The total loss in a building exposed to earthquake hazard

Part II: a hypothetical example

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SUMMARY – This paper contains an illustration of the method for estimation of the total loss (a sum of the direct and the indirect losses) for a building described in the companion paper, referred to as Part I. The method is applied to a hypothetical building of a university campus. The probability distribution functions of the subsystem direct and indirect losses, and of the total loss for the building are evaluated for a given value of the shaking parameter at the building site. Probability distribution functions for the physical damage of the elements at risk, and for the indirect loss proportionality factors for the subsystems, that are suitable for such a detailed analyses, are not available at present. Therefore, hypothetical, but physically admissible, analytical functions have been used. In particular, Beta probability distribution functions have been used for the losses of the elements at risk, with expected value and variance depending on the level of the input hazard and on the characteristics of the element. Resistance classes are introduced to discriminate between elements at risk of the same kind, but with different susceptibility to damage. The indirect loss proportionality factors are assumed to be uniformly distributed.

KEYWORDS: eartquake losses, assessment of losses, indirect losses, direct losses, optimum design level, optimum level of strengthening, probabilistic estimate, decision making tool, earthquake hazard.

Introduction

In Part I of this paper (Jordanovski et al. 1992), a methodology is presented for assessment of the total losses (direct and indirect losses) for a single building exposed to some natural or man made hazard. In the model, the building is represented by an integral system, consisting of subsystems. The subsystems contain elements at risk, which suffer physical damage caused by the hazard. Resistance classes are introduced to distinguish between elements with different susceptibility to damage, and indirect loss proportionality factors, to model the indirect losses. This work was motivated by the need for a more accurate model for assessment of losses caused by moderate to large earthquakes. This model, interfaced with database of the building inventory, residents, ongoing activities, and probability distribution functions for the damage of the elements at risk and for the indirect loss proportionality factors, can assist the decision making for long range planning related to mitigation of the financial consequences of damaging earthquakes.

Probability distribution functions for the losses of the elements at risk for such a detailed system are not available at present. At this time, the most complete set of damage probability matrices (ATC, 1985) that is applicable to buildings in the United States, is based on expert opinions, and can be used by engineers to estimate the generic loss for selected types of buildings. In this paper, the procedure of constructing damage probability functions for the elements at risk is discussed, and some physically admissible functions are suggested. To illustrate the method, it is applied to a hypothetical building of a Large Public or Private Organization (LPO) exposed to given earthquake motion at the base. Hypothetical, but physically admissible probability distribution functions for the losses of the elements at risk and for the indirect loss proportionality factors are used.

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Received january 1992, revised june 1992.

Admissible probability distribution functions for the losses of the elements at risk

The conditional distribution functions $F_{H|Y}$ of the input hazard level H for different groups of elements, and the conditional distribution function $F_{LER|H}$ of the losses for the elements at risk, can be determined by statistical regression analysis applied to

1. empirical data, such as compiled data on losses after particular earthquakes,
2. results of theoretical analyses, involving evaluation of linear and non-linear response of buildings and simulation, and
3. expert opinions.

Empirical data on losses gathered after earthquakes are often incomplete. For example, the available damage probability matrices obtained by regression of data on structural and nonstructural damage are incomplete. The expert opinions and the theoretical data are also not equally reliable for all values of the input hazard level H and the level of shaking Y. In addition to this, determining the probability distribution functions for all the possible values of the conditional variable is time consuming, and implementing these in the analysis is memory demanding. Therefore, another approach is recommended and used in this paper.

Often, in the engineering analyses, a theoretical distribution function is chosen that would best fit the data, and that would not violate the physical properties of the process. The theoretical distribution functions are often defined by two parameters: the expected value (the mean) and the variance (the standard derivation). Those are evaluated by fitting the theoretical distribution function to the data. Then, various tests are performed, such as the Kolmogorov-Smirnov test, to determine the «goodness of fit». For example, in the case of the conditional distribution function $F_{LER|H}$, the functions u_1 and u_2 have to be determined such that

$$u_1(h) = \mathrm{E}[LER|H = h] \tag{1}$$

and

$$u_2(h) = \mathrm{Var}[LER|H = h]. \tag{2}$$

Similarly, for $F_{H|Y}$, the functions v_1 and v_2 have to be determined such that

$$v_1(y) = \mathrm{E}[H|Y = y] \tag{3}$$

and

$$v_2(y) = \mathrm{Var}[H|Y = y]. \tag{4}$$

In Eqs. (1) through (4), E[·] and Var[·] indicate expected value and variance. The advantage of this procedure is that it makes it possible to fill-in the no-data regions within the interval of the data. The reliability of the results strongly depends on the quantity and quality of the available data and on the smoothness of u_1, u_2, v_1 and v_2. In this respect, u_1 and v_1 are smoother than u_2 and v_2 because of their nondecreasing nature.

In this study, the Beta probability distribution function is used to model the element losses probability distribution function (Jordanovski et al., 1992), i.e.

$$f_{LER|H}(\ell|h) = \frac{1}{(b-a)^{r+p+1}} \frac{(\ell-a)^{r-1}(b-\ell)^{p-1}}{B(r,p)} \tag{5}$$

where

$$B(r,p) = \int_0^1 x^{r-1}(1-x)^{p-1}dx \tag{6}$$

is the Beta-function, a and b are the lowest and the highest values that the element loss can take, and r and p are parameters that define the slenderness and skewness of the density function. r and p are related to the expected value and to the variance by

$$\mathrm{E}[LER|H = h] = \frac{br+pa}{r+p} = u_1(h) \tag{7a}$$

and

$$\mathrm{Var}[LER|H = h] = \frac{pr(b-a)^2}{(p+r)^2(p+r+1)} = u_2(h). \tag{7b}$$

In Fig. 1, examples of Beta probability distribution function $f(x)$ are shown for different values of r and p. In curve (1), $r = 0$ and $p >> 1$; in curve (2), $r, p \neq 0$ and $p >> r$ and $f(x)$ is skewed to the left; in curve (3), $r = p >> 1$ and $f(x)$ is symmetric; in curve (4), $r >> p \neq 0$ and $f(x)$ is skewed to the right; in curve (5) $r = p = 1$, and $f(x)$ is constant.

A desirable property of the Beta probability distribution function is that it is nonzero in a closed interval $[a, b]$, and with adequate choice of r and p different weight can be assigned to smaller or higher values of the losses. The minimum loss, a, is usually equal to 0 and the maximum loss, b, is usually equal to the replacement value of the element.

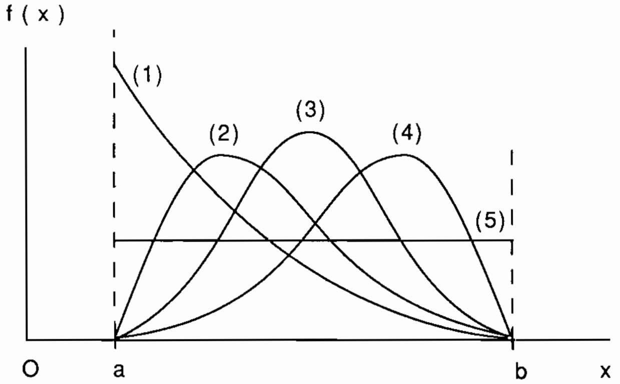

Fig. 1 – Examples of Beta-probability distribution functions, for five sets of values of the parameters r and p.

Examples of $u_1(h)$ and $u_2(h)$

In the hypothetical example in this paper, the functions $u_1(h)$ and $u_2(h)$ appearing in Eqs. (7a) and (7b) are assumed to be the following

$$u_1(h) = \mathrm{E}[LER|H = h] = b(1 - e^{-qh}) \tag{8}$$

and

$$u_2(h) = \mathrm{Var}[LER|H = h] = b(1 - e^{-qh})e^{-qh}, \tag{9}$$

where q is some constant. Sketches of $u_1(h)$ and $u_2(h)$ are shown in Figs. 2 and 3. These forms of $u_1(h)$ and $u_2(h)$ are physically admissible hypothetical functions, and are used only to illustrate the model. Even though, in reality, the damage of the structural components is not necessarily a continuous function of the building response, but may have jumps (components of the element suddenly break when certain level of h is reached), $u_1(h)$ in Eq. (8), as a monotonically increasing function of the input hazard level, does not violate the relationship between the damage of the element at risk and the input hazard level. From Eq. (9), it follows that the scatter of the data, $\mathrm{Var}[LER|H = h]$, is small when the input hazard level and the damage are small, and $\mathrm{Var}[LER|H = h] \to 0$ as $h \to 0$. When $h \to h_{max}$ and the loss due to the physical damage approaches the maximum loss, then $\mathrm{Var}[LER|H = h] \to 0$ also. For intermediate values of h, $\mathrm{Var}[LER|\mathrm{H} = \mathrm{h}] \neq 0$. Then, larger $\mathrm{Var}[LER|H = h]$ means that the loss can take comparable values in a larger interval about the mean value. The form of $u_2(h)$ in Eq. (9) is also physically admissible.

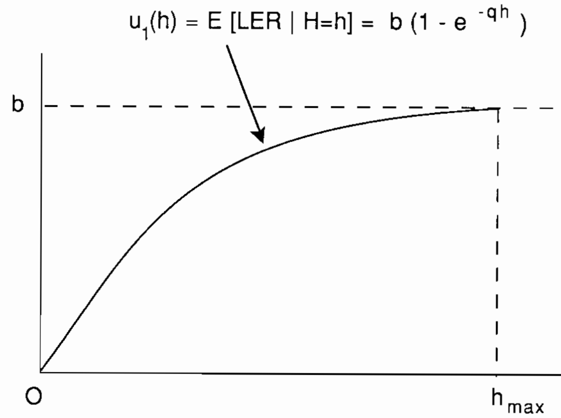

Fig. 2 – The assumed form of the function $u_1(h)$, representing the expected value of the loss of an element at risk for a given value of the input hazard level H, in the hypothetical example in this paper.

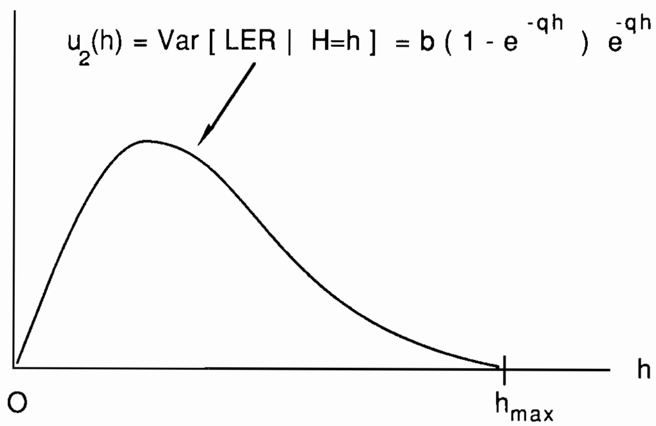

Fig. 3 – The assumed form of the function $u_2(h)$, representing the variance of the loss of an element at risk for a given value of the input hazard level H, in the hypothetical example in this paper.

Resistance classes

The structural elements of a building sometimes may not have the design strength, because of the human factor involved in the construction process. Elements of the same kind may have different vulnerability in different subsystems. Three possibilities can be suggested to account for this difference:

1. different distribution functions have to be defined for different elements or groups of elements,
2. one distribution function can be used for all the elements of a given kind, but with a larger standard deviation, and
3. same analytical representation of $u_1(h)$ can be used (as in Eq. (8), e.g.) for all the elements of the kind, but the values of some parameters of u_1 (e.g., q) should be different for elements belonging to different vulnerability classes.

In the hypothetical example that follows, the third possibility is employed.

First, three resistance classes are defined:

a) poor resistance class,
b) fair resistance class, and
c) good resistance class.

q defines the rate of growth of $u_1(h)$. Quantitatively, it is defined for each of these classes in terms of the value of h for which the expected value of the loss equals 90% of the maximum loss, b. In mathematical terms this could be expressed as

$$u_1(h) = 0.9b. \tag{10}$$

Then, from Eqs. (8) and (10) it follows

$$q = \frac{-\ln 0.1}{h}. \tag{11}$$

It is assumed in the examples that for a good resistance class $h = 0.9h_{max}$, for a fair resistance class $h = 0.8h_{max}$, and for the poor resistance class $h = 0.6h_{max}$. In Fig. 4, $u_1(h)$ are illustrated for the three resistance classes.

Accounting for the difference in the vulnerability of the elements of a given kind by assigning it to different resistance classes is physically more reasonable than by increasing the variance, because through the resistance classes the variance of the overall distribution function of the elements (including the distribution functions for the classes) is increased at all values of h, uniformly.

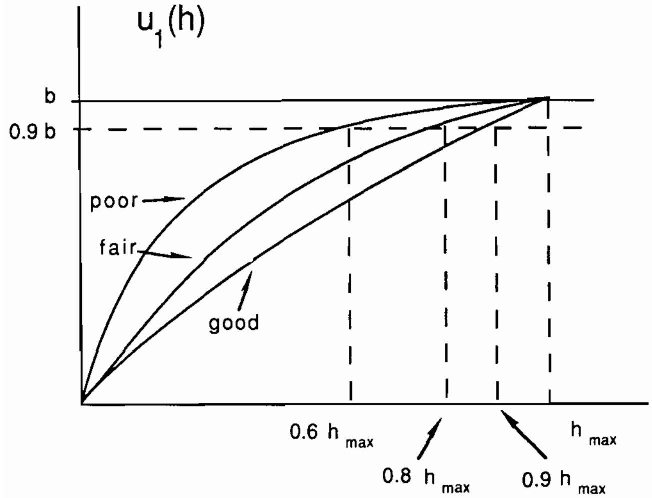

Fig. 4 – An illustration of the definition of the three resistance classes (poor, fair and good) for an element at risk. For example, for a «good» resistance class, the expected value of the element loss equals 90% of the maximum loss, b, when the input hazard level, H, equals 90% of its maximum value h_{max}.

This would not be the case if a standard shape, as in Eq. (9) e.g., is assumed.

In determining the function $u_1(h) = E[LER|H = h]$ by a regression analysis of empirical data, it may happen that $u_1(h)$ in Eq. (8) does not fit the data. Consequently, conditions other then Eqs. (10) and (11), have to be defined to determine the distribution function and to define the criteria for the resistance classes. In a general case, $u_1(h)$ can be defined with the help of the mean and the standard deviation, $\tilde{u}_1(h)$ and $\tilde{\sigma}(h)$, of the distribution function determined by a regression analysis of all the data for that kind. For example,

1) $u_1(h) = \tilde{u}_1(h)$ for a good resistance class,
2) $u_1(h) = \tilde{u}_1(h) - \tilde{\sigma}(h)$ for a fair resistance class, and
3) $u_1(h) = \tilde{u}_1(h) - 2\tilde{\sigma}(h)$ for a poor resistance class.

Assigning an element to a lower or to a higher resistance class may also express the confidence of the individual performing the analysis that the element will in reality perform as it was initially designed.

An application of the model to a hypothetical example

To illustrate the model described in Part I of this paper (Jordanovski et al., 1992), the computer program ESTIMATE was written and applied to a hypothetical building, exposed to a given earthquake ground motion described by a single parameter. Hypothetical analytical probability distribution functions for the damage of the elements at risk were used, as described in the previous section of this paper.

Description of the building

The hypothetical building is a two-story moment resisting frame building. It belongs to an LPO (e.g. a university campus) and it is used for lecturing, as an office building, and for research. The classrooms are on the first floor. On the second floor, the X-department has faculty and administrative offices and several computer laboratories. In the basement there are several experimental laboratories. The total cost of the building, including all the equipment, has been estimated to be equal to B monetary units (m.u.). The X-department is involved in several projects which bring income of B_X m.u./year to the university, and the lectures that take place in this building generate B_L m.u./year profit to the university. The experimental laboratories are also engaged in projects whose interruption may have long term impact on the university finances.

Choice of the subsystems and of the elements at risk

The subsystems are chosen to be the different levels of the building, i.e.

1. SS_1: the basement,
2. SS_2 the first floor, and
3. SS_3: the second floor.

The considered elements at risk for each of the subsystems are the following.

Basement:

B.1 Structural elements (columns, beams, shear walls...)

B.2 Non-structural elements (ceilings, partitioning walls, stairs, facade...)

B.3 Installations (telephone lines, electrical lines, air conditioning ducts, lights, elevators...)

B.4 Laboratory equipment (electronic microscope, optical lasers...)

First Floor:

F1.1 Structural elements (columns, beams, shear walls...)

F1.2 Non-structural elements (ceilings, partitioning walls, stairs, facade...)

F1.3 Installations (telephone lines, electrical lines, air conditioning ducts, lights, elevators...)

Second Floor:

F2.1 Structural elements (columns, beams, shear walls...)

F2.2 Non-structural elements (ceiling, partitioning walls, stairs, facade, roof...)

F2.3 Installations (telephone lines, electrical lines, air conditioning ducts, lights, elevators...)

F2.4 Equipment (two main-frame computers, 20 personal computers, 8 laserjet printers, 3 xerox and 3 FAX machines...)

A block diagram of the integral system, the subsystems, and the elements at risk is shown in Fig. 5.

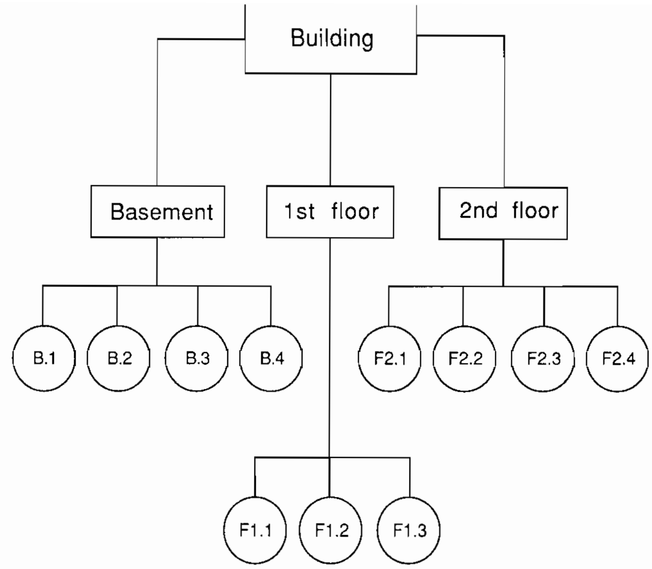

Fig. 5 – A block diagram of the integral system, IS, the subsystems, SS_j, and the elements at risk, $ER_{j,k}$, for the hypothetical example in this paper.

The Input

The shaking parameter

The shaking parameter, Y, could be the MMI intensity of shaking, the peak ground acceleration, or the response spectrum at the building site, for example. The site shaking parameter is a random variable, and a proper interface between the computer program ESTIMATE and a program that evaluates in a probabilistic way the ground response at the building site to motion generated at the surrounding faults (NEQRISK is an example of such a computer program, Lee and Trifunac 1985) is necessary. For the purpose of demonstrating the program ESTIMATE alone, the losses for the hypothetical building are estimated for a given value of the shaking parameter, (for example the maximum value expected to occur for exposure time of 80 years). This maximum value is assumed to be equal to 8 units of the shaking parameter.

Input parameters for the elements at risk

For each of the elements at risk, the following parameters are defined:

(i) the input hazard parameter, ihp, (the floor response parameter with which the damage of the element is best correlated)

$ihp = d$: interstory drift
$ihp = v$: peak velocity
$ihp = a$: peak acceleration
$ihp = s$: maximum shear force
$ihp = m$: maximum bending moment,

(ii) the input hazard level, H, as a function of the ground shaking at the site,

(iii) minimum and maximum losses for the element, L_0 and L_m, in monetary units,

(iv) probability distribution functions for the element losses, as functions of the input hazard level h (the Beta probability distribution function is used, defined on the interval $[L_0, L_m]$),

(v) resistance class parameter, rc,
$rc = g$: good resistance class
$rc = f$: fair resistance class
$rc = p$: poor resistance class.

Since the subsystems are the story levels, the input hazard level is assumed to depend only on the story height. For all the elements of the story, the input hazard level is assumed to be normally distributed with mean μ and standard deviation σ

$$\mu(y) = \mathrm{E}[H|Y = y] = k(y - 6)m^2 \qquad (12)$$

and

$$\sigma^2 = \mathrm{Var}[H|Y = y] = 0.15\mu(y)$$

where $m = 1$ for the basement, $m = 2$ for the first floor and $m = 3$ for the second floor, and k is a proportionality factor. It follows from Eq. (12) that the input hazard level is larger at higher levels of the building. The standard deviation is also larger at the higher floors, where the mean of the input hazard level is larger.

Quantitatively, the resistance classes are defined in the above text. Then, the expected value, and the variance of the Beta probability distribution function for the losses are defined as described in Eqs. (7). In Table 1, the values of the input parameters for all the elements at risk are summarized. It can be seen from this table that the damage of all the first three elements at risk at the basement is correlated with the floor displacement, and of the fourth element, to the floor acceleration. At

Tab. 1 Input parameters for the subsystems and for the elements at risk of the building

Floor	Element	Input Hazard Parameter	Resistance Class	Minimum Direct Loss [m.u.]	Maximum Direct Loss [m.u.]	Indirect Loss Class
Basement	B.1	d	p	0	300	h
	B.2	d	p	0	100	
	B.3	d	p	0	400	
	B.4	a	p	0	1,000	
First Floor	F1.1	d	p	0	300	h
	F1.2	d	p	0	200	
	F1.3	d	p	0	400	
Second Floor	F2.1	d	p	0	300	h
	F2.2	d	p	0	300	
	F2.3	d	p	0	400	
	F.2.4	a	p	0	600	

the first floor, the damage of all the elements at risk is correlated with the interstory drift. At the second floor, the damage of the structural and nonstructural elements, and the installations is correlated with the interstory drift, and of the equipment, with the floor accelerations.

The resistance class indicates how well the element is expected to perform during an earthquake, as compared with the average performance calculated from statistical data, or as compared with some expected performance. It reflects the past experience of the element (e.g., if the structural element has some cracks from past earthquake, then it is assigned to a lower resistance class). Also, equipment which is not bolted properly to the floor or to the wall, or which is placed where there is a higher probability that a heavy object can fall onto it and damage it, is assigned to a lower resistance class.

Because the example building is an older building, it is assumed that, for the present state, all the elements at risk belong to the poor resistance class.

The Indirect Loss Proportionality Factors

The Indirect Loss Proportionality Factors, *ilpf*, for the floors are assumed to take one of the following values:

ifpf = *l*: low indirect loss proportionality class,
ifpf = *a*: average indirect loss proportionality class,
ifpf = *h*: high indirect loss proportionality class.

It is assumed, for example, that the indirect losses can exceed at most three times the direct losses. The subsystems are assigned to one of the three indirect loss proportionality classes (low, average and high), as defined in the above text. All the three subsystem are assigned to the high indirect loss proportionality class.

NUMERICAL RESULTS

The minimum loss for all the elements is zero. Adding up the maximum direct losses, it follows that the maximum direct loss for the basement is 1,800 m.u., for the first floor 900 m.u., for the third floor 1,600 m.u., and for the whole building, 4,300 m.u.

The maximum indirect losses are three times larger than the maximum direct losses, and the maximum total losses are four times the maximum direct losses. In Table 2, for each floor and for the whole building, the maximum direct, indirect and total losses, the expected value of the losses for exposure time of 80 years, and the dispersion are shown. It can be seen that, for example, the maximum direct loss for the whole building (the replacement value) is 4,300 m.u., the maximum indirect loss is 12,900 m.u. and the maximum total loss is 17,200 m.u. The expected value of the total loss for the whole building is 11,900 m.u. and the dispersion is 811 m.u.

Tab. 2 A summary of the values of the maximum direct loss, the maximum indirect loss, the expected loss and the dispersion, for the individual stories and for the whole building

Floor	Maximum Direct Loss [m.u.]	Maximum Indirect Loss [m.u.]	Maximum Total Loss [m.u.]	Expected Total Loss [m.u.]	Dispersion [m.u.]
1	1,800	5,400	7,200	4,126	535
2	900	2,700	3,600	2,847	287
3	1,600	4,800	6,400	5,010	486
Total	4,300	12,900	17,200	11,900	811

Let us define the mean damage ratio, MDR, as the ratio of the expected value of the total losses and the replacement value of the building. Then, it follows that for the initial state MDR = 2.77.

In Fig. 6 the probability density distribution functions of the direct and total loss are shown for each floor, and, in Fig. 7, the cumulative and the density probability distribution functions of the losses for the whole building are shown. From Fig. 7, it can be seen that, given that the shaking parameter $Y = 8$, the most probable total loss is about 12,000 m.u. and the loss that will not be exceeded with confidence level of 90% is about 13,000 m.u.

This program can be used for simulation of the total loss for the building for different states, each corresponding to some level of strength and some cost to achieve that strength. Then, a cost-benefit analysis can be performed and the optimum strength can be chosen (Jordanovski et al. 1991).

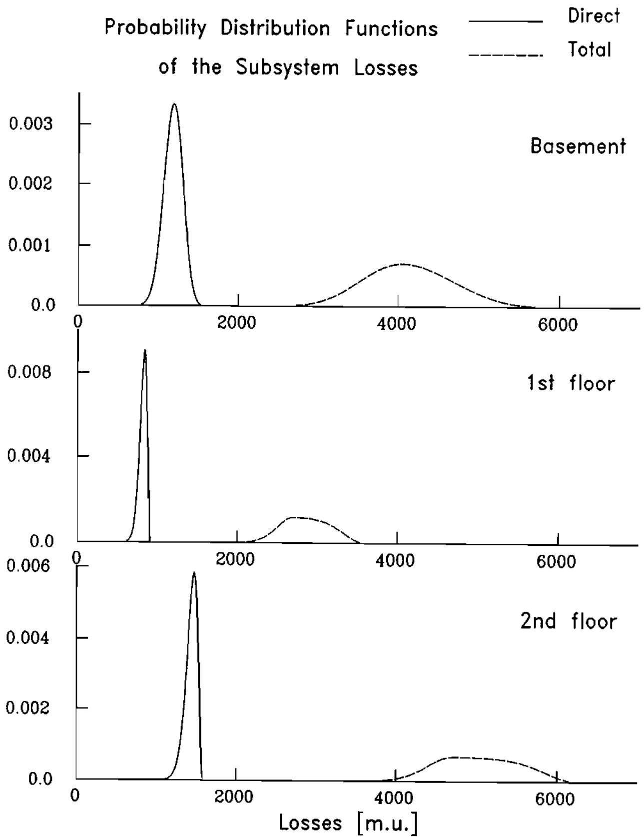

Fig. 6 – Density probability distribution functions of the direct (the solid line) and the indirect losses (the dashed line) for the three subsystems in the hypothetical example, evaluated for a given value of the shaking parameter.

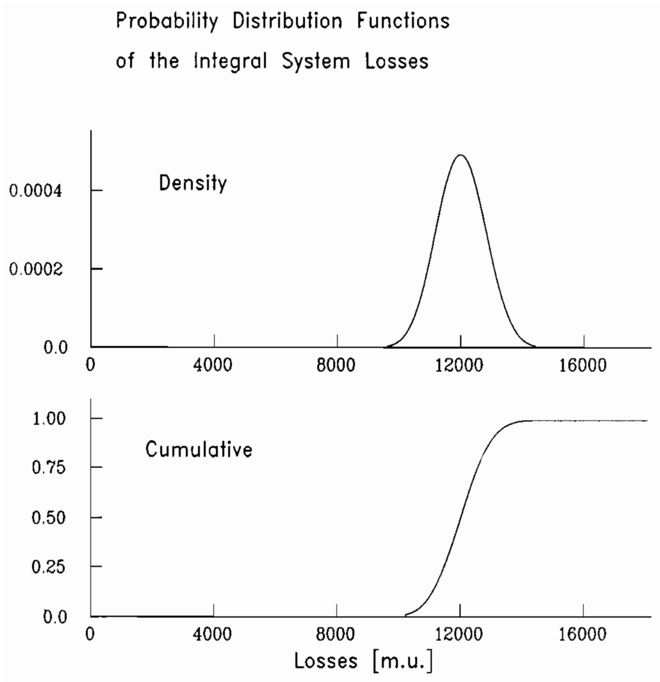

Fig. 7 – The density and the cumulative probability distribution function of the total loss for the integral system, evaluated for a given value of the shaking parameter.

Summary and conclusions

The method for assessment of the total loss of a building exposed to a hazard, presented in Part I of this paper (Jordanovski et al. 1992), is applied to a hypothetical building of a university campus, exposed to earthquake shaking, using hypothetical, but physically admissible analytical probability distribution functions for the losses of the elements at risk. The Beta probability distribution function is used as a convenient physically admissible probability distribution function, with expected value and variance specified for particular elements at risk as functions of the input hazard level. The example building is a two story moment resisting frame building housing offices, laboratories and classrooms. The subsystems are the two stories and the basement. The elements at risk are the structural and nonstructural elements, the installations and the laboratory and office equipment, for example. Three resistance classes (good, fair and poor) are defined both rigorously and descriptively. The indirect loss proportionality factor is assumed to be uniformly distributed over the interval of the losses. Three classes of indirect losses proportionality factor are defined (low, average and high), both exactly and descriptively. The losses of the subsystems are assumed to be a sum of the losses of the elements at risk, and the losses of the integral system (the building) to be a simple sum of the subsystem losses. The losses are estimated for the maximum possible value of the site response parameter in the next (for example) 80 years. An interactive computer program EQLOSS has been written to estimate the earthquake losses for an LPO (a university campus, in this example). This program can be interfaced with the bank of data on all the buildings of the LPO (on the campus), which can be easily updated by the user. It also allows graphical representation of the damage probability functions for the integral system. Such a computer program can be used by the owner or by an executive as a decision making tool for mitigation of the losses caused by future earthquakes. By executing the program for different scenarios, the optimum steps for future action can be determined. At present the program estimates the losses for given level of shaking at the site. However, it can be easily interfaced with the computer program NEQRISK (Lee and Trifunac 1985) so that, then, the expected value of the losses or the losses that will not be exceeded with a given level of confidence during the service time of the building can be estimated.

Acknowledgements

The work presented in this paper was done in the Summer of 1991, while Dr. Jordanovski was a visiting scholar at the University of Southern California. It was supported by a grant from the University of Southern California, Office of the President. This financial support, and the help of Dr. W. Regensburger (Office for Emergency Planning) who administered the grant in various organizational aspects of this work are gratefully acknowledged.

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