of the response. This limitation is particularly important when linear response spectra are modified to describe the response of nonlinear hysteretic systems. For linear systems, statistics of ordered peaks can be employed to describe the expected amplitudes of many peaks [33–35,38,72], but the analogous representation for nonlinear systems has not been developed thus far. Formulation of new design criteria based on the power of incident wave energy (demand) and the ability of structures to absorb that power (capacity) offers a rational way to consider amplitudes and durations of the pulses of incident motion, but this approach abandons RSM [68].

## 7. Generalization of RSM to differential motions

The common use of RSM implicitly assumes that all points of building foundations move synchronously and with the same amplitudes. This implies that the wave propagation in the soil can be neglected. Unless the structure is long (e.g., a bridge with long spans, a dam, a tunnel) or "stiff" relative to the underlying soil, these simplifications are justified, and can lead to selection of approximate design forces. Simple analyses of 2D models of long buildings suggest that when $a/\lambda < 10^{-4}$, where $a$ is wave amplitude and $\lambda$ is the corresponding wavelength, the wave propagation effects on the response of simple structures can be neglected [49,50].

Fig. 5(top) illustrates the "short" waves propagating along the longitudinal axis of a "long" building or a multiple-span bridge. For simplicity, the general incident

(a)




(b)

Fig. 5. (Top) Deformation of columns in a two-degree-of-freedom system, during out-of-plane response, excited by Love waves. (Bottom) Deformation of columns in a long structure during in-plane response, excited by Rayleigh waves.

wave motion has been separated into out-of-plane motion (Fig. 5(top)), consisting of SH and Love waves, and in-plane motion (Fig. 5(bottom)) consisting of P, SV and Rayleigh waves. The in-plane motion can further be separated into horizontal (longitudinal), vertical and rocking components, while out-of-plane motion consists of horizontal motion in a transverse direction and torsion along the vertical axis [61]. Trifunac and Todorovska [62] analyzed the effects of the horizontal in-plane component of differential motions and showed how RSM can be modified to include the first-order effects of differential motion on individual columns.

Designating by SDC($T$,$\delta$,$\zeta$,$\tau$) the relative displacement spectrum for column deformations, where $T$ is the period of the equivalent SDOF system, $\zeta$ is its fraction of critical damping, $\delta$ is the ratio of the peak relative response of the first floor to SD($T$,$\zeta$) and $\tau = Ax/\beta_{av}$ is the normalized travel time between "central" point $R$ of the foundation and a given column, at distance $x$ ($A$ is scaling parameter $\sim$1, and $\beta_{av}$ is the average shear wave velocity in the top 30 m of soil); for seismic waves propagating along the surface it can be shown that for in-plane motions (Fig. 5(bottom))

$$\mathrm{SDC}(T,\delta,\zeta,\tau) \sim \left\{[\delta\,\mathrm{SD}(T,\zeta)]^2 + (v_{\max}\tau^2)\right\}^{1/2}, \quad (5)$$

where SD($T$,$\zeta$) is the relative displacement spectrum (e.g. see Fig. 6) and $v_{\max}$ is the peak ground velocity associated with the corresponding excitation. An example of SDC($T$,$\delta$,$\zeta$,$\tau$) for S16W strong motion recorded at USC station #53, during the Northridge earthquake, is shown in Fig. 6 for $\tau = 0.001$ through 0.1 s, $\delta = 1$ and $\zeta = 0.05$.

In Eq. (5), SD($T$,$\zeta$) is representative of relative column displacement caused by inertial forces, while $v_{\max}\tau$ approximates the maximum relative column displacement arising from pseudo-static deformations in the soil associated with wave passage. It can be seen that for long structures (large $\tau$) pseudo-static deformation of columns can be large and can dominate in the contribution to SDC($T$,$\delta$,$\zeta$,$\tau$) for intermediate and short periods of oscillators ("stiff" structures).

For out-of-plane motion (Fig. 5(top)) and ground motion consisting of "long" waves, SDC must be calculated for a two-degree-of-freedom system, with translational period $T$, torsional period $T_T$ and their respective fractions of critical damping $\zeta$ and $\zeta_T$. For $T \sim T_T$ and $\zeta \sim \zeta_T$ it can be shown that [61]

$$\mathrm{SDC}(T, T_T, \zeta, \zeta_T, \tau, \delta) \approx \{[\delta\,\mathrm{SD}(T,\zeta)]^2 + 2(v_{\max}\tau)^2\}^{1/2}. \quad (6)$$

The SDC spectrum for in-plane motion is illustrated in Fig. 6 for horizontal component S16W of a recording in the near-field of the Northridge, California earthquake of 17 January 1994. The results indicate that during this earthquake the increase in the shear forces for peripheral columns (on individual foundations) caused by differential ground motion was significant, so that one must consider this effect in the design of new structures and in the retrofitting of existing structures. This shows that for high-frequency (stiff) structures, with moderate to large horizontal dimensions, the shear forces and the associated bending moments in the peripheral columns will exceed the estimates based on the relative displacement spectra SD($T$,$\zeta$) by factors that can be large.

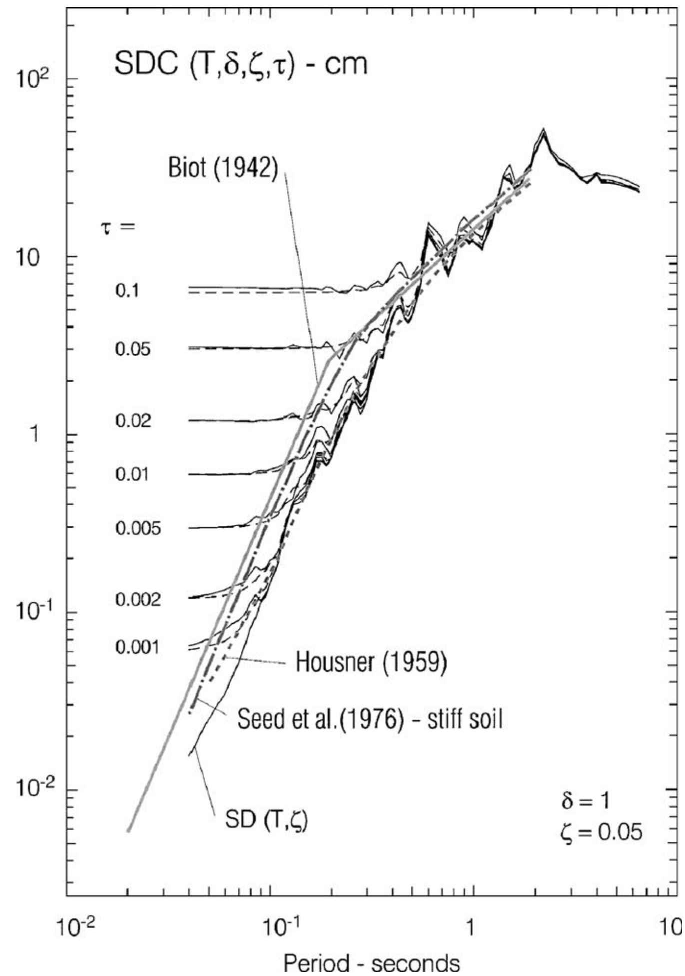



Fig. 6. Relative displacement spectrum for columns, SDC($T$,$\delta$,$\zeta$,$\tau$), for S16W component of acceleration recorded at USC station #53 of the Los Angeles Strong Motion Network [73], during Northridge, CA earthquake of 17 January 1994 ($M = 6.7$), at epicentral distance of 6 km for $\zeta = 0.05$ and $\delta = 1$ (one-story building). The solid lines correspond to SDC spectra computed exactly, and the dashed lines to the approximation, given by Eq. (3). "Standard" spectrum shapes of Biot [9], Housner [11] and Seed et al. [20], normalized to agree with recorded motions at long periods, are shown for comparison. Peak amplitudes of strong motion at this site were 12.4 cm, 59.8 cm/s and 381 cm/s$^2$.

In Fig. 6, we also compare the computed SD($T$,$\zeta$) with the "standard" spectral shapes of Biot, Housner and Seed. While all of these shapes agree favorably with SD($T$,$\zeta$) for this particular recording Biot's spectrum overestimates the classical SD($T$,$\zeta$) spectrum, and is more conservative than the other two.

## 8. Discussion and conclusions

The focus of this paper has been on the birth and early history of RSM [3,9], and on its linear vibrational framework, within which it continues to be introduced in the introductory courses on earthquake engineering. An outline of how this method has been modified during the past 40 yr to allow approximate analyses of nonlinear response has been left for a future paper. Here, we only note that in spite of the voluminous published work on such generalizations, and extensions of the RSM to the response of structures experiencing nonlinear deformations, no general method has been developed thus far. To guide the development of future methods for design of earthquake-resistant structures undergoing large nonlinear response, it will be necessary also to record the responses of many structures experiencing nonlinear deteriorating response. This will require a far more comprehensive instrumentation of structures than is available today. It will also require development and deployment of new instrumentation systems capable of recording permanent displacements and permanent rotations at many locations in the soil–structure systems [73,74].

In attempting to avoid the limitations of RSM, which result from its vibrational formulation, it is expected that in the future its use will become restricted to the design of structures that are expected to experience only linear response amplitudes. For performance-based design of structures that are expected to experience "controlled" nonlinear responses, it will be necessary to develop new design principles. These new methods will most likely be based on equating the maximum power demand with the design capacity of the structure to absorb a given energy per unit of time. In the meantime, Biot's RSM, which is so deeply and so ubiquitously interwoven into all aspects of earthquake engineering, will continue to be the central guiding concept in earthquake-resistant design.

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