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Modified Mercalli intensity scaling of the frequency dependent duration of strong ground motion

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New empirical models of the duration of strong ground motion in terms of the Modified Mercalli intensity at the recording station are presented. Two groups of regression equations are considered: one explicitly includes the dependence of the duration on the distance to the source, and the other excludes this dependence. The Modified Mercalli intensity serves as a parameter in both types of models. The models of the first type are more descriptive, but are also more region dependent, because the regional dispersion and attenuation laws are 'built into' the frequency dependent regression coefficients.

For a given site intensity, the duration grows when the distance from the source to the recording site increases. For a given distance from the source, the dependence of the duration on the site intensity is more complex. At low frequencies, the duration of strong motion decreases when the intensity increases, while at high frequency it grows with increasing intensity. A smooth transition from one type of dependence to another occurs at intermediate frequencies.

When compared to basement rock sites, the duration of strong motion at sedimentary sites is prolonged by about 5 s at frequencies near 1 Hz. The prolongation of the duration on the soft soils can be as much as 7 s. The influence of the type of soils on the duration is stronger at higher frequencies ($f = 0.3$–25 Hz), while the effect of the presence of sedimentary deposits can be observed at lower frequencies ($f = 0.15$–2 Hz).

The residuals of the empirical regression equations were also studied, and their distribution function is proposed.

INTRODUCTION

Instrumental data are essential for all investigations of the amplitude and duration characteristics of strong ground motion. However, it takes time to accumulate sufficient and homogeneous data on the regional estimates of the magnitude scale. As a substitute for the instrumental data, a qualitative description of the earthquake effects in terms of the Modified Mercalli Intensity (MMI) scale[1] or its equivalents can be used.

This work investigates the relationship between the frequency dependent duration of strong ground motion and the Modified Mercalli intensity at the recording site, I_{MM}. Regression equations, relating the duration of strong motion to I_{MM} can be found in the literature,[2–4] but in this paper we present models which include new parameters and consider more detailed analyses.

We develop two groups of regression equations. One of the groups includes the distance to the source as one of the model parameters. Those models can be compared with the empirical equations relating the duration of strong ground motion to the magnitude of the earthquake and the epicentral distance.[5–7] The comparison allows us to study the MMI scale through several narrow frequency 'windows', and find for which frequencies and distances the MMI is governed more by the magnitude of the earthquake, than by the attenuation with distance.

Historically, the MMI scale has evolved dealing with a description of the earthquake effects on older structures. The damage of the modern and in particular long-period (tall) structures, long span bridges or large dams is hard to describe by the MMI scale, because such structures are not even mentioned in the definition of the

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scale. Thus, the correlations of frequency dependent duration of strong ground shaking with I_{MM} should be useful not only for the duration studies, but also because of the possibility to understand better the 'new' characteristics of the MMI scale at low frequencies.

The geological conditions at the recording site have been shown to influence the duration of strong motion when the Modified Mercalli intensity is used as the 'main' parameter in the scaling equations.[2] In this work, we expand such analyses to include the scaling in terms of the local soil conditions as well.

DEFINITION OF DURATION AND THE STRONG MOTION DATA

Following the works of Trifunac and Brady,[8] Trifunac and Westermo,[2–4,9,10] and our study on the dependence of duration of strong ground motion on the earthquake magnitude,[5–7] we adopt the definition of duration of a function of motion $f(t)$ (where $f(t)$ is acceleration, velocity or displacement) as the sum of the time intervals during which the integral $\int_0^t f^2(\tau)\,d\tau$ has the steepest slope and gains 90% of its final value. We prefer this definition because the integral $\int_0^t f^2(\tau)\,d\tau$ has a specific physical meaning.[8] For example, $\int_0^t v^2(\tau)\,d\tau$ is proportional to the total energy transmitted by the seismic waves past the recording point, and the time derivative of this integral gives the power of the seismic excitation as a function of time. The time derivative of $\int_0^t a^2(\tau)\,d\tau$ gives $a^2_{rms}(t)$. The functional $\int_0^t a^2(\tau)\,d\tau$ is proportional to the work (per unit mass) done during the time interval from $t = 0$ to $t = t_0$ by all the forces acting on a single degree-of-freedom viciously damped oscillator, excited by the acceleration $a(t)$. While the length of the record t_0 is sufficient to capture all the significant motions at the recording site, the functional $\int_0^{t_0} a^2(\tau)\,d\tau$ is related to the Arias intensity. Also, the prediction of the response, $f(t)$, of a multi-degree-of-freedom structure can be expressed[11–18] in terms of the number of peaks of $f(t)$ during the entire history of the excitation, the width of the power spectrum of $f(t)$ and by the value of $((1/t_0)\cdot\int_0^{t_0} f^2(\tau)\,d\tau)^{1/2}$.

Our definition of strong motion duration can be made frequency dependent by considering each record filtered through several relatively narrow band-pass filters. In this work, we use 12 frequency bands with central frequencies changing from $f_0 = 0{\cdot}075$ Hz up to $f_0 = 21$ Hz. A useful feature of the definition of duration used here is that, unlike some other physically related definitions,[19,20] it considers the strong motion part as being composed of several separate strong motion 'pulses', and the beginnings and the ends of these 'pulses' can be specified. The definition of the duration of strong motion as one continuous time interval is not meaningful for some records. The information about the location in the record and the duration of each separate strong motion pulse can be used to study the source of the earthquake and the associated wave propagation phenomena.[5]

We use the same database as in Novikova and Trifunac.[5–7,21] This database includes approximately three times more records that what was available to Trifunac, Brady and Westermo some 15 years ago. The current database has 486 vertical and 984 horizontal components of acceleration, velocity and displacement, generated by 106 earthquakes and recorded by strong motion accelerometers at 283 different sites, in the Western USA and, primarily, in California. These data are described by Lee and Trifunac[22] and by Novikova and Trifunac.[5,21] The methods employed in band-pass filtering and in the calculation of the duration of strong ground motion in each of the 12 frequency bands are presented by Novikova and Trifunac.[5] The durations of strong motion, obtained from the acceleration, velocity and displacement are treated together as one homogeneous set of data. This is possible due to the narrow band nature of the frequency bands used. Only carefully selected data were included in the analysis. Each channel of acceleration, velocity and displacement of each record was analyzed separtely. Cases where the duration of strong motion was obviously longer than the length of the recording were not included in the anlaysis. Also, the cases with too low signal to noise ratio were disregarded.

HOW TO MODEL THE DURATION OF STRONG GROUND MOTION IN TERMS OF THE MODIFIED MERCALLI INTENSITY?

In the empirical models of the duration of strong ground motion in terms of the earthquake magnitude, the epicentral distance and the site specific parameters,[5–7] we assumed the duration can be represented as a sum of (1) the source rupture time, τ_0, which can be related to the magnitude, (2) the dispersive term τ_Δ, which represents the prolongation of the duration along the propagation path, and (3) the terms which describe the influence of the site specific features (for example, the geometry of the sedimentary basin and the local soil conditions) at the recording site. In the models of the duration in terms of the Modified Mercalli intensity, however, the first two terms, $\tau_0 + \tau_\Delta$, are treated together, because the source and the propagation effects cannot be uncoupled easily when the shaking at a site is measured by the MMI scale. Thus, we will consider two groups of regression equations. In the first group, the simplest possible form of the dependence of the duration of the strong ground motion, *dur*, on the intensity at the site, I_{MM}, is assumed:

$$\tau_0 + \tau_\Delta = \text{const}_1 + \text{const}_2 \cdot I_{MM} \tag{1}$$

where const_1 and const_2 are unknown frequency dependent coefficients. The other group will include the distance to the source as one of the model parameters. Although we assume that no instrumental data are available, the models where $\tau_0 + \tau_\Delta$ depends on both the intensity at the site and the source to site distance are worth considering. First, these models can provide some useful information on the nature of the MMI scale. Second, the position of the epicenter can be approximately located even if no instrumental data are available, by creating a map of the Modified Mercalli intensities and finding the point where I_{MM} reaches its maximum.

We must choose the definition of distance and the way of including it in the regression equations. The answer to this question depends on the correlations of the Modified Mercalli intensity with the magnitude of the earthquake and on various definitions of distance: epicenter, hypocentral, closest distance to the fault, and others. These definitions may be different for different seismic regions and for different databases. Lee and Trifunac,[23] for example, studied the database which is used in this work and obtained the following estimate for I_{MM} at a site:

$$I_{MM} = 1{\cdot}5 \cdot M + 1{\cdot}12 - 0{\cdot}856 \cdot \ln \tilde{\Delta} - 0{\cdot}015 \cdot \tilde{\Delta} - 0{\cdot}26 \cdot s \tag{2a}$$

where $\tilde{\Delta}$ is 'representative distance'

$$\tilde{\Delta} = \sqrt{\Delta^2 + H^2 + L^2} \tag{2b}$$

Δ designates the epicentral distance, H is the hypocentral depth, L stands for the 'effective' source dimension, and s is the geological site classification parameter: $s = 2$ for the sites, located on the basement rock, $s = 0$ for the sites on sediments and $s = 1$ for the intermediate sites.[24] The 'effective' source dimension represents the part of the source which can be 'felt' at an epicentral distance Δ, and might be approximated by:

$$L = L(M)\left\{1 - \exp\left(\frac{\Delta \cdot \ln 0{\cdot}1}{L(M)}\right)\right\} \tag{2c}$$

where $L(M)$ is an empirically determiend linear function of magnitude, M, such that

$$\begin{aligned} &\text{for } M = 3 \quad L(M) = 0{\cdot}2\,\text{km} \\ &\text{for } M = 6{\cdot}5 \quad L(M) = 17{\cdot}5\,\text{km} \end{aligned} \tag{2d}$$

Only about one third of the database had the Modified Mercalli intensity actually observed at the recording site. The remaining two thirds of data on I_{MM} were estimated using eqn (2) and the 'representative distance' in eqn (2b) in particular. Thus, it is logical to use this 'representative distance' from the source to the site in our regression equations, which relate the duration of strong ground motion and the Modified Mercalli intensity at the site. To take advantage of the 'representative distance', some estimate of the source dimension $L(M)$ should be available. However, we do not wish to use the magnitude of an earthquake in the development of the models of duration of strong ground motion in terms of the Modified Mercalli intensity. The use of magnitude would contradict the assumption that no instrumental data are available. Thus, this restriction prevents us from taking advantage of the 'representative distance' $\tilde{\Delta}$, and the best we can do is to consider the hypocentral distance $\Delta' = \sqrt{\Delta^2 + H^2}$ as an approximation to $\tilde{\Delta}$. Of course, the use of the hypocentral distance requires a knowledge of the hypocentral depth H, which is not available if there were no instrumental records of an earthquake. However, in the regions with a seismogenic zone which can be described reasonably well (like the San Andreas fault system in Central and Southern California), the prevailing hypocentral depth can be estimated. Also, detailed studies of the rate of attenuation of the intensity with distance may be used to estimate the hypocentral depth.[25,26]

As far as the functional form of the empirical dependence of $\tau_0 + \tau_\Delta$ on I_{MM} and Δ' is concerned, we can obtain it by combining eqn (2) with the established functional forms[6] of $\tau_0(M)$ and $\tau_\Delta(\Delta)$. This, however, would result in nonlinear regression analysis and complications when considering small distances due to the presence of the term which involves the logarithm of the distance. A useful observation from the comparison of eqn (2) with $\tau_0(M)$ and $\tau_\Delta(\Delta)$ is that the dependence of $\tau_0 + \tau_\Delta$ on the intensity and distance should be coupled. To avoid the usually unstable (especially for noisy data) procedure of nonlinear regression analysis, and to use the simplest possible functional form for $\tau_0 + \tau_\Delta$ which would allow, however, the above mentioned coupling, we use:

$$\begin{aligned} \tau_0 + \tau_\Delta = {} & \text{const}_3 + \text{const}_4 \cdot I_{MM} + \text{const}_5 \cdot \Delta' \\ & + \text{const}_6 \cdot I_{MM}\Delta' \end{aligned} \tag{3}$$

where const_i are some unknown frequency dependent coefficients.

THE REGRESSION MODELS

Several regression equations modeling the duration of strong ground motion in terms of the Modified Mercalli intensity and other (site specific) parameters are discussed in this section. Two groups of equations are presented, the first group uses $\tau_0 + \tau_\Delta$ as given by eqn (3), the second group does not explicitly include the dependence of the duration on the distance to the source and utilizes a representation in terms of eqn (1). The first group is more region dependent, because the 'distance related' coefficients of the models from this group will change if the attenuation properties of the region are altered.

First, we describe the models which only include the terms $\tau_0 + \tau_\Delta$. Then we examine the models which include the influence of the geological and local soil site conditions on the duration. Each model is studied independently in 12 frequency bands, and the unknown regression coefficients are obtained by the singular value decomposition technique, which provides good control over the accuracy of the solution.[27]

The duration of strong duration motion estimated from any of the models will be called the 'estimated duration'. The value obtained from the acceleration, the velocity or the displacement, according to our definition of duration of strong ground motion, will be called the 'observed duration'.

Models $dur = dur(I_{MM}, \Delta', I_{MM}\Delta')$ and $\mathrm{dur} = \mathrm{dur}(I_{MM})$

The first model only includes the dependence of the duration of strong ground motion on the Modified Mercalli intensity and the distance to the source in the form of eqn (3). Renaming the coefficients, we have:

$$dur(f) = a_1(f) + a_{19}(f)\cdot I_{MM} + a_4(f)\cdot\Delta' + a_{20}(f)\cdot I_{MM}\Delta' \tag{4}$$

The numbering of the coefficients a_i was chosen in this way to maintain correspondence with our previous studies of the duration.[5–7,21] Equation (4) was first fit to the data for the horizontal and for the vertical components separately. No significant differences in the behavior of $a_{19}(f)$, $a_4(f)$ and $a_{20}(f)$ for the two components of motion were observed. However, the constant coefficient $a_1(f)$ was found to be different in these two cases. Also, the estimated duration appears to be negative at high frequencies for small hypocentral distances and for very small intensities. This can be explained by the lack of data for this combination of parameters. The final equation for the first model is then:

$$\left\{\begin{matrix} dur^{(h)}(f) \\ dur^{(v)}(f) \end{matrix}\right\} = \max\left[\left(\left\{\begin{matrix} a_1^{(h)}(f) \\ a_1^{(v)}(f) \end{matrix}\right\} + a_{19}(f)\cdot I_{MM} + a_4(f)\cdot\Delta' + a_{20}(f)\cdot I_{MM}\Delta'\right), 1\right] \tag{5}$$

The hypocentral distance, Δ', is measured in kilometers. The superscripts (h) and (v), correspond to the horizontal and to the vertical components of motion respectively.

Equation (5) was fit to the data in two steps. In the first iteration, instead of the formula $dur = \max[(\cdot), 1]$, the simple equation $dur = (\cdot)$ (i.e. eqn (4)) was considered and the coefficients $\{a_1(f), a_4(f), a_{19}(f), a_{20}(f)\}_{\mathrm{first}}$ were obtained. This first set was used in the second iteration for the evaluation of the quantity $(\cdot)$. The data points for which $(\cdot) < 1$ s were not included in the second iteration of the regression analysis. The set $\{a_1(f), a_{19}(f), a_4(f), a_{20}(f)\}_{\mathrm{second}}$ is almost same as $\{a_1(f), a_{19}(f), a_4(f), a_{20}(f)\}_{\mathrm{first}}$, and either of them can serve as a solution. The similarity between those two sets of coefficients follows from the fact that the database does not include many cases for which $(\cdot) < 1$ s.

Table 1 gives the regression coefficients of the model in eqn (5) as $a_i(f) \pm \sigma_i(f)$, where $\sigma_i^2(f)$ are the variances of the values found. A zero value for a coefficient corresponds to the cases when $|\sigma_i/a_i| > 1$. The number of available data points $N(f)$ is very different at each channel, reflecting the statistical reliability of the regression analysis performed. The average observed duration, dur_{av}, and the standard deviation of the estimated duration from the observed value, σ_{dur}, are also listed. Note the strong dependence of dur_{av} on the frequency.

Graphical representation of the regression coefficients, plotted versus the central frequency of the channels, is shown in Fig. 1. The duration of strong ground motion does not depend on the Modified Mercalli intensity level for low frequencies of motion ($f \lesssim 0{\cdot}1$ Hz). At these frequencies, the duration of the source rupture is shorter than the period of the wave used to measure it. This results also in an absence of the dependence of the duration of strong motion on the earthquake magnitude.[6] There is no dependence on the distance to the source either, because essentially only one mode of propagation of surface waves exists for such low frequencies (at local distances), and no dispersion can be noticed.[6] Thus, no dependence of the duration on the Modified Mercalli intensity should be expected.

The isolines of the strong motion duration at higher frequencies, as predicted by eqn (5), are shown for the horizontal component of motion in Fig. 2. For comparison, the observed duration is also presented. The latter is shown (averaged over the intervals of Δ' and I_{MM}) by the shades of different density, with a denser shade representing longer duration. The empty 'boxes' correspond to the ranges of intensities and hypocentral distances where no data are available. For

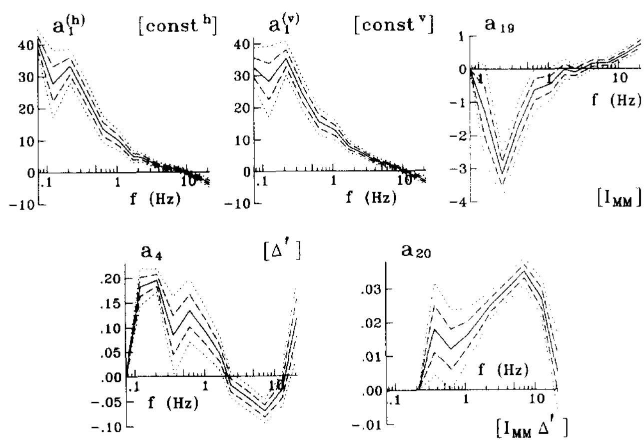

Fig. 1. The coefficients $a_i(f)$ in the model in eqn (5), plotted versus the central frequency of the channels (solid lines). The coefficients are bounded by their 'σ-intervals' (dashed lines) and by their 95% confidence intervals (dotted lines).

Table 1. Results of the regression analysis of the model in eqn (5)

Channel number	f_0 (Hz)	No. of data points $N(f)$	Coefficients a_i and their accuracy ('σ-interval')					σ_{dur} (s)	dur_{av} (s)
			$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	$a_1^{(v)}$ $\pm\sigma_1^{(v)}$	a_{19} $\pm\sigma_{19}$	a_4 $\pm\sigma_4$	a_{20} $\pm\sigma_{20}$		
1	0·075	37	40·8 ±2·0	32·5 ±3·1	0·0	0·0	0·0	10·2	38·3
2	0·12	311	27·7 ±5·4	28·2 ±5·7	−1·30 ±0·75	0·182 ±0·019	0·0	10·1	28·3
3	0·21	962	33·3 ±2·7	35·3 ±2·7	−3·17 ±0·37	0·195 ±0·012	0·0	7·8	21·4
4	0·37	1499	23·8 ±2·6	24·2 ±2·6	−1·73 ±0·39	0·084 ±0·040	0·018 ±0·007	7·4	21·0
5	0·63	2035	13·7 ±2·1	15·6 ±2·1	−0·62 ±0·32	0·134 ±0·033	0·012 ±0·006	7·8	18·7
6	1·1	2636	10·0 ±1·6	12·8 ±1·6	−0·44 ±0·25	0·089 ±0·025	0·016 ±0·004	7·0	15·6
7	1·7	3119	5·1 ±1·0	7·8 ±1·0	−0·03 ±0·16	0·046 ±0·018	0·021 ±0·003	5·4	12·4
8	2·5	3418	4·4 ±0·7	6·2 ±0·7	−0·11 ±0·11	−0·018 ±0·013	0·025 ±0·002	4·0	9·1
9	4·2	2739	1·7 ±0·6	3·1 ±0·6	0·16 ±0·10	−0·043 ±0·013	0·030 ±0·002	3·3	7·6
10	7·2	2576	1·0 ±0·5	1·6 ±0·5	0·18 ±0·08	−0·070 ±0·012	0·035 ±0·002	2·8	6·4
11	13	1584	−1·1 ±0·5	−1·0 ±0·5	0·46 ±0·08	−0·028 ±0·017	0·027 ±0·003	2·3	5·1
12	21	735	−3·4 ±0·7	−3·3 ±0·7	0·75 ±0·12	0·118 ±0·038	0·005 ±0·006	2·0	4·2
			1 horiz	1 vert	I_{MM}	Δ'	$I_{MM}\Delta'$		
			Corresponding parameters						

every fixed intensity, the duration grows with distance, due to the dispersion of the seismic waves.[6] The dependence of the duration of strong motion on the intensity at the site, for a fixed distance, is more complex. The intensity, by itself, is a function of the magnitude of the earthquake and of the distance to the source. Being directly proportional to the magnitude, the intensity grows when the magnitude increases. The intensity also grows when the distance to the source decreases. These two facts result in what might seem as a 'contradictory' behavior of the duration of strong motion as a function of the intensity at the site. On the one hand, the duration should increase when the intensity at the site increases, because this could correspond to the increase in the duration with the increase in magnitude. On the other hand, the duration should decrease with increasing intensity, because the increase of intensity could correspond to a shorter distance (with no change in magnitude). The resulting picture depends on which of those two effects prevails. One should also remember that the intensity at a site is often being assigned by estimating the damage to structures sensitive to the short period part of the spectrum at the site. Long and short period waves attenuate with different rates, so that a severe earthquake felt at a larger distance might have short period amplitudes smaller and long period amplitudes higher, than a smaller shock, recorded at a smaller distance. As a result, the behavior of the intensity scale, at low frequencies, may see 'contradictory' at first. For long period waves (channels No. 2 and 3, $f_0 = 0{\cdot}12$–$0{\cdot}21$ Hz), the influence of earthquake magnitude[6] is not 'felt', and the increase in I_{MM} (for fixed distance) causes the decrease in the duration. For high frequencies ($f_0 \geq 2{\cdot}5$ Hz), the dispersion does not play as important a role as for low and for intermediate frequencies.[6] As a result, the duration increases with increasing intensity, because the latter is caused primarily by a growing magnitude. In the intermediate frequency range (channels No. 4–7, $f_0 = 0{\cdot}37$–$1{\cdot}7$ Hz), the behavior of the duration as a function of the intensity for a fixed distance is of intermediate and dual nature. For large distances, it resembles the behavior typical for the high frequency channels, and for short distances $dur(I_{MM})$ appears to be similar to $dur(I_{MM})$ for low frequencies. The definitions of 'long' and 'short' should be scaled by the wavelength of the corresponding channel. Once this scaling is taken into account, it is easy to understand why the 'transition' distance (where 'short' distance borders with 'long' distance) moves towards the source when the frequency of vibration becomes higher.

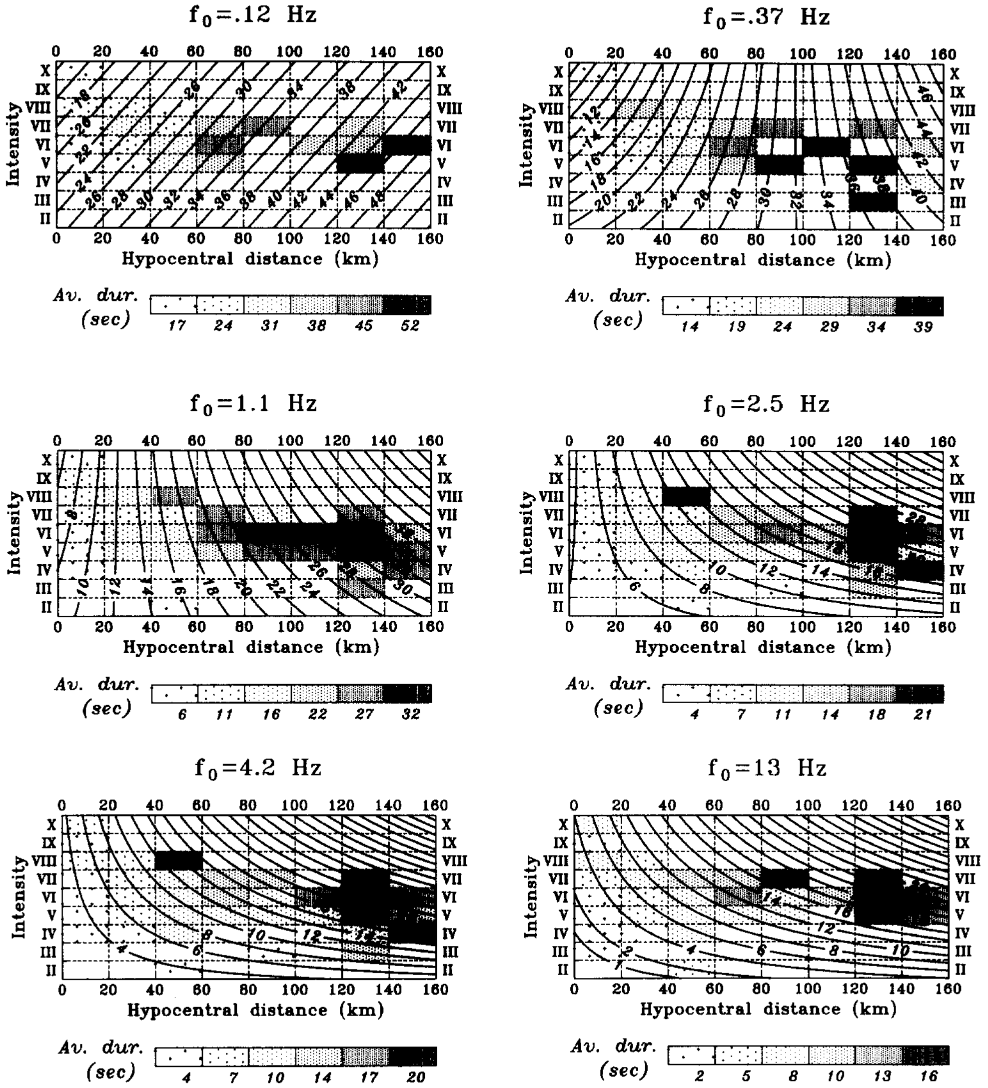

Fig. 2. Isolines of the duration (in seconds) of the horizontal component of strong earthquake ground motion as estimated from the model in eqn (5). Observed duration is shown averaged in the ranges of intensity and hypocentral distances, specified by the dashed mesh. The longer duration corresponds to a darker shade. For a fixed intensity, the duration grows with increasing distance. For a fixed distance, at low frequencies, duration tends to decrease when the intensity increases and at high frequencies, the duration grows with the increase in the intensity level. There is a smooth transition from one pattern to another in the moderate frequency range.

The model discussed above has the distance to the earthquake source as one of the parameters. We assume that the depth of the hypocenter can be obtained from teleseismic records, from the macroseismic field, or from geological field studies. The location of the epicenter can also be estimated if the intensities are known at many locations surrounding the epicenter. However, the model which does not include any distance to the source as a parameter, may appear to be more useful in practice. Such a model would also be less region dependent, as it does not assume (explicitly at least) any region specific dispersion or attenuation law. We next consider such a model:

$$\left\{ \begin{matrix} dur^{(\mathrm{h})}(f) \\ dur^{(\mathrm{v})}(f \end{matrix} \right\} = \max \left[\left(\left\{ \begin{matrix} a_1^{(\mathrm{h})}(f) \\ a_1^{(\mathrm{v})}(f) \end{matrix} \right\} + a_{19}(f) \cdot I_{\mathrm{MM}} \right), 1 \right] \tag{6}$$

Table 2 and Fig. 3 show the results of the regression analysis with this model, which was performed in two steps, similar to those for the model in eqn (5). The behavior of the coefficient $a_{19}(f)$ can easily be explained now by comparison with Fig. 2 (eqn (5)) and by recalling the discussion on the nature of the dependence of the

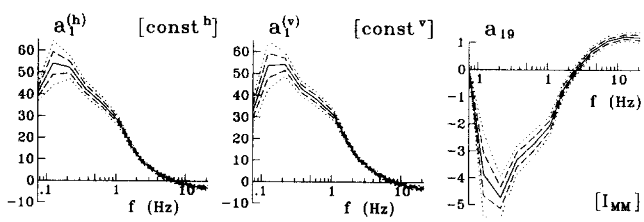

Fig. 3. The coefficients $a_i(f)$ in the model in eqn (6), plotted versus the central frequency of the channels (solid lines). The coefficients are bounded by their 'σ-intervals' (dashed lines) and by their 95% confidence intervals (dotted lines).

Table 2. Results of the regression analysis of the model in eqn (6)

Channel number	f_0 (Hz)	No. of data points $N(f)$	Coeff. a_i and their accuracy ('σ-interval') $a_1^{(h)}$ $\pm\sigma_1^{(h)}$	$a_1^{(v)}$ $\pm\sigma_1^{(v)}$	a_{19} $\pm\sigma_{19}$	σ_{dur} (s)	dur_{av} (s)
1	0·075	37	40·8 ±2·0	32·5 ±3·1	0·0	10·2	38·3
2	0·12	311	54·1 ±5·2	53·6 ±5·6	−3·88 ±0·79	11·5	28·3
3	0·21	962	52·3 ±2·7	54·2 ±2·8	−4·74 ±0·40	8·7	21·4
4	0·37	1499	42·3 ±1·8	43·2 ±1·8	−3·33 ±0·27	8·4	21·0
5	0·63	2035	35·8 ±1·6	37·9 ±1·7	−2·75 ±0·25	9·2	18·7
6	1·1	2636	27·7 ±1·2	30·9 ±1·3	−2·05 ±0·19	8·6	15·6
7	1·7	3119	15·9 ±0·9	18·8 ±0·9	−0·71 ±0·14	7·1	12·4
8	2·5	3418	9·2 ±0·6	11·2 ±0·6	−0·12 ±0·10	5·3	9·1
9	4·2	2739	3·1 ±0·6	4·7 ±0·6	0·66 ±0·10	4·9	7·6
10	7·2	2576	−0·3 ±0·5	0·5 ±0·6	1·06 ±0·09	4·5	6·4
11	13	1584	−2·5 ±0·5	−2·0 ±0·5	1·22 ±0·08	3·5	5·1
12	21	735	−3·2 ±0·6	−2·8 ±0·6	1·19 ±0·10	2·9	4·2
			1 horiz	1 vert	I_{MM}		
			Corresponding parameters				

duration on the intensity and the distance in the previous model. The duration decreases with the growth of the intensity for low and intermediate frequencies, when the increase of intensity corresponds to a decrease of distance. In the high frequency range ($f_0 \geq 2{\cdot}5$ Hz) the duration becomes longer when the intensity increases, and this shows that, for these frequencies, the intensity is governed more by the magnitude than by the distance to the source.

Models $dur = dur(I_{MM}, \Delta', I_{MM}\Delta', s)$, $dur = dur(I_{MM}, s)$

So far, we have considered the first two terms, $\tau_0 + \tau_\Delta$, from the general description of the duration of strong ground motion. We will call 'basic' the models that only include those two terms. Next we turn to the third term, which describes possible prolongation of duration due to some specific conditions at the recording site. It is known that the presence of deep sediments under the station may increase the duration of strong motion at some frequencies.[2] In this work, we will consider the influence of the geological condition, s, at the site using not one (as in the work cited above), but two independent coefficients. The parameter s is a qualitative indicator variable,[24] and needs to be considered in a different way to conventional quantitative variables.[28]

When eqn (5) is taken as a 'basic' equation, the model has the form:

$$\begin{Bmatrix} dur^{(h)}(f) \\ dur^{(v)}(f) \end{Bmatrix} = \max\left[\left(\begin{Bmatrix} a_1^{(h)}(f) \\ a_1^{(v)}(f) \end{Bmatrix} + a_{19}(f)\cdot I_{MM} + a_4(f)\cdot\Delta' + a_{20}(f)\cdot I_{MM}\Delta'\right), 1\right] + a_{13}(f)\cdot S^{(1)} + a_{14}(f)\cdot S^{(0)} \tag{7a}$$

where Δ' is measured in kilometers and

$$S^{(1)} = \begin{cases} 1, & \text{if } s = 1, \\ 0, & \text{if } s \neq 1, \end{cases} \quad S^{(0)} = \begin{cases} 1, & \text{if } s = 0 \\ 0, & \text{if } s \neq 0 \end{cases} \tag{7b}$$

Recall here that $s = 0$ corresponds to sites on sediments, $s = 2$ stands for the sites located on basement rock and $s = 1$ designates intermediate sites.[24]

The results of the regression analysis on eqn (7) are shown in Table 3. The set of coefficients $\{a_1(f), a_{19}(f), a_4(f), a_{20}(f)\}$ is similar to the same set from the model in eqn (5). The last two coefficients, $a_{13}(f)$ and $a_{14}(f)$, are shown in Fig. 4 as functions of the central frequency of the channel. The comparison of those coefficients with their counterparts from the model[6] where $\tau_0 + \tau_\Delta$ is expressed in terms of M and Δ, shows remarkable similarity between these two sets of coefficients. Such a

Table 3. Results of the regression analysis of the model in eqn (7)

Channel number	f_0 (Hz)	No. of data points $N(f)$	Coefficients a_i and their accuracy ('σ-interval')							σ_{dur} (s)	dur_{av} (s)
			$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	a_{19} $\pm\sigma_{19}$	a_4 $\pm\sigma_4$	a_{20} $\pm\sigma_{20}$	a_{13} $\pm\sigma_{13}$	a_{14} $\pm\sigma_{14}$		
1	0·075	37	40·8 ±2·0	32·5 ±3·1	0·0	0·0	0·0	0·0	0·0	10·2	38·3
2	0·12	311	27·7 ±5·4	28·2 ±5·7	−1·30 ±0·75	0·182 ±0·019	0·0	0·0	0·0	10·1	28·3
3	0·21	850	26·6 ±3·0	28·5 ±3·1	−2·51 ±0·41	0·191 ±0·013	0·0	0·0	2·83 ±0·64	7·9	20·7
4	0·37	1179	20·3 ±2·9	20·5 ±3·0	−1·51 ±0·43	0·088 ±0·042	0·016 ±0·007	0·0	2·98 ±0·51	7·7	20·5
5	0·63	1647	8·5 ±2·4	11·0 ±2·5	−0·55 ±0·35	0·127 ±0·035	0·012 ±0·006	2·84 ±0·85	5·76 ±0·74	8·1	18·3
6	1·1	2189	5·8 ±1·7	8·5 ±1·8	−0·23 ±0·26	0·093 ±0·026	0·014 ±0·004	1·44 ±0·63	3·83 ±0·54	7·1	15·1
7	1·7	2645	3·8 ±1·1	6·3 ±1·1	−0·09 ±0·17	0·039 ±0·018	0·021 ±0·003	1·10 ±0·44	2·36 ±0·38	5·6	12·2
8	2·5	2931	4·2 ±0·7	5·7 ±0·7	−0·19 ±0·12	−0·026 ±0·013	0·026 ±0·002	0·0	1·28 ±0·17	4·1	9·0
9	4·2	2464	1·2 ±0·6	2·4 ±0·6	0·16 ±0·10	−0·045 ±0·013	0·030 ±0·002	0.0	0·88 ±0·15	3·4	7·4
10	7·2	2576	1·0 ±0·5	1·6 ±0·5	0·18 ±0·08	−0·070 ±0·012	0·035 ±0·002	0·0	0·0	2·8	6·4
11	13	1584	−1·1 ±0·5	−1·0 ±0·5	0·46 ±0·08	−0·028 ±0·017	0·027 ±0·003	0·0	0·0	2·3	5·1
12	21	735	−3·4 ±0·7	−3·3 ±0·7	0·75 ±0·12	0·118 ±0·038	0·005 ±0·006	0·0	0·0	2·0	4·2
			1	1	I_{MM}	Δ'	$I_{mm}\Delta'$	$S^{(1)}$	$S^{(0)}$		
			horiz	vert	Corresponding parameters						

similarity provides additional support for the methods we use to construct our models, and for the models themselves. In the particular case, the assumption that $\tau_0 + \tau_\Delta$ can be modeled by eqn (3) receives some additional support.

The influence of the geological conditions at the site is noticeable for the intermediate frequencies only. At low frequencies, the wavelength of the waves is too long to 'feel' the presence of the sediments, and at high frequencies the attenuation effects may overshadow the prolongation which is caused by multiple reflections in the sediments.[29] These reflections may be the main reason for the duration on sediments being longer (on average) than the duration on rock sites.[5,6] $a_{14}(f)$ shows that, for the sites on sediments, this prolongation can be as much as about 5 s (channel No. 6, $f_0 = 1{\cdot}1$ Hz). $a_{13}(f)$ gives prolongation for intermediate sites of about 2·5 s at the same frequency.

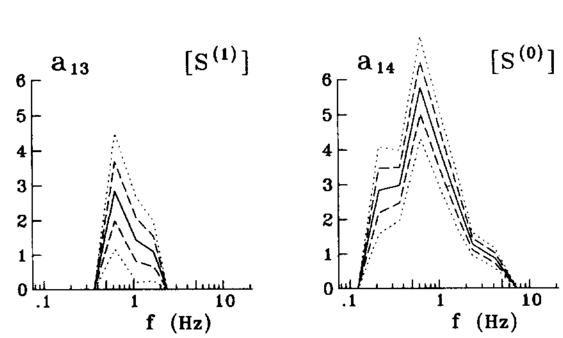

Fig. 4. The coefficients related to the site geologic classification in the model in eqn (7), plotted versus the central frequency of the channels (solid lines). The coefficients are bounded by their 'σ-intervals' (dashed lines) and by their 95% confidence intervals (dotted lines).

We now turn to the second model which includes the influence of the geological conditions at the recording site on the duration of strong ground motion. The 'basic' model in this case is the model in eqn (6). The preliminary analysis showed that the representation of the site-specific term in the form $a_{13}(f)\cdot S^{(1)} + a_{14}(f)\cdot S^{(0)}$ causes instability of the inversion. So, we consider then the geological parameter s as a 'regular' quantitative variable. The corresponding model equation is, then:

$$\left\{\begin{matrix} dur^{(h)}(f) \\ dur^{(v)}(f) \end{matrix}\right\} = \max\left[\left(\left\{\begin{matrix} a_1^{(h)}(f) \\ a_1^{(v)}(f) \end{matrix}\right\} + a_{19}(f)\cdot I_{MM}\right), 1\right] + a_{15}(f)\cdot(2-s) \tag{8}$$

We use here the term $a_{15}(f)\cdot(2-s)$ instead of $a_{15}(f)\cdot s$ because we want the coefficient $a_{15}(f)$ to be positive when showing that the duration on sediments ($s = 0$) and on intermediate sites ($s = 1$) is longer than the

Table 4. Results of the regression analysis of the model in eqn (8)

Channel number	f_0 (Hz)	No. of data points $N(f)$	Coefficients a_i and their accuracy ('σ-interval')				σ_{dur} (s)	dur_{av} (s)
			$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	a_{19} $\pm\sigma_{19}$	a_{13} $\pm\sigma_{13}$		
1	0·075	37	40·8 ±2·0	32·5 ±3·1	0·0	0·0	10·2	38·3
2	0·12	311	54·1 ±5·2	53·6 ±5·6	−3·88 ±0·79	0·0	11·5	28·3
3	0·21	850	44·4 ±3·3	46·3 ±3·4	−4·10 ±0·46	1·92 ±0·53	8·9	20·7
4	0·37	1179	37·2 ±2·2	38·0 ±2·2	−3·20 ±0·31	2·60 ±0·40	8·8	20·5
5	0·63	1647	28·7 ±1·9	31·3 ±2·0	−2·58 ±0·27	3·52 ±0·38	9·7	18·3
6	1·1	2189	21·2 ±1·4	24·4 ±1·5	−1·79 ±0·21	2·73 ±0·29	8·7	15·1
7	1·7	2645	12·7 ±1·0	15·4 ±1·0	−0·69 ±0·15	1·83 ±0·22	7·3	12·2
8	2·5	2931	7·5 ±0·7	9·3 ±0·7	−0·14 ±0·11	1·12 ±0·16	5·5	9·0
9	4·2	2464	1·8 ±0·6	3·3 ±0·7	0·64 ±0·10	0·74 ±0·14	4·9	7·4
10	7·2	2374	−0·8 ±0·6	−0·2 ±0·6	1·07 ±0·09	0·21 ±0·13	4·4	6·2
11	13	1500	−2·4 ±0·5	−2·0 ±0·6	1·20 ±0·09	0·04 ±0·13	3·4	5·1
12	21	735	−3·2 ±0·6	−2·8 ±0·6	1·19 ±0·10	0·0	2·9	4·2
			1 horiz	1 vert	I_{MM}	s		
			Corresponding parameters					

duration on basement rock ($s = 2$). The results of the analysis of this model are given in Table 4. All the results agree with the previously discussed models. The first three coefficients are very similar to those for the model in eqn (6), and

$$a_{15}(f) \approx 0{\cdot}5 \cdot \{a_{13}(f) + 0{\cdot}5 \cdot a_{14}(f)\}$$

where $a_{13}(f)$ and $a_{14}(f)$ are taken from the model in eqn (7) and $a_{15}(f)$ belongs to the model in eqn (8). The last relationship is what might be expected when the effect of the geology is described in two different ways; one of the ways accounts for the qualitative nature of the parameter s (and gives coefficients a_{13} and a_{14}), and the other one treats the parameter s as any other quantitative parameter.

Models $dur = (I_{MM}, \Delta', I_{MM}\Delta', s, s_L)$ and $dur = dur(I_{MM}, s, s_L)$

We next consider the influence of the local soil conditions at the recording site. This influence was not considered before in the regression models of the duration of strong ground motion with the Modified Mercalli intensity as the main scaling parameter. We use the soil classification factor s_L, which describes the sites on a local geotechnical engineering scale.[30] For deep soil sites, $s_L = 2$ (soil layer deeper than 100 m), and $s_L = 1$ for stiff soil (soil layer 15–70 m deep). Both $s_L = 2$ and $s_L = 1$ sites have a shear wave velocity less than 800 m/s. If the shear wave velocity in the soil material exceeds 800 m/s, the site is classified as 'rock', and $s_L = 0$.

The scaling of the duration in terms of the 'geological' parameter s and the 'local' soil parameter s_L should be added to the 'basic' model in the form

$$a_{11}(f) \cdot S_L^{(1)} + a_{12}(f) \cdot S_L^{(2)} + a_{13}(f) \cdot S^{(1)} + a_{14}(f) \cdot S^{(0)}$$

where $S^{(1)}$ and $S^{(0)}$ are indicator variables for $s = 1$ (intermediate sites) and $s = 0$ (sites on sediments), defined by eqn (7b), and $S_L^{(1)}$ and $S_L^{(2)}$ are the corresponding indicator variables for $s_L = 1$ (stiff soil sites) and $s_L = 2$ (soft soil sites). In both cases, the geological rock ($s = 2$) or the 'local soil rock' ($s_L = 0$), are chosen as a reference. Unfortunately, the instability of the regression analysis (due to the small number of stations with known s_L and the small amount of data on geological rock sites) does not allow us to include the indicator variables $S^{(1)}$ and $S^{(0)}$ in this model. So we use the model

$$\left\{ \begin{array}{c} dur^{(h)}(f) \\ dur^{(v)}(f) \end{array} \right\} = \max \left[\left(\left\{ \begin{array}{c} a_1^{(h)}(f) \\ a_1^{(v)}(f) \end{array} \right\} + a_{19}(f) \cdot I_{MM} + a_4(f) \cdot \Delta' + a_{20}(f) \cdot I_{MM}\Delta' \right), 1 \right] + a_{15}(f) \cdot (2 - s) + a_{11}(f) \cdot S_L^{(1)} + a_{12}(f) \cdot S_L^{(2)} \quad (9a)$$

Table 5. Results of the regression analysis of the model in eqn (9)

Channel number	f_0 (Hz)	No. of data points $N(f)$	Coefficients a_i and their accuracy ('σ-interval')								σ_{dur} (s)	dur_{av} (s)
			$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	a_{19} $\pm\sigma_{19}$	a_4 $\pm\sigma_4$	a_{20} $\pm\sigma_{20}$	a_{15} $\pm\sigma_{15}$	a_{11} $\pm\sigma_{11}$	a_{12} $\pm\sigma_{12}$		
1	0·075	37	40·8 ±2·0	32·5 ±3·1	0·0	0·0	0·0	0·0	0·0	0·0	10·2	38·3
2	0·12	311	27·7 ±5·4	28·2 ±5·7	−1·30 ±0·75	0·182 ±0·019	0·0	0·0	0·0	0·0	10·1	28·3
3	0·21	850	27·5 ±3·2	29·3 ±3·2	−2·58 ±0·42	0·194 ±0·013	0·0	0·95 ±0·48	0·0	0·0	8·0	20·7
4	0·37	1179	19·3 ±3·0	19·5 ±3·0	−1·47 ±0·43	0·090 ±0·043	0·016 ±0·007	1·81 ±0·35	0·0	0·0	7·7	20·5
5	0·63	1139	15·1 ±3·1	17·5 ±3·2	−1·73 ±0·46	0·059 ±0·040	0·027 ±0·007	1·29 ±0·53	1·99 ±0·89	4·42 ±1·01	7·7	19·0
6	1·1	1376	12·1 ±2·5	15·1 ±2·6	−1·51 ±0·38	0·075 ±0·032	0·019 ±0·006	0·23 ±0·42	3·80 ±0·70	7·22 ±0·80	7·0	16·7
7	1·7	1550	8·0 ±1·9	10·4 ±1·9	−0·97 ±0·29	0·048 ±0·025	0·020 ±0·004	0·0	3·78 ±0·44	5·47 ±0·42	5·6	14·1
8	2·5	1574	5·8 ±1·4	7·4 ±1·4	−0·65 ±0·21	−0·008 ±0·019	0·023 ±0·003	0·0	2·40 ±0·33	3·45 ±0·31	4·2	10·5
9	4·2	1159	1·2 ±1·4	2·5 ±1·4	0·04 ±0·22	0·005 ±0·021	0·022 ±0·004	0·0	2·02 ±0·32	2·36 ±0·30	3·7	9·3
10	7·2	1093	2·2 ±1·3	2·8 ±1·3	0·04 ±0·20	−0·056 ±0·022	0·029 ±0·004	0·0	0·92 ±0·29	0·74 ±0·27	3·2	8·3
11	13	628	−2·5 ±1·8	−2·6 ±1·8	0·80 ±0·27	0·025 ±0·045	0·015 ±0·007	0·0	0·27 ±0·31	0·92 ±0·29	2·7	7·1
12	21	284	−4·7 ±1·5	−4·7 ±1·5	1·02 ±0·21	0·034 ±0·010	0·0	0·0	0·0	0·96 ±0·30	2·5	6·3
			1 horiz	1 vert	I_{MM}	Δ'	$I_{MM}\Delta'$	s	$S_L^{(1)}$	$S_L^{(2)}$		
			Corresponding parameters									

where the hypocentral distance, Δ', is measured in kilometers and

$$S_L^{(1)} = \begin{cases} 1, & \text{if } s_L = 1, \\ 0, & \text{if } s_L \neq 1, \end{cases} \quad S_L^{(2)} = \begin{cases} 1, & \text{if } s_L = 2 \\ 0, & \text{if } s_L \neq 2 \end{cases} \tag{9b}$$

All the terms which include s or s_L were chosen in such a way, that the corresponding coefficients, coming out positive as a result of the regression analysis, would show prolongation of the duration on sedimentary and soft soil sites, as compared with basement rock locations or 'rock' sites.

The results of the analysis of the model in eqn (9) are shown in Table 5. The last three coefficients, a_{15}, a_{11} and a_{12}, are also presented in Fig. 5, as functions of the central frequencies of the channels. The first six coefficients have functional forms similar to those found in the previous models. The new coefficients $a_{11}(f)$ and $a_{12}(f)$, can be compared with their counterparts in the model which accounts for the geological and local soil conditions and has the magnitude of the earthquake as the 'master' parameter of the model.[6] The behavior of $a_{11}(f)$ and $a_{12}(f)$ in the current model and in this 'magnitude' model are remarkably similar, and the additional duration (in both models) at the soft soil sites (compared with 'rock') is about 7 s for frequencies about 1 Hz, and about 3·5–4 s at stiff soil sites for frequencies 1–2 Hz. The influence of the geological parameter is more prominent at lower frequencies, with maximum contributions at channel No. 4 ($f_0 = 0{\cdot}37$ Hz). The difference in the range of frequencies where the influence of the geological and local soil conditions is noticeable might come from the difference in the characteristic depth of the corresponding 'soft' layer (up to several kilometers in the case of sediments and not more than several hundred feet in the case of local soils). Notice that the consideration of the influence of the local soil condition on the duration of the strong ground motion at the site is important. The prolongation of the duration due to the presence of soft

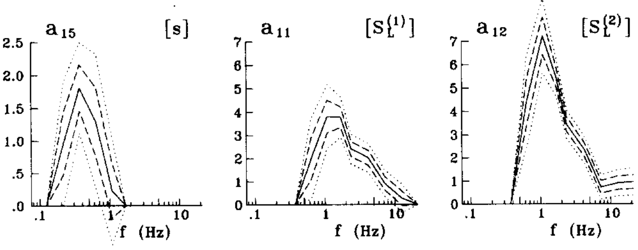

Fig. 5. The coefficients related to the site geologic and soil classification in the model in eqn (9), plotted versus the central frequency of the channels (solid lines). The coefficients are bounded by their 'σ-intervals' (dashed lines) and by their 95% confidence intervals (dotted lines).

Table 6. Results of the regression analysis of the model in eqn (10)

Channel number	f_0 (Hz)	No. of data points $N(f)$	Coefficients a_i and their accuracy ('σ-interval')					σ_{dur} (s)	dur_{av} (s)
			$a_1^{(h)}$ $\pm\sigma_1^{(h)}$	$a_1^{(v)}$ $\pm\sigma_1^{(h)}$	a_{19} $\pm\sigma_{19}$	a_{15} $\pm\sigma_{15}$	a_{13} $\pm\sigma_{13}$		
1	0·075	37	40·8 ±2·0	32·5 ±3·1	0·0	0·0	0·0	10·2	38·3
2	0·12	311	54·1 ±5·2	53·6 ±5·6	−3·88 ±0·79	0·0	0·0	11·5	28·3
3	0·21	850	44·4 ±3·6	46·3 ±3·6	−4·10 ±0·49	1·92 ±0·57	0·0	9·6	20·7
4	0·37	1179	37·2 ±2·5	38·0 ±2·5	−3·20 ±0·35	2·60 ±0·45	0·0	9·9	20·5
5	0·63	1647	28·7 ±1·9	31·3 ±2·0	−2·58 ±0·27	3·52 ±0·38	0·0	9·7	18·3
6	1·1	1376	28·7 ±1·8	31·8 ±1·9	−2·96 ±0·27	3·17 ±0·46	0·84 ±0·42	8·4	16·7
7	1·7	1550	24·3 ±1·4	26·5 ±1·4	−2·06 ±0·21	0·0	1·66 ±0·26	7·3	14·1
8	2·5	1574	15·5 ±1·0	17·0 ±1·1	−1·07 ±0·16	0·0	0·96 ±0·19	5·5	10·5
9	4·2	1159	10·5 ±1·2	11·9 ±1·2	−0·35 ±0·18	0·0	0·43 ±0·20	5·2	9·3
10	7·2	2576	−0·3 ±0·5	0·5 ±0·6	1·06 ±0·09	0·0	0·0	4·5	6·4
11	13	1584	−2·5 ±0·5	−2·0 ±0·5	1·22 ±0·08	0·0	0·0	3·5	5·1
12	21	735	−3·2 ±0·6	−2·8 ±0·6	1·19 ±0·10	0·0	0·0	2·9	4·2
			1 horiz	1 vert	I_{MM}	s	s_L		
			Corresponding parameters						

soils under the station may be more prominent at some frequencies than the prolongation due to the presence of deep sedimentary deposits.

The last model we consider is the simplified form of the previous one (eqn (6) is again taken as the 'basic' model):

$$\begin{Bmatrix} dur^{(h)}(f) \\ dur^{(v)}(f) \end{Bmatrix} = \max\left[\left(\begin{Bmatrix} a_1^{(h)}(f) \\ a_1^{(v)}(f) \end{Bmatrix} + a_{19}(f)\cdot I_{MM}\right), 1\right] + a_{15}(f)\cdot(2-s) + a_{16}(f)\cdot s_L \qquad (10)$$

The results of the regression analysis of this model are presented in Table 6. This 'rough' model is hardly able to detect the influence of the local soil conditions on the duration of strong ground motion, although the previous model in eqn (9) allows one to detect this influence.

Distribution function of the residuals

It is of interest for earthquake engineering applications not only to be able to predict the expected value of the duration of strong ground motion, but also to evaluate the probability of exceedance of any given duration at a particular frequency. A study of the residues allows one to estimate this probability from the distribution functions of the observed residuals.

We define the residual factor ρ (relative residual) of a model prediction from a data point as the ratio of the observed duration of strong ground motion, dur_{obs}, to the duration, predicted by a model, dur:

$$\rho = \frac{dur_{obs}}{dur}$$

We found that this quantity is easier to deal with than, for example the difference $dur_{obs} - dur$, because ρ has a well defined lower bound (zero) and has very similar distributions for all the frequency channels. We also found that the distribution function of ρ does not depend on the parameters of the models, such as the Modified Mercalli intensity, the distance to the source and the site conditions. This distribution function, $q(\rho)$, is very similar for different models. We approximate it by:

$$q(\rho) = \frac{1}{\eta}\cdot\frac{\rho^b}{a+\rho^c} \qquad (11a)$$

where η is the normalizing coefficient:

$$\eta = a^{(b+1/c)-1}\cdot\frac{\pi}{c}\cdot\left[\sin\frac{(b+1)\pi}{c}\right]^{-1} \qquad (11b)$$

Table 7. Coefficients in the distribution function $q(\rho)$ (see eqn (11)) for the models in eqns (5)–(10)

Ch. number	f_0 (Hz)	For eqn (5)			For eqn (6)			For eqn (7)			For eqn (8)			For eqn (9)			For eqn (10)		
		a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
1	0·075	2·3	3·5	12·0	2·3	3·5	12·0	2·3	3·5	12·0	2·3	3·5	12·0	2·3	3·5	12·0	2·3	3·5	12·0
2	0·12	0·6	2·7	7·4	1·9	1·7	7·4	0·6	2·7	7·4	1·9	1·7	7·4	0·6	2·7	7·4	1·9	1·7	7·4
3	0·21	0·4	3·1	7·4	0·4	2·7	6·9	0·3	3·2	7·2	0·4	2·6	6·8	0·4	3·0	7·4	0·4	2·6	6·8
4	0·37	1·2	2·5	8·3	1·1	2·3	7·6	1·1	2·4	7·9	1·5	1·8	7·1	1·2	2·4	8·1	1·5	1·8	7·1
5	0·63	2·1	1·5	7·1	2·7	1·0	6·1	1·4	1·5	6·4	4·0	0·8	6·1	1·9	1·6	7·0	4·0	0·8	6·1
6	1·1	3·5	1·1	7·1	3·5	0·7	5·6	2·7	1·1	6·6	3·7	0·6	5·4	2·7	1·3	6·9	3·5	0·9	6·1
7	1·7	2·6	1·4	7·3	4·0	0·6	5·6	2·2	1·3	6·8	2·8	0·6	5·1	2·5	1·6	7·5	2·8	1·0	6·2
8	2·5	2·4	1·6	7·6	2·1	0·8	5·3	1·7	1·5	6·8	1·6	0·8	5·0	1·8	1·7	7·2	1·5	1·3	6·0
9	4·2	4·2	1·2	7·7	2·2	0·5	4·6	1·7	1·5	6·8	2·2	0·5	4·7	2·0	1·7	7·3	2·2	1·0	5·9
10	7·2	1·7	1·4	6·5	2·0	0·3	4·1	1·7	1·4	6·5	1·9	0·4	4·3	1·7	1·8	7·2	2·0	0·3	4·1
11	13	1·3	1·6	6·5	1·4	0·5	4·2	1·3	1·6	6·5	1·4	0·5	4·2	3·1	1·5	7·7	1·4	0·5	4·2
12	21	1·4	1·5	6·4	1·0	0·6	4·2	1·4	1·5	6·4	1·0	0·6	4·2	1·1	2·1	7·4	1·0	0·6	4·2

The coefficients a, b and c should be adjusted for each model at every frequency channel. We choose these coefficients so that the cumulative distribution function

$$P(\rho) = \int_0^{\rho} q(\rho)\,\mathrm{d}\rho \qquad (12)$$

stays close to the observed cumulative distribution function $P_{\mathrm{obs}}(\rho)$ (Kolmogorov–Smirnov test).

Note that the mean value of the residuals ρ is equal to 1 by the construction of all our duration models (unbiased estimate). Thus, we could have reduced the number of coefficients in the distribution function (11) by setting its mean equal to unity. We, however, decided to allow additional flexibility in the coefficients to achieve a better fit in terms of the Kolmogorov–Smirnov test. As a result, the mean of the proposed distribution differs slightly from 1 at some channels.

Table 7 gives the 'best' values for coefficients a, b and c for the distribution (11) for the models in eqns (5)–(10). Having the distribution function of ρ, we can predict the duration of strong ground motion which will not be exceeded with any given probability at the site with known properties during an earthquake with given parameters. For a probability P, the value of ρ_{P}, such that $P = P(\rho_{\mathrm{P}})$ can be found from eqns (11) and (12). The duration not to be exceeded with probability P is then $dur_{\mathrm{P}} = dur \cdot \rho_{\mathrm{P}}$, where dur is the duration of strong motion predicted by the model we have chosen.

SUMMARY

In this work, new empirical models of the duration of strong earthquake ground motion in terms of the Modified Mercalli intensity and other (site specific) parameters were investigated. These models differ from those in the literature by the consideration of the distance to the source as one of the scaling parameters (in some of the models), by the presence of the site specific soil parameter (s_{L}), not considered in the previous studies, and by the more complete database.

We used the definition of duration of a function of motion $f(t)$, where $f(t)$ is acceleration, velocity or displacement, as the sum of the time intervals during which the integral $\int_0^t f^2(\tau)\,\mathrm{d}\tau$ gains a significant portion of its final value.[9] All records were band-pass filtered through 12 narrow frequency bands, and the duration of strong ground motion was studied separately in these frequency bands.

Two groups of regression equations were considered, one of which explicitly includes the dependence of the duration of strong ground motion on the distance to the source, and the other does not consider this dependence. The models of the first type are more descriptive, but they are also more region dependent, because the dispersion and attenuation laws are 'built into' the frequency dependent regression coefficients.

It was found that the modelling of the duration as a function of the Modified Mercalli intensity and the distance to the source allows one to see some interesting features of the duration and of the MMI scale itself. For a given intensity, the duration grows when the distance between the source and the recording site becomes large. For a fixed distance, the dependence of duration on intensity is more complex. At low frequencies, the duration of strong motion decreases when the intensity increases. At high frequencies, the duration grows with increasing intensity. A smooth transition from one type of dependence to another can be found at the intermediate frequencies. A combination of several trends is responsible for such a behavior. An increase in intensity may come from an increase of the magnitude, and then the duration of strong motion should increase. A growth of I_{MM} at the site may be the result of a decrease of the source-to-site distance, and then the duration decreases. Different contributions prevail at different frequencies, which results in what appears as a 'contradictory' behavior of the duration as a function of the Modified Mercalli intensity and the distance to the source. One may also remember that the MMI scale evolved as a descriptive scale which considers some measure of the damage of typically older structures, and tends to be more sensitive to the

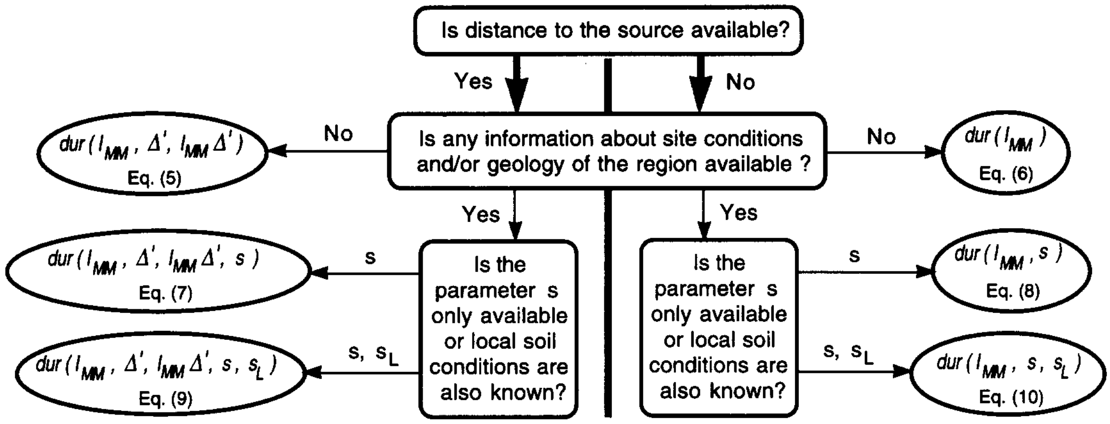

Fig. 6. A flow chart for selecting the proper scaling model of the duration of strong motion in terms of the Modified Mercalli intensity at the site.

high frequency end of the earthquake excitation. The attenuation of the seismic waves is very different at low and at high frequencies, and this also contributes to the observed behavior of the MMI scale at the low frequency end.

The influence of the geological conditions at the recording site on the observed strong motion duration was also studied. The special treatment of the qualitative indicator variable s allowed us to resolve the prolongation of duration on sedimentary ($s = 0$) and at intermediate ($s = 1$) sites with respect to the sites located on basement rock ($s = 2$). It was shown that, when the soil condition parameter, s_L, is available, it should be included in the regression equations. This parameter was not considered previously in the regression models of the duration of strong motion, when the 'main' parameter in the equation was the Modified Mercalli intensity. The influence of the type of soils on the duration of strong ground motion is stronger at higher frequencies ($f = 0{\cdot}3$–25 Hz), while the presence of sedimentary deposits can be noticed at lower frequencies ($f = 0{\cdot}15$–2 Hz).

The frequency dependent coefficients, describing the influence of the geological and of the local soil site conditions on the duration of strong ground motion were found to be very similar in the following two cases: (1) when the 'basic' duration (i.e. the portion which only depends on the 'size' of the earthquake and the source to site distance) is described in terms of the magnitude and epicentral distance,[6] and (2) when the 'basic' duration is described in terms of the Modified Mercalli intenisty (and, possibly, hypocentral distance). This similarity and internal consistency provide additional support for our models.

Six regression equations relating the duration of strong ground motion and the earthquake and site parameters were studied. Figure 6 provides an overview for choosing the proper model in each particular case, depending on what parameters are available. Each model is shown in this figure by specifying the set of parameters it considers. The equation number of each model is also given for easy reference to the main text. The regression coefficients were obtained at the specific frequencies only. If an estimate of the duration of strong motion at some frequency, not present in this set, is required, the duration can be computed by interpolation.

A distribution function of the residuals of the predicted duration has been proposed. This, allows, for example, estimation of the duration of strong ground motion which will not be exceeded with any given confidence level during an earthquake and at a site with known properties.

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