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Brief history of computation of earthquake response spectra

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Abstract

The methods for computation of response spectrum amplitudes are reviewed for the period preceding the modern digital computer age. The mechanical and electrical analog methods that preceded the modern digital calculations were time consuming, inaccurate, and difficult to verify. Modern studies of response and of the nature of strong ground motion became possible after mid-1960 with accumulation of strong-motion records and with accurate digitization and digital data processing.

1. Introduction

The response spectrum method (RSM) developed by Biot [1–5] remained in the academic sphere of research for about 40 years, finally gaining wide engineering acceptance during the early 1970s. There were two main reasons for this. First, the computation of structural response to irregular ground motion led to "certain rather formidable difficulties" [6], and, second, there were only a few well-recorded accelerograms that could be used for response studies (Fig. 1).

All of this started to change in the mid-1960s with appearance of digital computers and with commercial availability of strong-motion accelerographs [7,8]. Before the digital computer age, the computation of response was extremely time consuming, and the results were so unreliable that many studies from that period, that used response spectrum amplitudes, must be treated with caution [9]. By the late 1960s and early 1970s, the digitization of analog accelerograph records [10] and the digital computation of ground motion and of the response spectra were developed completely and tested for accuracy [11,12]. Then, in 1971, with the occurrence of the San Fernando, California, earthquake, the modern era of RSM was launched. This earthquake was recorded by 241 accelerographs, including more than 175 in the Los Angeles area, where a large number of instruments had been installed at various levels in high-rise buildings. By combining the data from the San Fernando earthquake with all previous strong-motion records, it become possible to launch the comprehensive empirical scaling analyses of spectral amplitudes [13–15].

2. Computation of response spectra

2.1. Historical Review

Computation of response spectra requires the solution of Duhamel's integral ([11]; Appendix A) and then selection of the maximum response. Prior to the age of digital computers, execution of these tasks was difficult and very time consuming. For example, before the 1940s, direct numerical integration [16] and semi-graphical procedures using Intergraph instruments [17] had been used.

"The first use of a mechanical analyzer for finding oscillator response to an earthquake motion was by Frank Neumann [18,19] of the U.S. Coast and Geodetic Survey in 1936. In this work, the earthquake displacement curve, obtained by double integration of an accelerogram, was used to govern the motion of a torsional pendulum" (see discussion by M.P. White [20]).

Response spectra were evaluated mechanically at Stanford University, as follows. "The acceleration record was integrated twice to give ground displacements. A cam cut

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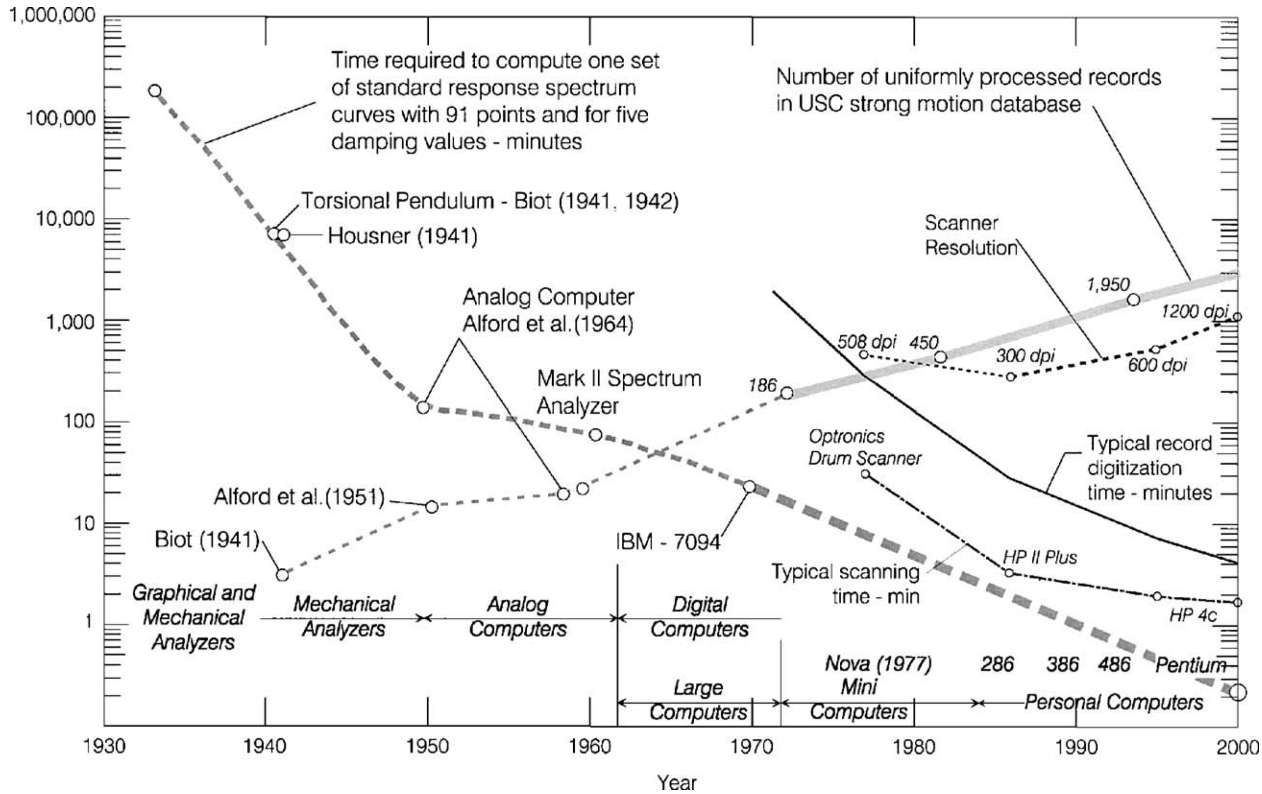

Fig. 1. Trends in the capabilities of accelerogram digitization and data processing, between 1970 and 2000: time required to compute one set of standard response spectrum curves (in minutes), and the cumulative number of accelerograms in strong-motion data bases (light dashed line for the period prior to 1970), and in the uniformly processed strong-motion data bases (wide gray line for the period after 1970).

in the pattern of these displacements actuated a shaking table upon which a simple oscillator was placed.'' The maximum relative displacement of such an oscillator multiplied by its natural frequency, ω_n, then gave the required value of pseudo-spectral velocity [21,22].

White and Byrne [23] suggested a method by which an accelerogram can be used directly to actuate a mechanical analyzer. This principle is the same as the one later employed by Biot [4,5], and Housner [21,24].

The first practical method for computation of spectral amplitudes was based on the torsional pendulum analog [4,25]. In this method, an oscillator is represented by an eccentric mass supported by stretched wire, one end of which is forced to twist through angles proportional to the acceleration amplitude, versus time [4,21,26]. The most time-consuming difficulty associated with the use of such a torsional pendulum was the inconvenience of changing the natural period of torsional response. Gross changes in period were made by using torsional wire of different diameters. Fine changes were made by selecting the excentricity of the mass on the inertia bar. Damping was also difficult to control. At first, it was thought to be zero, but later it was discovered to be in the range of a few percent of critical. The damping in the torsional pendulum came from the internal friction of the torsional spring and from air damping of the inertia bar [26]. With Biot's torsional pendulum at Columbia University, it took about 8 h to construct one spectrum curve consisting of about 30 points [5]. At Caltech, it took about 15 min to construct one spectrum point [26]. Prorating these durations to computation of spectra at 91 period points for five damping values [11] results in a duration of work of about 7000 min (167 h; Fig. 1).

At the Earthquake Research Institute of Tokyo University, a moving coil galvanometer element was used as the mechanical torisonal system [27]. It had a torsional element with fixed frequency, and the period changes were effected by changing the speed of the film drive mechanism in the ground motion generator. By energy input into the torsional system, through electrical feedback loop, effective zero damping of the system was possible.

The idea of using analog computers for computation of response spectra can be traced back to 1934: ''The direct computation of … spectra might be tedious, but automatic electrical methods can be easily imagined, such as a photographic record passing in front of a photoelectric cell acting upon a tuned circuit'' [3]. This idea was finally implemented, 20 years later, during the 1950s [26,28].

In the late 1940s, an analog computer technique was proposed for solving the response of a single-degree-of-freedom system to arbitrary excitation $F(t) = -M\ddot{z}$ [29,30]. The differential equation of motion of a mechanical single-degree-of-freedom system,

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{z}, \tag{1}$$

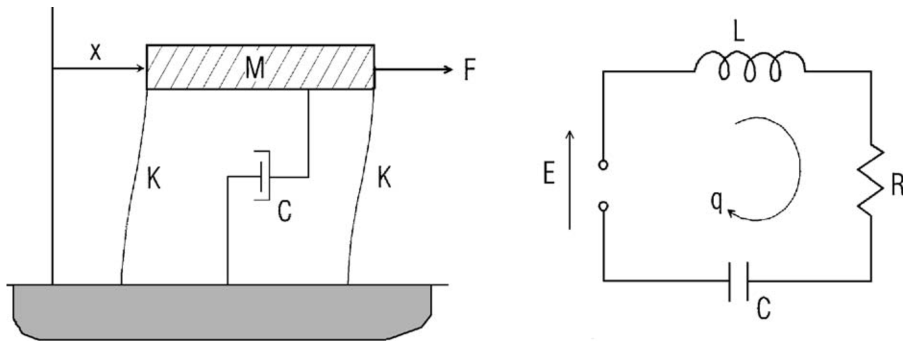

Fig. 2. Basic electromechanical analog (redrawn from [26]).

can be represented via its electrical analog as

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = E, \tag{2}$$

where M, C, and K are mass, damping coefficient, and stiffness, respectfully, x is relative displacement of M, and z is absolute ground displacement. In Eq. (2), L is inductance, R is resistance, and C is capacitance. E is applied variable voltage, and q is electrical charge (Fig. 2 and Table 1). The time dependent voltage input E was introduced through a photoelectric cell, which scanned a rotating film disc (Fig. 3).

The significance of the analog computer was that it enabled, for the first time, systematic calculation of response spectra with assigned damping values. It was about 30 times faster than the torsional pendulum analog (Fig. 1). Crede et al. [31] showed how a commercial electronic differential analyzer could be used for determination of response spectra. Then a special-purpose spectrum analyzer using electronic operation techniques was described by Morrow and Riesen [32]. Using these ideas, a small special-purpose analog computer system, Mark II, designed for computation of response spectra, was developed in 1954 and tested through the mid-1950s [28]. Using this electric analog, response spectra were calculated for a series of strong-motion earthquakes in the western United States [17].

In the early 1960s the methods for computation of response spectra started to change, following the general availability of digital computers. Digitized accelerograms could be used in Duhamel integral and integration could be performed numerically. Assuming that acceleration data can be approximated by piece-wise straight line segments between equally spaced points in time, the Duhamel integral can be integrated exactly over each time interval, thus reducing numerical integration to a sequential application of 2×2 matrices and two 2-component vectors. This required eight multiplications and six additions for each time step, or 14N operations for an accelerograms defined by N points ([33]; see Appendix A).

Through the 1980s, proposals were made to speed up these calculations by using digital filter simulations of response [34–36]. Lee [37] showed that by using the digital

Table 1
Mechanical–Electrical relations for analog[a]

Mechanical system	Electrical analog
M = mass of system	L = inductance = $(a/N^2)m$
K = spring constant	C = capacitance = $1/ak$
C = damping constant	R = resistance = ac/N
τ = period of vibration	τ^1 = simulated period = τ/N
F = exciting force	E = applied voltage
x = displacement	q = electrical charge
v = velocity	i = current
$x = a(F/E)q$	
N = time-scale change factor	
a = impedance change factor	

[a]From Alford et al. [26].

impulse and step invariant simulations of a continuous system, calculations of response displacement and velocity can be reduced to two multiplications and three additions per time step and to two multiplications and four additions per time step if displacement, velocity, and acceleration responses are computed simultaneously.

The speed of digital computers has increased remarkably since 1970 (Fig. 1), eliminating the need to increase the speed of response calculation. Therefore, in USC's Strong Motion Data Processing Laboratory, we have chosen to continue with use of the exact response calculations based on equally spaced points in time and linear interpolation of acceleration between the digitized points [11,38,39].

In summary, before introduction of the torsional pendulum analog, computation of response spectra was so long and difficult that spectra of only several recorded accelerograms for "zero" damping could be considered [21,24]. Between 1940 and 1950, the torsional pendulum method [4] "was a big advance, because it was about 30 times quicker than doing it graphically" [40]. The introduction of the analog computer method in the early 1950s reduced the time of computing spectral amplitudes about 60 times, but the conversion of recorded accelergrams into film disk records [26,28], the selection and calibration of required electrical constants, and the reading of maximum responses from an oscilloscope complicated and delayed the process. With introduction of digital

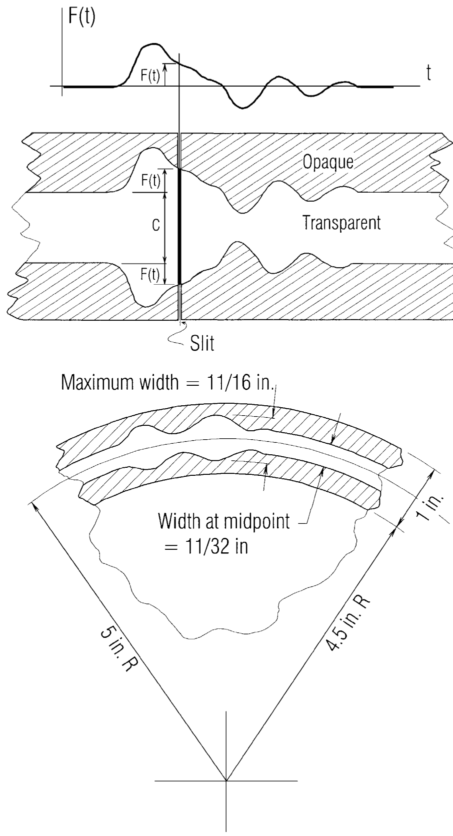

Fig. 3. Top: Transformation of input function to variable-width film trace. Bottom: Dimensions of the standard film disk record for the function generator (redrawn from [28]).

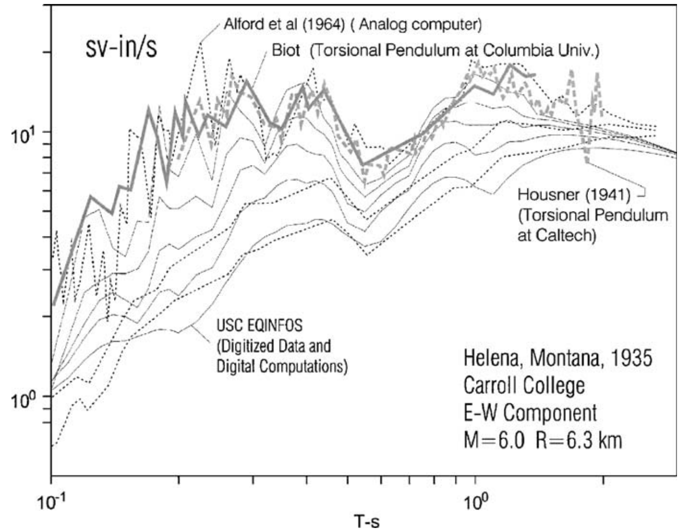

Fig. 4. Comparison of relative velocity response spectrum amplitudes for E–W component of strong motion recorded at Carroll College, during Helena, Montana earthquake of 1935. Spectra computed by torsional penduli of Biot and Housner (damping not specified) are compared with spectra computed by analog computer (damping values 0.0, 0.10, and 0.20), and digital computer (for five damping values 0., 0.02, 0.05, 0.10, and 0.20 [41]).

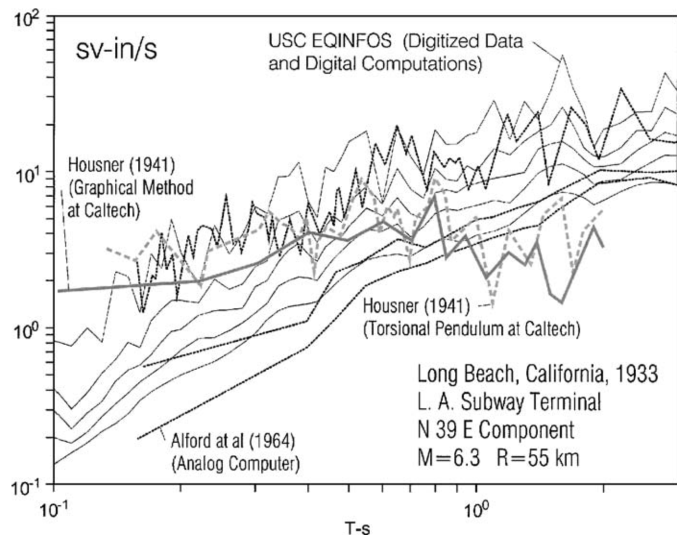

Fig. 5. Comparison of relative velocity response spectrum amplitudes for N 39 E component of strong motion recorded at Los Angeles Subway Terminal, during Long Beach, California earthquake of 1933. Spectra computed by graphical method (zero damping), torsional pendulum (damping not specified), analog computer (damping values 0.0, 0.10, and0.20), and digital computer (for five damping values 0., 0.02, 0.05, 0.10, and 0.20 [41]).

computers in the 1960s, it became necessary to convert analog film records of acceleration into digital points, and until the late 1970s this slowed down the process considerably. Since the early 1990s, digitization of analog records has become fast, efficient, and accurate. At present, it is the organization of the whole process and the archiving and distribution of data that limit the speed with which response spectra can be obtained in their final form.

2.2. Accuracy

Figs. 4–6 illustrate the accuracy of different old methods for computing response spectra. Fig. 4 compares relative velocity spectra computed by (1) Biot, using a torsional pendulum at Columbia University [21]; (2) Housner, using a torsional pendulum at Caltech [21]; (3) [26], using an analog computer; and (4) modern digital computing ([11], from digitized accelerogram of Helena, Montana, earthquake of 1935; [42]). It can be seen that all spectra follow the same broad trends (large amplitudes near 0.2–0.4 s and near 1 s, and small amplitudes near 0.5–0.6 s), but the local peaks fluctuate in a random manner. For most periods, zero damped spectral amplitudes computed by modern digital methods, are smaller than all three "old" spectra. This may be related to the selection of scaling constants used in electrical analog computing and in the two analyses based on the torsional pendulum. Two spectral curves, for $\zeta = 0.1$ and 0.2, computed by Alford et al. [26] have amplitudes and trends similar to those of digitally computed spectra, but their details do not agree.

Figs. 5 and 6 reveal far worse agreement. In both figures, the spectra computed by Alford et al. [26] follow the trend of modern results, but detailed comparison shows serious

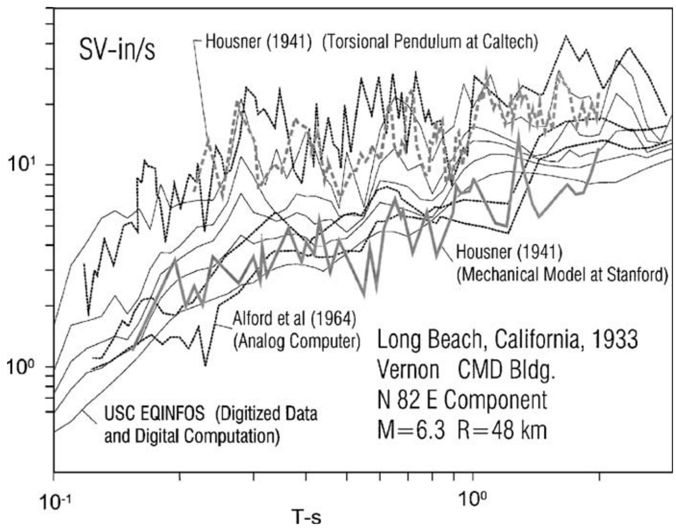

Fig. 6. Comparison of relative-velocity response spectrum amplitudes for N 82 E component of strong motion recorded at Vernon, CMD Building, during Long Beach, California earthquake of 1933. Spectra computed by mechanical model at Stanford (damping not specified), torsional pendulum at Caltech (damping not specified), Analog Computer (three damping values 0., 0.10, and 0.20), and digital computer (for five damping values 0., 0.02, 0.05, 0.10, and 0.20 [41]).

discrepancies at various periods. For the Vernon record [42], spectra computed by torsional pendulum at Caltech [21] have a correct overall trend, but local peaks do not correlate with those in modern calculations. The spectra computed by the mechanical method at Stanford have the wrong amplitudes and are depleted of high-frequency amplitudes. The spectra computed by the graphical method and by torsional pendulum at Caltech [21], for the Los Angeles subway terminal record [43], shown in Fig. 5, have erroneous amplitudes and trends. The differences are so large that the explanation probably involves an erroneous selection of scaling constants or possibly the use of an incomplete record. These major discrepancies, together with other similar discrepancies in spectral amplitudes reported by Trifunac et al. [9], show that the accuracy and reliability of the methods used to compute response spectra prior to the introduction of digital computer methods were so bad that all pre-1960s spectra must be considered with great caution. These examples also show that the estimates of accuracy "to within 10 percent" using an analog computer, claimed by Alford et al. [26], were not realistic. The speed, efficiency, and accuracy of computing response spectrum amplitudes by analog computer were apparently offset by noise introduced into the process while (1) making the film disc records [28] and (2) reading the peak response from a cathode ray tube display. It is unfortunate that there are no published reports on analyses and properties of these errors.

Biot [2] had unmistakable vision when he stated that "the peaks of spectral curves will reveal the presence of certain characteristic frequencies of the soil at given locations." With today's accurately digitized data and modern digital processing, response spectra can be used for such studies [44,45]. Unfortunately, as illustrated by Figs. 4–6, the accuracy of the "old" methods for response calculation was not adequate for such analyses.

3. Preparation of accelerograms for processing

In the old graphic method of digitization, the record was first drawn on a large scale. This was followed by multiplications and integration, or the use of an Intergraph instrument.

The introduction of the torsion pendulum [4,5] presented an advantage in that it was not necessary to convert the recorded accelerogram to a different analog record. A point of suspension of a torsional pendulum was given angular displacements by rotating its arm through an angle proportional to recorded acceleration. The accelerogram was placed on a table traveling with constant velocity, while the arm, connected to the torsional wire at one end and a tracer on the other, was forced to follow the acceleration trace.

The introduction of analog computers required conversion of inertial force $-M\ddot{z}$ in the mechanical system into applied voltage $E(t)$ in the electrical analog. This was accomplished by designing a "plotting table" to prepare film disc records, which were then used in a forcing function generator to produce $E(t)$ (Fig. 3). According to Caughey et al. [28]: "The earthquake ground acceleration record drawn to a suitable scale is wrapped around a drum, which is slowly rotated by an electric motor around a vertical axis. The curve is manually traced by a follower mechanism, which is converted through a selsyn system to a shutter, thus exposing a photographic negative, which rotates along with the drum. In this way, a variable-width film trace is produced … where the overall slit width is seen to be equal to a constant plus twice the acceleration function. A similar slit system, along with a light source and a photocell, is then used in the function generator to reproduce the original ground acceleration curve" (See Fig. 3).

With the appearance of digital computers, it became necessary to convert analog acceleration traces into a sequence of digital points representing acceleration versus time. The first digitization system in California capable of digitizing a large number of accelerograms and converting the digitized data into computer punch cards was described by Hudson [46]. Similar-hand operated digitization tables were in operation at that time in Japan, the Soviet Union, New Zealand, and Yugoslavia. This digitization method was accurate but time consuming. Digitization, plotting to verify the accuracy of digitization, and the conversion of data from computer punch cards to files on magnetic tapes took, on the average, four days per record. However, subsequent data processing was relatively fast and efficient. It consisted of: (1) preparation of scaled digitized accelerograms (vol. I; [47]); (2) instrument and baseline correction, followed by computation of velocity and displacement curves (vol. II; [48]); (3) computation of response spectra (vol. III; [49]); and (4) computation of Fourier amplitude spectra using a fast Fourier algorithm (vol. IV; [50]).

The next major step forward in digitization of strong-motion accelerograms occurred in 1978, when the first automatic system for digitization (based on a rotating drum scanner by Optronics, controlled by a Nova mini-computer) was developed at the University of Southern California [38]. With this system, digitization of a typical accelerogram was reduced to 1–2 h. Because checking for the accuracy of digitization now became an integral part of running the digitization software, and because the digitized and corrected data resided on the same computer disk, this system was about 50 times faster than the hand-operated digitizers. During late 1980s with the development of high-resolution flatbed scanners (HP II with 300 dpi resolution, and HP 4C with 600 dpi resolution) and the commercial availability of fast personal computers, automatic digitization was converted to operate on this new hardware [39,44]. At present, it takes less than 15 min to digitize and prepare the vol. I data of good quality accelerograms that are less than 28s long. This is about 380 times faster than with the hand-operated digitizers of the late 1960s and early 1970s [46].

4. Data distribution

Before the 1970s, the number of digitized and processed accelerograms and their response spectra was small (less than about 100), and the data distribution could easily be organized through personal contacts and mail. During the 1970s and through the early 1980s, magnetic tapes with data could be ordered from major centers performing data processing (United States Geological Survey (USGS); the California Division of Mines and Geology (CDMG); and the University of Southern California's Strong Motion Group). These groups also contributed their data for archiving and distribution by the National Geophysical Data Center in Boulder, Colorado.

With the development of the Internet and the creation of specialized Web sites dealing with strong-motion data, it is possible today to download large volumes of strong-motion data at no cost. Useful links to Web sites that offer such data can be found at http://www.usc.edu/dept/civil_eng/Earthquake_eng/.

5. Discussion and conclusions

Before the digital computer age, computation of response spectra of strong-motion accelerograms was difficult and labor intensive, and the results had very uncertain accuracy. This, in combination with a very small number of available recorded accelerograms, made it impossible to carry out empirical studies on the scaling of earthquake spectral amplitudes. Also, it was difficult to explore the governing laws and to link the physical nature of the earthquake source mechanism with the amplitudes and shape of the response spectrum. It was primarily for these reasons that the response spectrum method was confined largely to the realm of academic research for almost 40 years (1932–1972).

As mentioned earlier, all of this changed during the early 1970s. Not only did digital computers became widely available, but also the number of recorded strong-motion accelerograms grew rapidly. Since the early 1980s it has become possible to carry out sophisticated and complex regression analyses of the recorded data, to search for intricate and detailed properties of the physical nature of strong earthquake ground motion, and to discover how this nature affects the response spectrum amplitudes. Today, we understand all of the principal factors that determine the overall amplitudes and shapes of the response spectra in Southern California [15,51–55]. In the future, when sufficient strong-motion data have been recorded in other seismically active areas of the world, it will be possible to develop such area-specific empirical scaling equations in these locations, as well.

Appendix A

For the standard corrected accelerograms [48], with amplitudes specified at equally spaced time intervals Δt, an approach based on the exact analytical solution of the Duhamel integral, for the successive linear segments of excitation, appears to be the most practical. This approach is described in [33]. For completeness of this paper, the important features of this method are briefly summarized here.

The differential equation for the relative motion $x(t)$ of a single-degree-of-freedom oscillator subjected to base acceleration $a(t)$ is

$$\ddot{x} + 2\omega\varsigma\dot{x} + \omega^2 x = -a(t), \tag{A.1}$$

where, $\zeta =$ fraction of critical damping and $\omega =$ the natural frequency of vibration of the oscillator. For $a(t)$, given by a segmentally linear function for $t_i \leq t \leq t_{i+1}$, (A.1) becomes:

$$\ddot{x} + 2\omega\varsigma\dot{x} + \omega^2 x = -a_i + \frac{\Delta a_i}{\Delta t}(t - t_i), \tag{A.2}$$

where

$$\Delta t = t_{i+1} - t_i = \text{const.} \tag{A.3}$$

and

$$\Delta a_i = a_{i+1} - a_i. \tag{A.4}$$

The solution of Eq. (A.2), for $t_i \leq t \leq t_{i+1}$, is

$$x(t) = \mathrm{e}^{-\varsigma\omega(t-t_i)}[C_1 \sin\omega_\mathrm{d}(t - t_i) + C_2 \cos\omega_\mathrm{d}(t - t_i)] - \frac{a_i}{\omega^2} + \frac{2\varsigma}{\omega^3}\frac{\Delta a_i}{\Delta t} - \frac{1}{\omega^2}\frac{\Delta a_i}{\Delta t}(t - t_i), \tag{A.5}$$

where

$$\omega_\mathrm{d} \equiv \omega\sqrt{1 - \varsigma^2}. \tag{A.6}$$

Setting $x = x_i$ and $\dot{x} = \dot{x}_i$ at $t = t_i$, C_1 and C_2 become

$$C_1 = \frac{1}{\omega_\mathrm{d}}\left(\varsigma\omega x_i + \dot{x}_i - \frac{2\varsigma^2 - 1}{\omega^2} + \frac{\varsigma}{\omega}a_i\right), \tag{A.7}$$

$$C_2 = x_i - \frac{2\varsigma}{\omega^3}\frac{\Delta a_i}{\Delta t} + \frac{a_i}{\omega^2}. \tag{A.8}$$

Substituting C_1 and C_2 into Eq. (A.5) and setting $t = t_{i+1}$ leads to the recurrence relationship for x_i and $\dot{x}_i$, given by

$$\begin{Bmatrix} x_{i+1} \\ \dot{x}_{i+1} \end{Bmatrix} = [A(\varsigma, \omega, \Delta t)]\begin{Bmatrix} x_i \\ \dot{x}_i \end{Bmatrix} + [B(\varsigma, \omega, \Delta t)]\begin{Bmatrix} a_i \\ a_{i+1} \end{Bmatrix}. \tag{A.9}$$

The elements of matrices A and B are:

$$\begin{aligned}
a_{11} &= \mathrm{e}^{-\varsigma\omega\Delta t}\left(\frac{1}{\sqrt{1-\varsigma^2}}\sin\omega_\mathrm{d}\Delta t + \cos\omega_\mathrm{d}\Delta t\right),\\
a_{12} &= \frac{\mathrm{e}^{-\varsigma\omega\Delta t}}{\omega_d}\sin\omega_\mathrm{d}\Delta t,\\
a_{21} &= -\frac{\omega}{\sqrt{1-\varsigma^2}}\mathrm{e}^{-\varsigma\omega\Delta t}\sin\omega_\mathrm{d}\Delta t,\\
a_{22} &= \mathrm{e}^{-\varsigma\omega\Delta t}\left(\cos\omega_\mathrm{m}\Delta t - -\frac{1}{\sqrt{1-\varsigma^2}}\sin\omega_\mathrm{d}\Delta t\right).
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
b_{11} &= \mathrm{e}^{-\varsigma\omega\Delta t}\left[\left(\frac{2\varsigma^2-1}{\omega^2\Delta t} + \frac{\varsigma}{\omega}\right)\frac{\sin\omega_\mathrm{d}\Delta t}{\omega_\mathrm{d}}\right.\\
&\quad \left. + \left(\frac{2\varsigma}{\omega^3\Delta t} + \frac{1}{\omega^2}\right)\cos\omega_\mathrm{d}\Delta t\right] - \frac{2\varsigma}{\omega^3\Delta t},\\
b_{12} &= \mathrm{e}^{-\varsigma\omega\Delta t}\left[\left(\frac{2\varsigma^2-1}{\omega^2\Delta t}\right)\frac{\sin\omega_\mathrm{d}\Delta t}{\omega_\mathrm{d}} + \frac{2\varsigma}{\omega^3\Delta t}\cos\omega_\mathrm{d}\Delta t\right]\\
&\quad - \frac{1}{\omega^2}\frac{2\varsigma}{\omega^3\Delta t},\\
b_{21} &= \mathrm{e}^{-\varsigma\omega\Delta t}\left[\left(\frac{2\varsigma^2-1}{\omega^2\Delta t} + \frac{\varsigma}{\omega^2}\right)\left(\cos\omega_\mathrm{d}\Delta t - \frac{\varsigma}{\sqrt{1-\varsigma^2}}\sin\omega_\mathrm{d}\Delta t\right)\right.\\
&\quad \left. - \left(\frac{2\varsigma^2}{\omega^2\Delta t} + \frac{1}{\omega^2}\right)(\omega_\mathrm{d}\sin\omega_\mathrm{d}\Delta t + \varsigma\omega\cos\omega_\mathrm{d}\Delta t)\right] + \frac{1}{\omega^2\Delta t},\\
b_{22} &= -\mathrm{e}^{-\varsigma\omega\Delta t}\left[\frac{2\varsigma^2-1}{\omega^2\Delta t}\left(\cos\omega_\mathrm{d}\Delta t - \frac{\varsigma}{\sqrt{1-\varsigma^2}}\sin\omega_\mathrm{d}\Delta t\right)\right.\\
&\quad \left. - \frac{2\varsigma}{\omega^2\Delta t}(\omega_d\sin\omega_\mathrm{d}\Delta t + \varsigma\omega\cos\omega_\mathrm{d}\Delta t)\right] - \frac{1}{\omega^2\Delta t}.
\end{aligned} \tag{A.11}$$

Therefore, if the displacement and velocity of the oscillator are known at t_i, the complete response can be computed by a step-by-step application of Eq. (A.9). The advantage of this method lies in the fact that for a constant time interval Δt matrices A and B depend only upon ζ and ω and are constant during the calculation of the response.

To calculate and plot complete response spectra, maximum values of relative displacement $SD = |x(t)|_{\max}$, relative velocity $SV = |\dot{x}(t)|_{\max}$, and absolute acceleration $SA = |\ddot{x}(t) + a(t)|_{\max}$ are stored for each period $T = 2\pi/\omega$ and a fraction of critical damping ζ. The calculation of these maxima is approximate because the displacement $x(t)$, velocity $\dot{x}(t)$, and acceleration $\ddot{x}(t)$ are found only at discrete points, where the values are x_i, $\dot{x}_i$, and $\ddot{x}_1 + a_i$ for $i = 1, 2, \ldots, N$ (N is the total number of discrete, equally spaced points at which the input accelerogram is given). For standard spectrum calculations, the choice of the interval of integration ΔT is selected to be

$$\Delta t \le \frac{T}{10}, \tag{A.12}$$

but it is always less than or equal to $\Delta t_{\max} = 0.02$ s. Here, T is the period of the oscillator for which the spectrum point is calculated. For such a choice of integration interval the discretization error is less than 5%.

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