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VARIATIONS OF APPARENT BUILDING FREQUENCIES - LESSONS FROM FULL-SCALE EARTHQUAKE OBSERVATIONS

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SUMMARY

A summary is presented of analyses of variations of the system frequency of 21 instrumented buildings in the Los Angeles area, which recorded the Northridge earthquake (M_S= 6.7) of January 17, 1994, and some of its aftershocks. Some of these buildings also recorded other earthquakes, e.g. the 1971 San Fernando (M_L = 6.6) and the 1987 Whittier Narrows (M_L = 5.9) earthquakes and some its aftershocks. All the three earthquakes occurred within the metropolitan area and caused strong shaking and damage. The system frequencies were found to be the lowest during the strongest shaking from the main shock, suggesting system softening, and then increased during the aftershocks, suggesting system recovery. The observed temporary changes varied from one building to another, but did not exceed 30% for this data set. The system "recovery" was interpreted to be due to dynamic compaction of the soil during the (weak) aftershock shaking.

1. INTRODUCTION

In most earthquake resistant design codes, the design shear forces are quantified using the seismic coefficient $C(T)$, where T is the "fundamental vibration period of the building," and various scaling factors that depend on the seismic zone, type of structure, soil site conditions, importance of the structure etc. As the building period cannot be measured for a particular structure before its construction, most codes provide simplified empirical formulae for its estimation, based on past experience and recorded response of existing buildings. The problem of estimation of T has been considered by many investigators, based on theory [Biot, 1942], small amplitude ambient and forced vibration tests of full-scale structures [Carder, 1936], and recorded earthquake response [Li and Mau, 1979].

The most reliable are the estimates of building periods obtained from recorded earthquake response. Such data are, however, extremely limited, both in quantity and in quality. The number of well-documented instrumented buildings that have recorded at least one strong earthquake is typically less than 100. When the recorded data is grouped by structural systems (moment resistant frame, shear wall etc.) and building materials (reinforced concrete, steal, etc.), the number of records per group becomes too small to control the accuracy of regression analyses, or to separate "good" from "bad" empirical models [Goel and Chopra, 1997; Stewart et al., 1999]. This problem is further complicated by the nonlinearity of the foundation soil even for very small strains [Hudson, 1970; Luco et al., 1987]. During strong earthquake shaking, the apparent period of the soil-foundation-structure system can lengthen significantly [Udwadia and Trifunac, 1974], and it may or may not return to its pre

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earthquake value. Environmental factors, such as temperature and heavy rainfall have also been found to cause small but systematic temporary changes [Clinton et al., 2006; Todorovska and Al Rjoub, 2006]. All of these factors contribute to the scatter in empirical regression analyses of building periods, and to ambiguity in choosing a representative T for evaluation of $C(T)$ [Trifunac, 1999; 2000].

For further improvements and developments of the building codes, it is essential to understand the amplitude dependent period lengthening (as function of the level of response of the structure and strain in the soil), and estimate its range. This can be best accomplished by analysis of building periods from *multiple* earthquake recordings in buildings—of *both* small and large levels of shaking. The first step towards this goal is to augment the database of multiple earthquake records in buildings, which is very limited, because most buildings records have been recorded on film, and, mostly only those with larger amplitudes have been digitized and released. Data of small amplitude response is being generated fast from instrumented buildings with a digital recording system, but it may take many years before they record larger amplitude response. Hence, as far as the building design codes are considered, the use of small amplitude data from *newly* instrumented buildings is quite limited. While small amplitude data are useful in those buildings in which large amplitude response has already been recorded by analog recorders, replaced by the digital system, smaller amplitude analog recordings of past earthquakes are also very valuable—for understanding of the variations of building periods with time, which may be temporary or permanent. In the Los Angeles metropolitan area, there have been many small earthquakes and aftershocks of larger earthquakes that have been recorded in buildings and archived but not digitized and released.

To this effect, an effort was initiated at the University of Southern California to augment the database of building periods estimated from multiple earthquake recordings, with the immediate objective to trace their variations with time and as a function of the level of response and understand their nature, and with the ultimate objective—to improve the code formulae for estimation of building periods. The effort consisted of digitization and processing of strong motion records in building in the Los Angeles area that have been archived at the U.S. Geological Survey (USGS), gathering of already processed data for the same buildings, and analysis of the building frequencies as a function of the level of response. This paper presents a summary of results for 21 buildings. Results for the first set of 7 buildings analyzed can be found in Todorovska et al. [2004a,b]. Similar analysis for a 7-story reinforced concrete hotel building in Van Nuys, damaged by the 1994 Northridge earthquake, can be found in Trifunac et al. [2001a,b], and for other instrumented buildings can be found in Hao et al. [2004], and Trifunac et al. (2001c,d,e).

2. METHODOLOGY

The instantaneous frequency was estimated by two methods: (a) zero-crossing analysis, and (b) from the ridge of the Gabor transform, both applied to the relative roof displacement when there was a record at the base, or to the absolute displacement when only the roof response was recorded, and considered as an approximation of the relative displacement in the neighborhood of the first system frequency. Both methods were applied to the filtered displacement, such that it contained only motion in the neighborhood of the first system frequency, and resembled a chirp signal. The zero-crossing analysis consists of measuring the time between consecutive zero crossings of the displacement, and assuming this time interval to be a half of the system period (see Trifunac et al. [2001c,d,e]. The Gabor transform is a time-frequency distribution, which is up to a phase shift identical to a moving window analysis with a Gaussian time window. The instantaneous frequency was determined from the ridge of the transform, and the corresponding amplitude was estimated from the skeleton of the transform, which is the value of the transform along the ridge [Todorovska, 2001]. The results by both methods were found to be consistent, within the scatter.

3. DATA

The data processed and analyzed was recorded in buildings in the Los Angeles metropolitan area that have been instrumented either by the USGS and partner organizations, or by the building owner (as required by the Los Angeles and state building codes), the latter being commonly referred to as “code” buildings. The data from these buildings have been archived by USGS, and are referred to in this paper as “USGS instrumented buildings,” identified by their station number. Figure 1 shows a map of the Los Angeles metropolitan area and locations of such buildings that were instrumented at the time of the 1994 Northridge earthquake.

The sensors in these buildings have been either three-component SMA-1 or multi-channel CR-1 accelerographs, both recording on film. Many of the "code" buildings (about 30 buildings total) have only one instrument, at the roof, due to a change in the original ordinance for Los Angeles, such that only one instrument at the roof was required, which lead to removal or neglect of the instruments at the ground floor and intermediate levels. This unfortunate fact limits considerably the use of these records, especially for analyses of soil-structure interaction. The recorded (absolute) roof motions can be used to estimate the apparent building period, as approximations of the relative roof motion near the first system frequency.

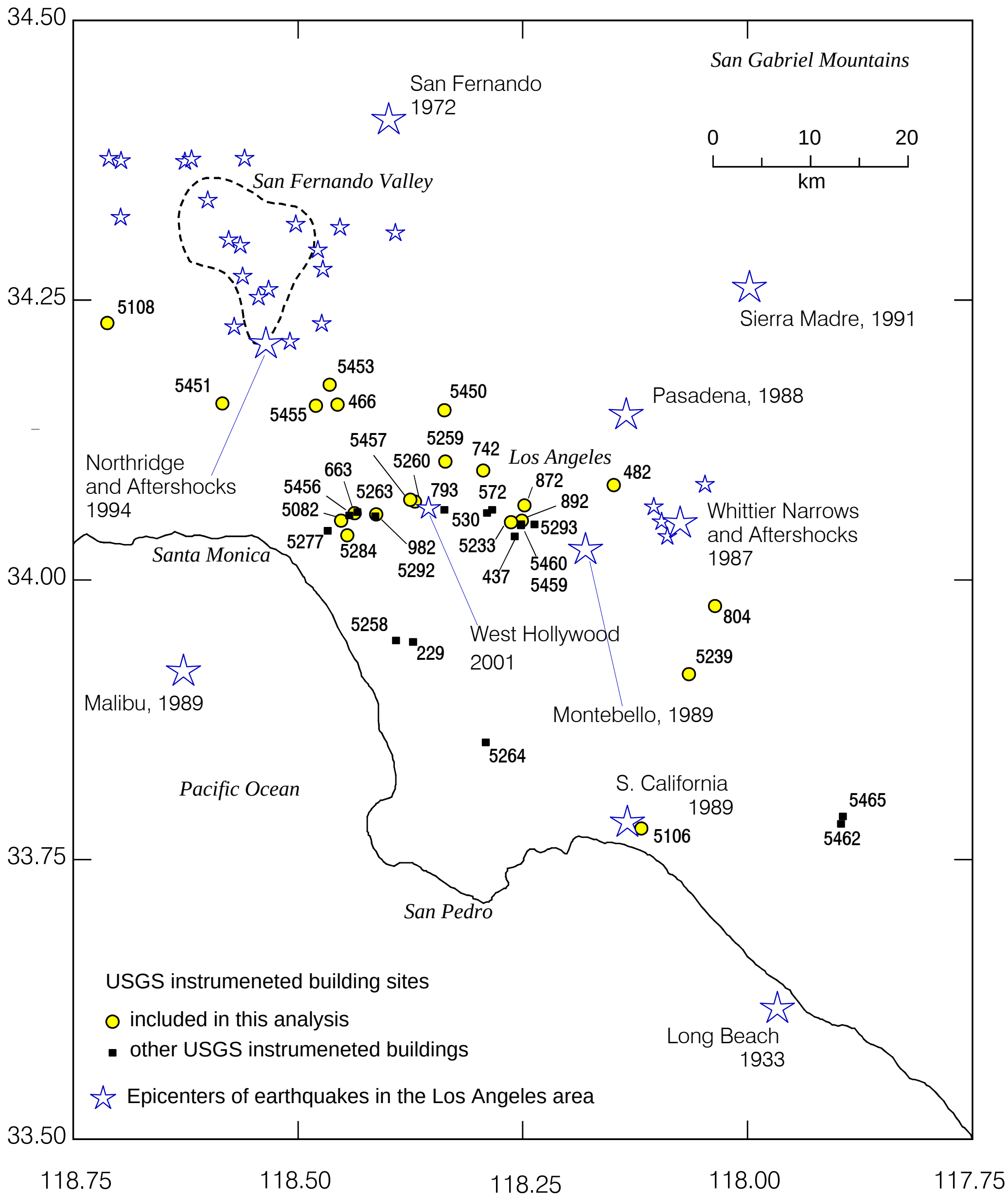

Figure 1 Locations of instrumented buildings in the Los Angeles metropolitan area at the time of the 1994 Northridge earthquake, for which the data is archived by USGS. The building sites are identified by their station number.

After the Northridge earthquake, the analog strong motion instrumentation is being gradually replaced by digital, and additional buildings are being instrumented. For some of these buildings, data of smaller local earthquakes and distant larger earthquakes has been recorded and released. The recorded level of response for these events, however, is much smaller than that for the Northridge earthquake. Figure 1 also shows the epicenters of earthquakes that have been recorded in these buildings. The Northridge main event was followed by a large number of aftershocks (9 of these had M > 5, and 55 had M > 4). Many of these larger magnitude aftershocks, as well as smaller magnitude but closer aftershocks, were recorded in the instrumented buildings. The aftershock of March 20, 1994 (M = 5.2; “aftershock 392”) was the one recorded by the largest number of (ground motion) stations [Todorovska et al., 1999]. The Northridge sequence was recorded on several films archived separately. The largest number of recorded aftershocks known to the authors of this paper is 86—at station USGS #5455, and about 60 at several other stations. Unfortunately, it turned out that the number of aftershock records useable for estimation of the building apparent frequency was small—up to 11.

This paper shows results for 21 buildings for which there were three or more adequate records of both strong and weak shaking (mostly the 1994 Northridge sequence or the Whittier-Narrows sequence) to estimate the apparent building frequency. These stations are marked by open (yellow) dots in Fig. 1. The stations marked by solid rectangles, less than three “adequate” records for such analysis were known to exist, and were not included in this analysis.

Table 1. Earthquakes recorded by USGS instrumented buildings (1971 to 2001).

Event	Date	Time	M_L	Latitude	Longitude	Depth (km)
San Fernando	02/09/1971	06:00	6.6	34 24 42N	118 24 00W	--
Whittier-Narrows	10/01/1987	14:42	5.9	34 03 10N	118 04 34W	14.5
Whittier-Narrows, 12th Aft.	10/04/1987	10:59	5.3	34 04 01N	118 06 19W	13.0
Whittier-Narrows, 13th Aft.	02/03/1988	15:25	4.7	34 05 13N	118 02 52W	16.7
Pasadena	12/03/1988	11:38	4.9	34 08 56N	118 08 05W	13.3
Malibu	01/19/1989	06:53	5.0	33 55 07N	118 37 38W	11.8
Montebello	06/12/1989	16:57	4.4	34 01 39N	118 10 47W	15.6
Upland	02/28/1990	23:43	5.2	34 08 17N	117 42 10W	5.3
Sierra Madre	06/28/1991	14:43	5.8	34 15 45N	117 59 52W	12.0
Landers	06/28/1992	11:57	7.5	34 12 06N	116 26 06W	5.0
Big Bear	06/28/1992	15:05	6.5	34 12 06N	116 49 36W	5.0
Northridge	01/17/1994	12:30	6.7	34 12 48N	118 32 13W	18.4
Northridge, Aft. #1	01/17/1994	12:31	5.9	34 16 45N	118 28 25W	0.0
Northridge, Aft. #7	01/17/1994	12:39	4.9	34 15 39N	118 32 01W	14.8
Northridge, Aft. #9	01/17/1994	12:40	5.2	34 20 29N	118 36 05W	0.0
Northridge, Aft. #100	01/17/1994	17:56	4.6	34 13 39N	118 34 20W	19.2
Northridge, Aft. #129	01/17/1994	20:46	4.9	34 18 04N	118 33 55W	9.5
Northridge, Aft. #142	01/17/1994	23:33	5.6	34 19 34N	118 41 54W	9.8
Northridge, Aft. #151	01/18/1994	00:43	5.2	34 22 35N	118 41 53W	11.3
Northridge, Aft. #253	01/19/1994	21:09	5.1	34 22 43N	118 42 42W	14.4
Northridge, Aft. #254	01/19/1994	21:11	5.1	34 22 40N	118 37 10W	11.4
Northridge, Aft. #336	01/29/1994	11:20	5.1	34 18 21N	118 34 43W	1.1
Northridge, Aft. #392	03/20/1994	21:20	5.2	34 13 52N	118 28 30W	13.1
Hector Mine	10/16/1999	09:46	7.1	34 36 00N	116 16 12W	3.0
West Hollywood	09/09/2001	23:59	4.2	34 04 30N	118 22 44W	3.7

Table 1 shows a list of earthquakes recorded in “USGS” instrumented buildings. For the Northridge sequence, only the aftershocks are shown for which there is an adequate record that has been used in the analysis presented in this paper. For most of the buildings, the contributing aftershocks have not been identified. For this analysis, however, the amplitude of response and their chronological order were sufficient. This table also lists the 2001 West Hollywood earthquake (M = 4.2), which occurred close to many of the instrumented buildings (see Fig. 1), and which were likely recorded by these buildings.

4. RESULTS

For each record considered for this analysis, the instantaneous system frequency was estimated and plotted versus time, and also versus the instantaneous amplitude of response. The latter curves were plotted on the same plot for all of the events, which made it possible to observe the variations of the system frequency as function of the amplitude of response *during a particular earthquake*, and also *from one earthquake to another*. Figure 2 shows such a plot for station USGS 5108 (Santa Susana ETEC Building No. 462), for data from the Northridge earthquake and its aftershocks, and for instantaneous frequency estimated by zero-crossing analysis. The horizontal axis shows the instantaneous frequency, and the vertical axis shows the amplitude of relative roof response expressed as a rocking angle in radians. The amplitudes of the response are those of the signal band-pas filtered near the first system frequency. The rocking angle was computed by dividing by the distance between the top and bottom instruments, estimated using average floor height of 12.5 feet (1 foot=30.48 cm) the amplitude of the relative (roof minus base) response, if motion at the base was recorded, or otherwise—the absolute horizontal response of the roof or top floor. It is noted here that this rocking angle includes the *rigid body* rocking, associated with soil-structure interaction, which could not be separated because of insufficient number of instruments at the base, in addition to motion resulting from *deflection* of the structure.

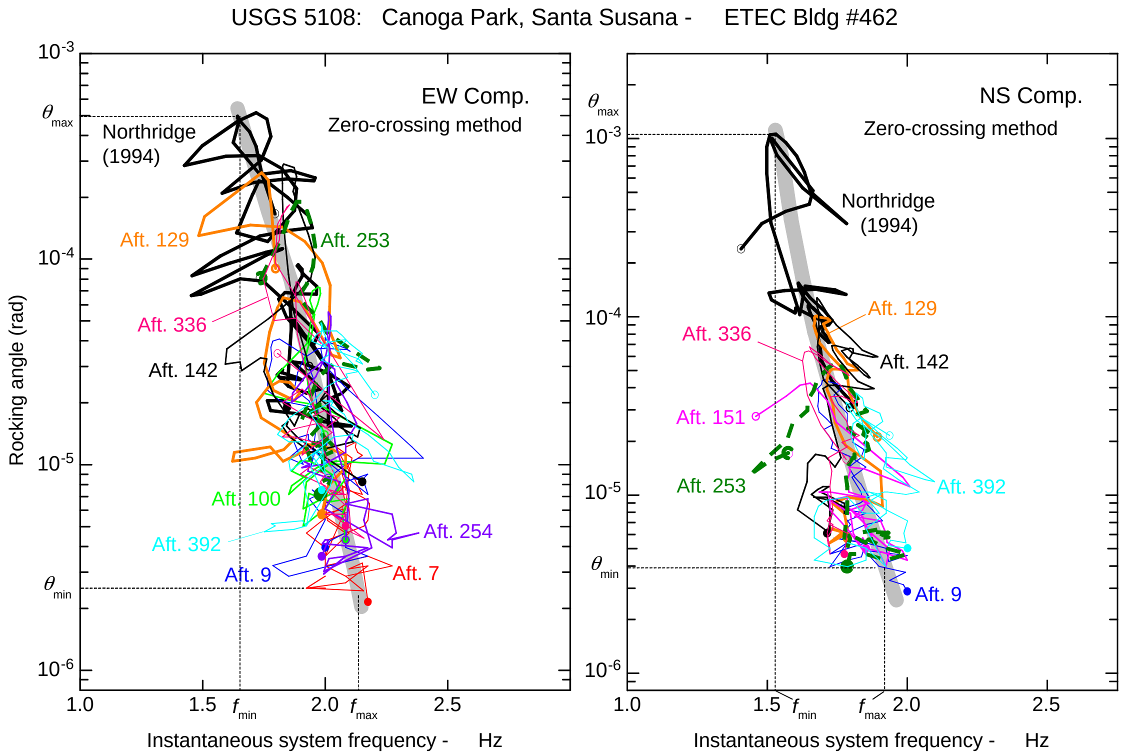

Figure 2 Instantaneous frequency versus amplitude of motion for station USGS 5108.

Each point in Fig. 2 corresponds to a particular instant in time, and the points corresponding to consecutive instants of time are connected by a line. Different lines are used for different earthquake events. The first and last point for each event are marked respectively by an open and a closed circle. There is a considerable scatter in the estimates, mostly caused by violations of the assumption that the signals analyzed (the relative or absolute roof displacement) are asymptotic, which is the basic assumption for virtually all nonparametric methods for estimation of instantaneous frequency [Todorovska 2001; Todorovska and Trifunac 2006]. Asymptotic signals are such signals whose variation in time is mostly due to change in phase rather than change in amplitude. The asymptoticity assumption is violated most in the instants when the amplitudes of the signal are small and the amplitude modulation varies significantly.

Despite the scatter in the data, the trend of the variation of system frequency with amplitude of response can be seen clearly in Fig. 2, and is marked by a backbone curve drawn approximately by hand. Such curves were drawn for all the stations. The common trend seen for most stations is a decrease of system frequency at the

Table 2. Maximum and minimum system frequencies and maximum and minimum rocking angles for 21 instrumented buildings.

Station No.	No. floors	Comp.	No. rec.	f_{max} Hz	f_{min} Hz	$\Delta f / f_{max}$ %	θ_{max} $\times 10^{-3}$ rad	θ_{min} $\times 10^{-3}$ rad	Comp.	No. rec.	f_{max} Hz	f_{min} Hz	$\Delta f / f_{max}$ %	θ_{max} $\times 10^{-3}$ rad	θ_{min} $\times 10^{-3}$ rad
0466	13	N00E	3	0.38	0.31	17.2	4.746	0.123	W00N	3	0.30	0.22	27.2	4.664	0.316
0482	12	NS	5	0.52	0.48	8.7	1.660	0.059	EW	5	0.51	0.46	8.8	2.818	0.115
0663	12	S72W	3	0.52	0.51	1.2	3.449	0.070	S18E	2	0.525	0.50	4.8	2.000	0.115
0742	8	E00S	12	2.52	2.15	14.7	0.400	0.003	N00E	10	1.69	1.18	30.2	1.820	0.005
0793	11	S00W	5	0.98	0.79	19.4	2.754	0.008	E00S	6	0.92	0.72	21.7	2.344	0.010
0804	10	NS	2	0.80	0.68	15.0	0.500	0.017	EW	2	1.22	1.14	6.2	1.000	0.022
0872	8	N12W	3	0.63	0.57	9.5	1.698	0.042	S78W	3	0.76	0.63	17.1	1.995	0.021
0892	55	N83E	2	0.53	0.52	1.9	0.240	0.010	W83N	2	0.53	0.52	1.9	0.027	0.013
5082	6	N35W	2	1.16	1.01	12.9	1.820	0.013	S55W	2	1.19	0.94	21.0	1.622	0.016
5106	11	NS	3	1.84	1.73	6.0	0.331	0.006	EW	3	1.85	1.76	4.9	0.260	0.010
5108	6	E00S	9	2.13	1.65	22.6	0.496	0.003	N00E	7	1.90	1.52	19.7	1.056	0.004
5233	32	N62W	2	0.62	0.46	25.8	0.630	0.015	S28W	2	0.65	0.48	26.2	0.468	0.007
5239	7	N90E	5	0.90	0.81	10.0	0.741	0.005	S00W	5	0.81	0.76	6.2	1.622	0.008
5259	8	N00E	8	1.95	1.51	22.6	1.023	0.002	W00N	6	3.62	2.89	20.2	0.219	0.002
5260	12	W65N	7	0.83	0.71	14.5	0.300	0.016	S65W	9	0.80	0.68	15.0	4.169	0.020
5263	19	E70S	5	0.61	0.49	19.7	2.239	0.013	N70E	3	0.62	0.49	21.0	1.585	0.014
5450	9	N00E	5	0.69	0.61	11.2	3.088	0.039	W00N	5	0.67	0.58	13.5	5.166	0.038
5451	12	N00E	3	0.33	0.27	17.2	7.384	0.180	W00N	3	0.43	0.37	14.1	9.573	0.134
5453	9	N00E	4	0.61	0.43	29.2	7.870	0.061	W00N	8	0.74	0.71	5.7	4.881	0.026
5455	13	E30S	5	0.43	0.41	3.9	5.394	0.056	N30E	5	0.43	0.36	14.6	5.361	0.099
5457	10	N00E	8	0.68	0.57	15.8	6.340	0.029	S00W	7	0.87	0.70	18.6	3.625	0.013

time of and shortly following the largest amplitudes of response to the main shock, and a recovery during the shaking by the aftershocks.

The backbone curves were used to read roughly the range of the frequency changes and amplitudes of response, which are shown for the 21 buildings in Table 2. The percentage changes for all 21 buildings are plotted in Fig. 3. It is seen that, for the range of responses in this database, the change for most of the buildings does not exceed more than 25%, and it did not exceed 30% for any of the buildings included in this analysis.

Results on further progress in this study will be posted at http://www.usc.edu/dept/civil_eng/Earthquake_eng/.

5. SUMMARY AND CONCLUSIONS

This paper presents results on the changes of the apparent building frequency of 21 buildings in the Los Angeles area during multiple earthquake excitation, which caused both large and small amplitude response. Most of these buildings recorded the 1994 Northridge earthquake and some of its aftershocks. Although the number of recorded aftershocks in these buildings was large (up to about 80), only a small number of records were useable for this analysis, because of the small signal to noise ratio at short frequencies, especially for the tall buildings. The objective of the analysis was to estimate the trends and roughly the range of changes of the system frequencies during multiple earthquake shaking, *both strong and weak*. The system frequency was estimated by two methods—zero crossing and Gabor analysis. The results by both methods were found to be consistent. The general observed trend of the variation of the system frequency is decrease during the main event and recovery during the aftershocks. For most buildings, the frequency changed up to 25%, and it did not exceed 30% for this data set. The "recovery" is consistent with an interpretation that the change was mainly due to changes in the soil (rather than in the structure itself), or changes in the bond between the soil and the foundation. Other possible causes of the temporary changes are: contribution of the nonstructural elements to the total stiffness in resisting the seismic forces, and opening of existing cracks in the concrete structures during larger amplitude response. The degree to which each of these causes contributed to the temporary changes cannot be determined from the current instrumentation and is beyond the scope of this analysis.

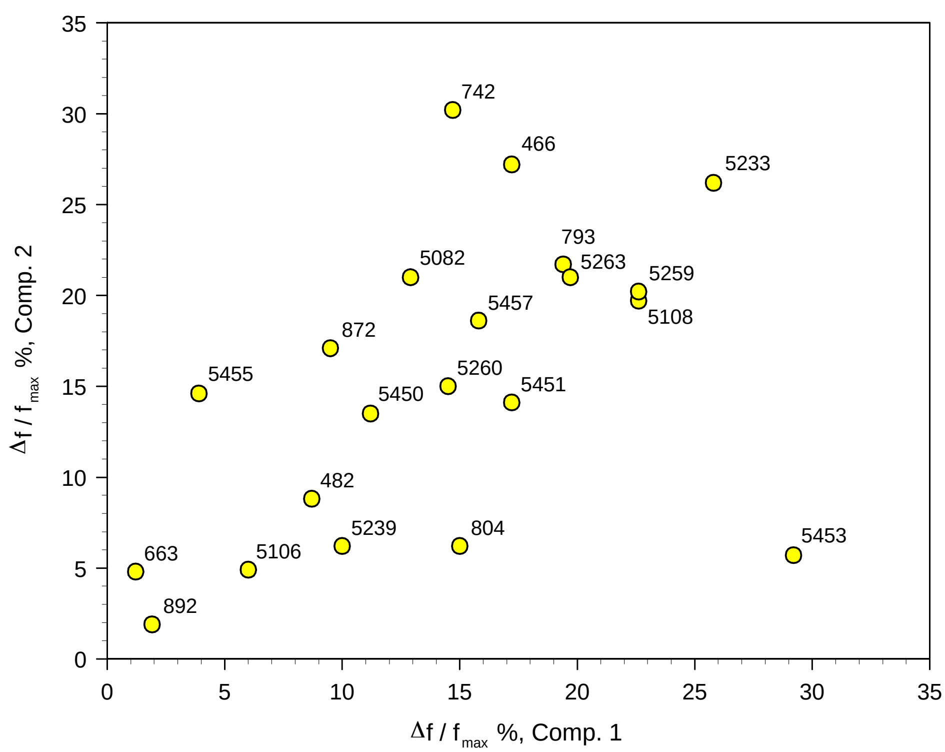

Figure 3 Observed changes in system frequency for 21 buildings.

6. ACKNOWLEDGEMENTS

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