The total loss in a building exposed to earthquake hazard

Part I: the model

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SUMMARY – In this paper, a probabilistic model for assessment of the losses of a single building exposed to strong earthquake ground motion is presented. The model is general and can be applied to other systems exposed to natural hazards, such as fire, tsunami, wind etc. The losses include structural and non-structural damage, damage to equipment, loss of function, and other indirect losses that can be translated into monetary costs.

In the model, the building is considered as a system consisting of subsystems which, themselves, consist of elements at risk. The elements at risk are those that suffer physical damage and contribute to the monetary loss of the subsystem to which they belong. The total loss of the system is represented as a sum of the losses of the subsystems. The subsystems can be different floors of a building, or functional units such as telephone lines, electrical lines, or air conditioning and heating systems. The losses are treated as random variables, and are described by their probability distribution functions. So far, empirical probability distribution functions are not available for such a detailed analysis. Therefore, analytical physically admissible probability distribution functions are suggested to be used on an interim basis. In the subsequent paper, these functions have been used in an application of the method to a hypothetical building.

The presently used procedures and computer programs estimate only generic losses of buildings belonging to one of previously defined classes. The presented method, interfaced with an appropriate database, will be able to estimate the losses of a particular building. Therefore, it can be used by large private or public organizations as a decision making tool in developing their own standards for the optimum design strength of new buildings (beyond the minimum requirements specified by the building codes), and for the optimum investment in strengthening of existing buildings, that would reduce possible losses from future earthquakes.

KEYWORDS: earthquake losses, assessment of losses, indirect losses, direct losses, optimum design level, optimum level of strengthening, probabilistic estimate, decision making tool, earthquake hazard.

Introduction

Motivation for the Work Presented in this Paper

The purpose of this paper is to present the first phase of our investigations, for assessment of the total financial loss to a Large Private or Public Organization (LPO, for example, a university campus, a major hospital complex, a manufacturing facility, a freeway system including bridges and tunnels, aqueducts, communication systems, etc.) caused by earthquakes. The work is motivated by the need of LPO administrations for a tool that could provide a rational basis, and that could assist in decision making and in long range planning for mitigation of the financial losses that could be caused by future earthquakes. To be specific, our hypothetical LPO is located in a seismically active region (Los Angeles, California) and strong earthquakes are a serious threat.

The state of California, and particularly the City of Los Angeles, have strict code requirements for design of new buildings. The City of Los Angeles building code also requires strengthening of old, unreinforced or inadequately reinforced masonry buildings. However, the required structural strength is only to the level of preventing total collapse and eliminating hazard to human life. It is up to the building owner to decide whether to invest in higher strength than the minimal

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values specified in the building code, that would save human life only. To comply with the code requirements, we will assume that many unreinforced buildings have been already demolished by the LPO, and those of historical importance have been strengthened or are in the process of strengthening. Therefore, the threat to human life has been reduced. At this stage, of primary concern are the possible financial losses caused by structural and nonstructural damage during possible future earthquakes, loss of equipment, and, more important, the long and short term indirect losses caused by loss of working space and interruption of work. The interruption of the work will seriously imbalance the LPO budget, and fatalities will have serious and, maybe, irreparable long term consequences. Continuity in the operation in various clinics, for example of the LPO's hospitals, are vital not only for the LPO's finances, but also for the proper functioning of the whole metropolitan area. In modern technologically developed societies, the indirect losses may often exceed, by several times, the direct losses (Tiedemann 1984a, b, 1987, 1989). In spite of this fact, the indirect losses are often not considered in the prediction of the losses, probably because of the lack of empirical postearthquake data.

An example of how large and serious the consequences of a moderate earthquake may be for an LPO is the experience of a private university during the 1989, Loma Prieta, California, earthquake. This earthquake, with magnitude 7.1 and with epicenter at about 50 km from the university, produced extensive damage, and 24 of 240 major buildings had to be closed. While no deaths or serious injures occurred, the damage was estimated at 160 million dollars.

A way to reduce the high financial stress immediately following a destructive earthquake, is to acquire earthquake insurance, which is equivalent to distributing the future losses over a longer period of time. However, the earthquake insurance in California appears to be very expensive. For an LPO, the earthquake insurance might be of the order of several million dollars per year. In the long run, it may be financially better either to create an emergency fund (self-insurance), or to invest in better performance of the buildings during future earthquakes.

The funds that the LPO may spend for new construction and for rehabilitation of existing buildings that do not meet the code requirements can be extensive. The total cost of new buildings could be of the order of $ 50 million/year, for example. The construction of the building structures (typically 0.20 to 0.40 of the total cost) is usually only a small fraction of the total financial losses, that may result from earthquake shaking. The indirect losses, caused mainly by interruption of work and by loss of working space, can exceed many times the replacement value of the whole building. In spite of this, so far, the indirect losses have not received adequate attention, and have not been considered as a significant factor in decisions on the standards for new construction. Results of previous studies have shown that, by a small additional investment, to increase the strength beyond the code level, the future losses can be reduced significantly.

The LPO should also consider developing its own standards for new construction, and for strengthening of existing buildings. The questions that need to be answered are what these standards should be like, and what the optimum sum for the self-insurance fund would be. This constitutes a difficult n-dimensional optimization problem. A decision making tool is needed, a system consisting of a methodology and a database that could assess the losses for individual buildings with accuracy beyond the current engineering practice.

In Part I of this paper a methodology is presented for assessment of the losses of individual buildings, and possible applications are suggested. The method is general and can be applied for various natural and manmade hazards. In Part II, some admissible probability distribution functions for the losses are suggested and discussed, and, to illustrate the method, it is applied to a hypothetical building, exposed to a given ground motion at the base.

Optimum Design Loads for Buildings

The code provisions for earthquake safer structures have evolved several times since the 1930's (Leslie and Biggs 1972). The most recent provisions are described in the 1988 Uniform Building Code. The survey of damage after the San Fernando, 1971, California, earthquake showed that those buildings that have been built following the most recent provisions have suffered less damage than the buildings built prior to 1930's when there were no such provisions (Whitman et al. 1973). Guidelines have been established for seismic evaluation of existing buildings (ATC, 1987), and provisions have been defined for strengthening of existing buildings that do not meet the code requirements (Sabol et al. 1988a, b). However, the code requires strengthening only to the nominal level which improves the safety of human life. Considering the high cost of labor and the cost of interruption of work during the time of the construction (Sabol et al. 1988a, b), the code required level of strengthening is probably not the optimum one in the long range. Once the rehabilitation process of a building is started, with a slight increase of the investment, the financial losses from future earthquakes may be significantly reduced. The question, then, arises what is the optimum level of strengthening from the financial point of view, and what is the optimum design level for new buildings (Whitman et al. 1974, Ferrito 1984). Also, building owners and administrators have to decide on the priority in the order and in the distribution of the available funds for strengthening of a group of buildings, to minimize possible future losses from earthquakes. One of the key motivations for the work presented in this paper has been to develop a method, based on a detailed analysis, that could help guide such decisions and answer the related questions.

Design Spectra for Various Building Codes and Site Specific Spectra for an LPO

So far, the standards for new construction and for rehabilitation of older buildings are determined by the requirements of the building code. The code provides for the minimum requirements, that take into consideration mainly the life safety. These code requirements change with time, and, in many instances, may not be even adequate for the expected ground motion at the building site. To illustrate this, Fig. 1 compares the amplitudes of the design acceleration spectra (in units of $G = 9.81$ m/sec^2, and plotted versus the first natural period of building vibration in seconds) for the Uniform Building Code (UBC) of 1979, with spectra of representative (recorded) strong earthquake ground motions, and with the Uniform Risk Spectra (URS) computed for the site of the LPO, assumed to be just south of downtown Los Angeles. The URS represent weighted and balanced contributions to shaking for all possible earthquake sources surrounding the LPO site and for the assumed exposure during the next 50 years (Lee and Trifunac 1987, Trifunac 1990). The URS spectral amplitudes are shown for the probabilities of exceedance equal to 0.1, 0.5 and 0.9. It is seen that the amplitudes of future shaking can be 10 times larger than the amplitudes for which the buildings are typically designed (UBC).

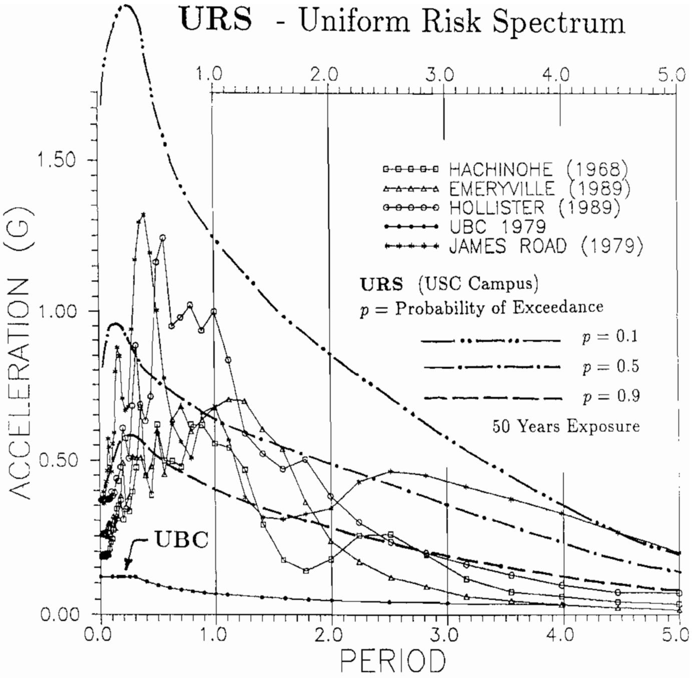

Fig. 1 – Comparison of the Uniform Building Code (UBC 1979) design spectrum with spectra of four recorded earthquakes, and with three uniform risk spectra (URS), corresponding to probability of being exceeded p =0.1, 0.5 and 0.9 for 50 years of exposure, computed for a site at about 5 miles south of downtown Los Angeles (University of Southern California main campus).

Vulnerability of Existing Buildings

There are, typically, three approaches to estimate the damage of existing buildings caused by earthquakes: 1) using theoretical analysis, 2) using analyses of empirical data, and 3) using judgment of experts. The theoretical approach consists of estimating the structural response to the prescribed ground motion first, and, then, correlating it with the damage of the individual elements (Blejwas and Bresler 1979, McCabe and Hall 1987). The second method consists of developing vulnerability matrices or indices for selected types of buildings using actual earthquake damage data (Whitman 1973, Benedetti et al. 1988, Petrovski and Milutinović 1987, Coburn et al. 1987). By the third method, damage probability matrices are developed on the basis of iterated expert opinion (ATC, 1985). The theoretical models can be used to calculate the structural response to any level of loading. However, these are limited in the sense that they represent idealized image of the real structure and cannot handle all possible details and real life situations. The second approach is conceptually the most appropriate, but it is not sufficient. The compiled damage data is incomplete (empirical data is missing for some ranges of the input ground motion and is insufficient for some types of structures). Also, results from one part of the world are not directly applicable to another part of the world, because of the differences in the construction technology and in the prevailing type of structures, and because of the differences in the code provisions. Therefore, the other two approaches have to be used to fill in the regions of missing or insufficient data. The third method could be used as a complement of the first method. Expert opinions are, however, often biased and limited by the experience and by the imagination of the experts.

At present, a fairly complete set of damage probability matrices (including physical damage of the structure and of its contents, as well as indirect losses), that is applicable to buildings in the United States, can be found in the Applied Technology Council Report No. 13 (ATC, 1985). These have been constructed on the basis of iterated expert opinions, and can be used by engineers to estimate the generic loss for types of buildings and of lifelines. An expert system has been constructed (Shah et al. 1987) that uses these damage probability matrices as input. Even though this is premature at present, this expert system is meant by the authors to be used for insurance and investment risk assessment.

A model for assessment of the total loss of a building

Definition of the Basic Concepts

In the model, the building will be referred to as the integral system (*IS*), which is composed of more subsystems (*SS*). Each of the subsystems (*SS*) is itself a system consisting of elements at risk (*ER*). The elements

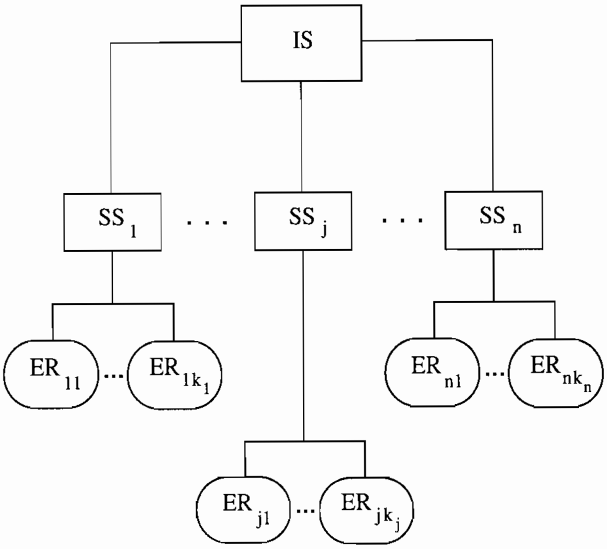

Fig. 2 – A block diagram of the integral system, *IS*, the subsystems, SS_j, and the elements at risk, .

at risk are the finest subsystems in the decomposition. In Fig. 2, a block diagram is shown of the integral system (*IS*), of the subsystems (SS_i, $i = 1, ..., n$) and of the elements at risk for each of the subsystems (ER_{ij}, $i = 1, ..., n$, $j = 1, k_i$, where k_i is the number of elements at risk for the i-th subsystem).

The subsystems could be physical divisions of the integral system, such as floors of a multi-story building, or functional units that run throughout the whole building such as, e.g., electrical installations, telephone lines, heating and air-conditioning systems. A subsystem could be a laboratory with expensive equipment and, perhaps, toxic materials that could be released as a direct or indirect consequence of the shaking, and which can represent additional hazard and, possibly, cause additional losses. The elements at risk are those that suffer physical damage, and contribute directly and indirectly to the physical damage of the subsystem to which they belong. Elements at risk are, for example, structural elements, such as columns, beams and shear walls, or particular pieces of laboratory equipment.

The input ground motion at the site of the building is described by the shaking parameter Y. Y is a random variable which can be a scalar or a vector, depending on the level of sophistication of the description of the ground motion. Y can be or can have as components the earthquake intensity at the site, the peak acceleration, the uniform risk spectrum (URS), the duration of shaking etc.. The damage of the elements at risk depends on the level of their input hazard. The input hazard level (H) for a subsystem is the level of some parameter of the response of the integral system (to the level of shaking Y) that is best correlated with the damage of the elements at risk of the subsystem. In the model, it represents the input excitation that can cause damage to the elements at risk. For example, the inter-story drift at the floor can be used as the input hazard level for the structural and nonstructural elements of a particular floor. The equipment may be sensitive to the absolute floor acceleration. The input hazard level is a random variable and it is a function of the shaking parameter Y. In simplified analyses, for assessment of generic losses for example, the shaking parameter Y would be used as the input hazard

The following convention in the notation will be used in the subsequent mathematical expressions. F_V and f_V will indicate the cumulative and the density probability distribution functions of the random variable V, and $P\{\cdot\}$ will indicate probability of the event in the brackets. $E[V]$ and $\mathrm{Var}[V]$ will indicate the expected value and the variance of the random variable V. All the random variables will be denoted by capital letters and the values that they can take by lower case letters.

Probability Distribution Functions of the *ER*'s, the *SS*'s, and of the *IS*

The Loss Associated with an Element at Risk

The loss *LER* associated with the physical damage of an element at risk *ER*, is a continuous random variable that depends on the input hazard level. It has a conditional cumulative probability distribution function (dependent upon the input hazard level, H)

$$F_{LER|H}(\ell|h) = P\{LER \leq \ell|H = h\} \qquad (1a)$$

and a density probability distribution function

$$f_{LER|H}(\ell|h) = \frac{d}{d\ell} F_{LER|H}(\ell|h). \qquad (1b)$$

The input hazard level, H, for en element at risk depends on the level of the site shaking parameter, Y. Let

$$F_{H|Y}(h|y) = P\{H < h|Y = y\} \qquad (2a)$$

and

$$f_{H|Y}(h|y) = \frac{d}{dh} F_{H|Y}(h|y). \qquad (2b)$$

be the conditional cumulative and conditional density probability distribution functions (dependent upon the input shaking parameter, Y) of H. Then, the total density probability function of the loss of the element at risk, *LER*, conditioned upon the level of shaking at the site, Y, is

$$f_{LER|Y}(l|h) = \int_0^\infty f_{LER|H}(l|h) \cdot f_{H|Y}(h|y) \cdot dh. \qquad (3)$$

Further in the text, for the purpose of brevity, the condition upon Y will be omitted in the notation. It should be implicitly understood until stated otherwise.

The Loss Associated with a Subsystem

The loss LSS associated with the physical damage of a subsystem SS is some function g of the losses associated with the damage of its elements at risk ER_i, $i = 1, ..., n$

$$LSS = g(LER_1, LER_2, ..., LER_n) \tag{4}$$

This functional relationship, in a real life situation, is not a simple function such as summation. For example, it would cost less to repair a group of elements all at one time, than to repair them one by one, separately.

Let the element losses, LER_i $i = 1, ..., n$, be jointly continuous with joint density function $f_{LER_1, ..., LER_n}(\ell_1, \ell_2, \ldots, \ell_n)$. Then, the probability that the subsystem loss will be less or equal to ℓ is

$$P\{LSS \le \ell\} = P\{g(LER_1, \ldots, LER_n) \le \ell\} \tag{5}$$

$$= \int\int\cdots\int_{g(LER_1, ..., LER_n)\le\ell} f_{LER_1, ..., LER_n}(\ell_1, \ell_2, \ldots, \ell_n) d\ell_1 d\ell_2 \ldots d\ell_n.$$

If the element losses LER_i, $i = 1, ..., n$ are independent, then their joint probability distribution function is a product of the individual probability distribution functions of the element losses, f_{LER_i} $i = 1, \ldots, n$

$$f_{LER_1, LER_2, ..., LER_n} = f_{LER_1} \cdot f_{LER_2} \cdots f_{LER_n}. \tag{6}$$

Recalling that by definition

$$P\{LSS \le \ell\} = F_{LSS}(\ell) \tag{7}$$

where $F_{LSS}(\ell)$ is the cumulative distribution function of the subsystem loss LSS, from Eqs. (5) and (6) it follows that

$$F_{LSS}(\ell) = \int\int\cdots\int_{g(LER_1, LER_2, ..., LER_n)\le\ell} f_{LER_1}(\ell_1)\cdots, f_{LER_n}(\ell_n) d\ell_1 \ldots d\ell_n. \tag{8}$$

Then the density distribution function of the subsystem losses can be calculated as

$$f_{LSS}(\ell) = \frac{d}{d\ell} F_{LSS}(\ell). \tag{9}$$

The simplest form of the function g is a simple summation. Until g is more precisely defined, the simplest form will be assumed here.

Losses of the Integral System

The probability distribution function of the total loss due to physical damage of the integral system can be derived similarly, from the probability distribution function of the subsystems. If the system loss LS is a function G of the subsystem losses $LSSj$, $j = 1, ..., N$

$$LIS = G(LSS_1, LSS_2, ..., LSS_N), \tag{10}$$

then the cumulative distribution function of LS, $F_{LIS}(s)$, and the corresponding density function, $f_{LIS}(s)$, are

$$F_{LIS}(s) = \int\int\cdots\int_{G(LSS_1, ..., LSS_N)\le s} f_{LSS_1, LSS_2, ..., LSS_N}(\ell_1, \ell_2, \ldots, \ell_N) d\ell_1 d\ell_2 \ldots d\ell_N. \tag{11}$$

and

$$f_{LIS}(s) = \frac{d}{ds} F_{LIS}(s). \tag{11a}$$

where $f_{LSS_1, LSS_2, ..., LSS_N}(\ell_1, \ell_2, \ldots, \ell_N)$ is the joint distribution function of all the subsystem losses.

The subsystem losses are, in general, not independent of each other. For example, if the subsystems represent different stories in a building, then extensive damage at the first floor can cause interruption of work at the other floors, which will induce indirect losses at these floors. At this time, neither the function G nor the joint probability density function $f_{LSS_1, LSS_2, ..., LSS_N}(\ell_1, \ell_2, \ldots, \ell_N)$ are known for buildings subjected to damaging earthquakes, and, therefore, assumptions have to be made in order to develop further the model. Suppose that the interaction of the subsystem losses with each other is negligible (the subsystem losses are independent), and that the total loss of the integral system is a sum of the losses of the subsystems. The assumption of the independence implies

$$f_{LSS_1, LSS_2, ..., LSS_N}(\ell_1, \ell_2, \ldots, \ell_N) = = f_{LSS_1}(\ell_1) \cdot f_{LSS_2}(\ell_2) \cdots f_{LSS_N}(\ell_N), \tag{12}$$

and the additional assumption that $G(LSS_1, LSS_2, ..., LSS_N)$ is a summation of the LSS_i $i = 1, ..., N$ implies that the integral on the right hand side of Eq. (8) is a convolution of the losses of the subsystems.

Identification of the Integral System and of the Subsystems

In the previous section, Eqs. (1) through (12) are applicable to any integral system, subsystems and their elements at risk, regardless of what they actually are. This way, in the implementation of the theory, flexibility is allowed in the selection of those elements. Also, the theory can be further generalized so that the integral system defined here is one of the subsystems of some higher order integral system.

In this study, the integral system is the whole building. The subsystems can be selected so that they represent either logical physical units of the system or functional units. In this study the subsystems are the individual floors and the basement of the building. This choice of the subsystems seems logical, because the shear and moment envelopes and the building response (relative displacement, absolute acceleration), which would be used as input hazard levels for the elements at risk, are normally estimated at the floor levels. Also, the direct and indirect losses, such as loss of equipment and interruption of work, heavily depend on the type of occupants of the subsystem and on the type of activities. The fact that different type of residents of the building may occupy different floors supports this choice.

The elements at risk of a given floor are then grouped into classes according to the following general criteria:

1. they belong to the same functional class, and
2. they respond and are vulnerable to the same input hazard parameters.

In the examples in this study, it is assumed that the losses associated with different elements and with different floors are not correlated, so that Eqs. (8) and (12) hold.

The Input to the Model

The input to the model, in general, consists of: 1) the conditional probability distribution functions, $F_{H|Y}$, of the input hazard level H for different elements at risk, 2) the conditional probability distribution functions of the element losses, $F_{LER|H}$, 3) the joint probability distribution function of the losses of all the elements at risk in a subsystem, for all the subsystems $f_{LER_1, LER_2, \ldots, LER_n}$, and 4) the functions g and G in Eqs. (4) and (10).

In the examples in this study it is assumed that the functions g and G are simple summations, i.e.

$$LSS = \sum_{i=1}^{n} LER_i \tag{13}$$

and

$$LIS = \sum_{j=1}^{N} LSS_j . \tag{14}$$

It is also assumed that the losses associated with different elements in a subsystem, and with the different subsystems in the integral system, are independent, so that Eq. (8) and (12) hold. Then, from Eqs. (8), (11) and (12), and from Eqs. (13) and (14) it follows that

$$F_{LSS}(\ell) = f_{LER_1} * f_{LER_2} * \ldots * f_{LER_n} \tag{15}$$

and

$$F_{LIS}(s) = f_{LSS_1} * f_{LSS_2} * \ldots * f_{LSS_n}, \tag{16}$$

where the symbol * indicates convolution.

Resistance Classes

The structural elements of a building sometimes may not have the design strength, because of the human factor involved in the construction process. Elements of the same kind may have different vulnerability in different subsystems. Three possibilities can be suggested to account for this difference:

1. independent probability distribution functions have to be defined for different elements or groups of elements,
2. one distribution function can be used for all the elements of a given kind, but with a larger standard deviation, and
3. same analytical probability distribution function can be used, e.g., the Beta probability distribution function, but with parameters that depend on the vulnerability classes.

In the hypothetical example in Part II of this paper, the third possibility is employed, with the following three resistance classes:

a) poor resistance class,
b) fair resistance class, and
c) good resistance class.

Accounting for the difference in the vulnerability of the elements of a given kind by assigning it to different resistance classes is physically more reasonable than by increasing the variance, because through the resistance classes the variance of the overall distribution function of the elements (including the distribution functions for the classes) is increased at all values of h, uniformly. This would not be the case if a standard shape, is assumed.

Modeling of the Indirect Losses

So far, in the description of the model, only the losses (in monetary units) due to physical damage of the elements at risk have been considered. These losses may include, for example, structural and nonstructural damage of the building, and loss or damage of stock and equipment, and are referred to in the text as primary or direct losses. The losses because of interruption of work, legal fees, renting temporary space, lost opportunities, disability premiums, medical expenses to treat injury, and other losses of similar nature, are called indirect or secondary losses. The loss of life has not been included so far in the model, because of the difficulty in transforming it into monetary units (this task raises many ethical questions). We suggest that the loss of life could be treated separately, rather than together with the monetary losses.

The indirect losses are correlated with the direct losses, but also depend on other factors of local or regional nature. For example, the loss due to interruption of work depends on

1. the type of activities that are interrupted and the amount of income that they generate, and
2. how soon the facilities can be repaired.

The time required to repair the building depends on the degree and on the distribution of damage, but also on the availability of construction companies and required materials and equipment at the time immediately after the earthquake, which depends on the extent of the overall damage in the region. Because of this complexity, the indirect losses must also be treated as random variables and their probability distribution functions have to be defined.

It would be most appropriate if the probability distribution functions for the indirect losses are defined by regression analyses of empirical data. Unfortunately, so far only limited post-earthquake data on indirect losses is available. Damage probability matrices for the indirect losses for several ranges of values of the Modified Mercalli Intensity have been developed for different classes of buildings only on the basis of iterated expert opinions (ATC 1985), and, so far, it has not been possible to verify those. Because the available damage probability matrices for the indirect losses have been developed for large classes of buildings, the estimated indirect losses are very sparse and can be used only for very general studies, to estimate regional losses, for example.

Collecting data on indirect losses and making it available to the whole engineering community is a very difficult if not an impossible task, mainly because of the fact that most of the valuable information is confidential and is not disclosed even to the engineers performing the assessment. Therefore, for practical purposes it would be convenient if some simple procedure could be developed which could be used by the building owners directly, without the need to disclose confidential information. The following procedure has been suggested.

Indirect Loss Proportionality Factor

The indirect losses of a subsystem, ILSS, can be expressed as a product of a factor $ILF \geq 0$ and the direct losses LSS, i.e.

$$ILSS = ILF \cdot LSS$$

where the factor ILF can be a given number or a random variable. The value of ILF (or its expected value, if it is a random variable) should be larger for a floor with expensive laboratory equipment and multi-million-dollar projects going on, than for some other floors. ILF could be called the Indirect Loss Proportionality Factor, and it could be assigned by the building owner or by his representative.

The Indirect Loss Proportionality Factor may significantly influence the total loss of the building and, therefore, its nature should be carefully studied using actual post earthquake damage data or through simulation. In general, ILF is a function of the direct loss of the subsystem. However, in the examples that follow, it is assumed that ILF is a uniformly distributed random variable over the interval of losses. For the user, the ILF can be defined descriptively. For example, three classes could be defined:

1) low indirect loss proportionality class with $ILF \in [0, a]$,
2) average indirect loss proportionality class with $ILF \in (a, b]$ and
3) high indirect loss proportionality class with $ILF \in (b, c]$.

The density probability distribution functions for these three classes could be, for example, $f_{ILF} = \dfrac{1}{a-0}$, $f_{ILF} = \dfrac{1}{b-a}$ and $f_{ILF} = \dfrac{1}{c-b}$, respectively. In general, ILF needs not to be uniformly distributed, and it can be defined on any closed interval. This approach leaves open the possibility to apply the fuzzy sets theory in the further development of the model. In the example discussed in Part II of this paper we choose $a = 1$, $b = 2$ and $c = 3$.

The Total System Loss

The Total Direct and Indirect Losses

The total loss of the integral system, $TLIS$, is a sum of the total direct loss, LIS, and of the total indirect loss, $ILIS$. Assuming that it is a simple sum of the subsystem losses, it can be written that

$$TLIS = \sum_{j=1}^{N} (1 + ILF_j) LSS_j \tag{17}$$

The total integral loss of the system can have values in the interval (ℓ_{min}, ℓ_{max}) where

$$\ell_{min} = \sum_{j=1}^{N} [1 + r_{j,min}] \ell_{j,min} \tag{18a}$$

and

$$\ell_{max} = \sum_{j=1}^{N} [1 + r_{j,max}] \ell_{j,max}, \tag{18b}$$

with $r_{j,\,min}$ and $\ell_{j,\,min}$ being the lowest values that ILF and the direct loss LSS can take for the j-th subsystem, and with $r_{j,\,max}$ and $\ell_{j,\,max}$ being the highest values of the indirect loss factor, ILF, and of the direct losses LSS, for the j-th subsystem. Rearranging the terms on the RHS of Eq. (17), it follows that

$$TLIS = LIS + ILIS \tag{19}$$

where

$$ILIS = \sum_{j=1}^{N} ILSS_j \tag{20}$$

is the indirect loss of the integral system, and where

$$ILSS_j = ILF_j \cdot LSS_j \tag{21}$$

is the indirect loss of the j-th subsystem. The cumulative probability distribution function of $ILSS_j$, $F_{ILSS_j}(s)$, then is

$$\begin{aligned} F_{ILSS_j}(s) &= P\{ILSS_j \leq s\} = \\ &= P\{ILF_j \cdot LSS_j \leq s\} \\ &= \int\int_{r \cdot \ell \leq s} f_{ILF_j,\, LSS_j}(r, \ell) \cdot dr \cdot d\ell, \end{aligned} \tag{22a}$$

where $f_{ILF_j,\, LSS_j}(r, \ell)$ is the joint density function of ILF and LSS for the j-the subsystem. If ILF_j and LSS_j are not correlated, then

$$f_{ILSS_j}(s) = \int\int_{r \cdot \ell \leq s} f_{ILF_j}(r) \cdot f_{LSS_j}(\ell) \cdot dr \cdot d\ell. \tag{22b}$$

The cumulative distribution function of the indirect losses of the integral system, given in Eq. (20), can be evaluated in a similar manner as for the direct losses, i.e. by convolution

$$F_{ILIS} = f_{ILSS_1} * f_{ILSS_2} * \dots * f_{ILSS_N}. \tag{23}$$

Similarly, the cumulative distribution function of the total integral system loss, $TLIS$, defined in Eq. (19), can be calculated as

$$F_{TLIS} = f_{ILIS} * f_{LIS} \tag{24}$$

In Eq. (23),

$$f_{ILSS_j}(s) = \frac{dF_{ILSS_j}(s)}{ds} \tag{25a}$$

is the density probability function of the indirect losses of the j-th subsystem, and in Eq. (24)

$$f_{ILIS}(s) = \frac{dF_{ILIS}(s)}{ds} \tag{25b}$$

and

$$f_{LIS}(s) = \frac{dF_{LIS}(s)}{ds} \tag{25c}$$

are the density probability functions of the indirect and direct loss, respectively, of the integral system.

Total System Losses for a Given Exposure Time

We recall that the probability distribution function F_{TLIS} of the total losses of the integral system and the previously defined subsystem losses distribution functions are, in fact, conditioned on the value of the site response parameter Y, but «$|Y = y$» has been omitted for the purpose of brevity.

The losses caused by earthquakes are usually assessed for a limited time interval, T, the exposure time, which is usually the expected service time for the building, e.g. $T = 40$, 50 or 100 years. This requires the probability of occurrence of Y to be determined for that exposure time. Let

$$F_{Y|T}(y|t) = P\{y \leq y|t_o \leq T < t_o + t\} \tag{27}$$

be the cumulative distribution function of Y for a given exposure time, and let

$$f_{Y|T}(y|t) = \frac{d}{dy} F_{Y|T}(y|t). \tag{28}$$

be the corresponding density function. Then the probability distribution function of the total losses for that time interval is

$$f_{TLIS|T}(\ell|t) = \int f_{TLIS|Y}(s|Y = y) f_{Y|T}(y|t) dy. \tag{29}$$

For decision making in optimum seismic design, optimum investment in strengthening, emergency planning and other related areas the following two quantities are usually calculated

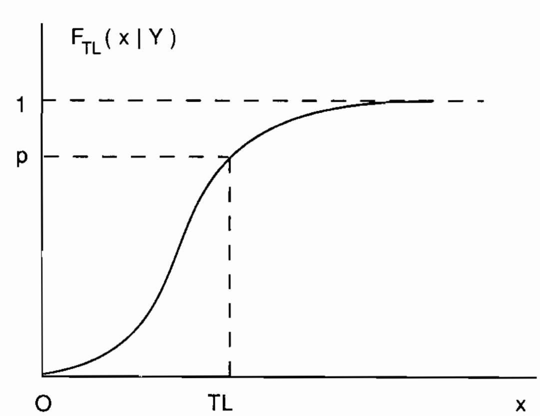

Fig. 3 – The cumulative probability distribution function of the total loss for a building, $F_{TL}(x|Y)$, for the next Y years. $F_{TL}(x|Y)$ is the probability that the total loss will not exceed x monetary units in the next Y years. TL in this figure is the loss that will not be exceeded with probability p.

1. The expected value of the total system losses for the exposure time t

$$E[TLIS|T=t]=\int_0^\infty \ell f_{TLIS|T}(\ell|t)d\ell, \tag{30}$$

2. The value of the losses, ℓ_p, that will not be exceeded with confidence level p during the exposure time, satisfies the following equation

$$p=\int_0^{\ell_p} f_{TLIS|T}(\ell|t)d\ell. \tag{31}$$

and can be graphically determined from the cumulative probability distribution function of the total loss as shown in Fig. 3.

Possible applications of the model

To implement the proposed methodology (i) an extensive database is needed on the buildings of the LPO, as well as (ii) computer programs to evaluate the building response at different levels in the linear and nonlinear range, (iii) realistic fragility curves for the elements at risk, (iv) a computer program to estimate site specific ground motions, and (v) a computer program that would calculate the probability distribution functions of the total loss for the individual buildings.

The building database should contain information about the building structure and history of prior strengthening, as well as information on the inventory and on the various activities in the buildings. This information is specific for the LPO. Computer programs for linear and nonlinear analyses of response of buildings have already been developed. Fragility curves for various structural and nonstructural elements and sensitive equipment can be constructed from available information in the literature, based on results of laboratory experiments or via theoretical investigations. The probability distribution functions of the damage can be constructed, for example, by theoretical estimation of the uncertainty, or by simulation. To estimate in a probabilistic manner the site specific motions, the «EQRISK» approach (Lee and Trifunac 1985, Trifunac 1990) can be used, for example. The original computer program EQRISK and the updated version NEQRISK evaluate site specific uniform risk spectra with a given probability of being exceeded and for a specified time of exposure. To estimate the losses for the whole building the interactive computer program EQLOSS has been developed by the authors of this paper, based on the model described in this paper. The different steps required to estimate the losses of a building are illustrated in Fig. 4.

The anticipated uses of this approach are many. The important applications will be:

1) In detection of the buildings at an LPO that may represent relatively higher risk during future earthquake shaking.

2) In the development of general standards for earthquake design criteria for the LPO facilities.

3) As a decision making tool to:

a) optimize the strength of new buildings and the investments in strengthening the existing buildings (by providing the facilities construction standards via the proposed analyses, leading to optimum design criteria, which can be incorporated into future contract specifications for new facilities, or for rehabilitation of the existing structures),

b) estimate the losses from future earthquakes,

c) plan additional space that can be used in emergency situations after a larger earthquake,

d) enable estimation of required funding for self earthquake insurance.

4) For quick post-earthquake evaluation, to find if the buildings at the LPO are safe for occupancy.

Optimization of the Total Cost

To reduce the physical damage of the building and of its contents and, consequently, all of the direct and indirect monetary losses (Fig. 4) to the building owner, it is necessary to strengthen the structural elements and their connections, as well as to revise and make necessary modifications to the configuration of the equipment inside (for example, to bolt equipment and furniture to the floor or to the wall). A more radical improvement (and more costly) of the susceptibility of the

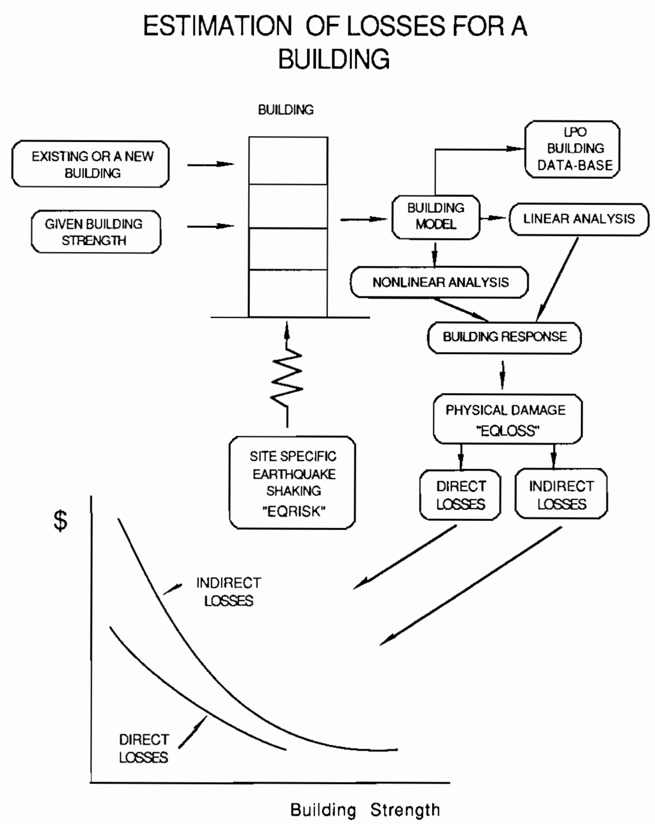

Fig. 4 – Steps required to estimate the indirect losses and the direct losses in a building as functions of the building strength.

building to damage would be by base isolation of the whole structure, or base isolation of expensive equipment inside the building, for example.

The improvements of the quality of the elements at risk and reduction of their vulnerability to the earthquake hazard require investment, and a proper balance between the investment and the reduction of the losses should be made. The optimization process of the investment in rehabilitation consists of finding the state for which the total cost to the building owner, equal to the sum of the investment in rehabilitation and the expected total losses (or the losses that will happen with some given level of confidence) is at a minimum.

To optimize the investment in rehabilitation of the building, the minimum of the utility function $\Phi(\alpha) = E[TLIS|\alpha] + B(\alpha)$ has to be found over all possible values of α, where α is a vector whose components are the input parameters for the building (the resistance class and the indirect loss proportionality class for the elements at risk), $E[TLIS|\alpha]$ is the expected value of the total loss, and $B(\alpha)$ is the investment in rehabilitation. The solution of the optimization problem is a vector α^* such that

$$\Phi(\alpha^*) \leq \Phi(\alpha), \qquad \text{for all } \alpha \in A \qquad (32)$$

where A is the set of admissible values of α.

A simplified representation of required analyses for finding the optimum design standards for new buildings and for strengthening of the existing buildings is shown in Fig. 5. In the graph at the bottom of the figure, the total cost of the construction, and the total financial losses are shown as functions of the building strength. The total cost to the LPO is the sum of the building construction cost and the expected losses. As the building strength increases, the construction costs increase, but the expected losses decrease. Because of uncertainties in the size of the largest earthquake that can occur on the nearby faults, and because of the smaller probability of occurrence of very large earthquakes during an interval of 50 – 100 years, the additional strength is effective in reducing possible losses only up to some limit. Thus, there is a critical point, where the additional investment is larger than the anticipated financial return. With further increase of the structural strength, the expected losses will reach some threshold level of «acceptable loss». The total cost to the LPO has a minimum at some building strength. This is the optimum design strength that has to be determined. This can be done by a computer search estimating the losses for different scenarios, until the optimal configuration is reached.

Summary and conclusions

As a result of the advancements in the construction technology and of the more strict code provisions for the design forces, the human casualties and the material damage caused by earthquakes in a modern society have been reduced significantly. However, in spite of this, the modern society is still vulnerable, even to moderate earthquakes, because of the large losses that may result from interruption of work, legal fees, loss of important equipment etc. To abate the damaging consequences of earthquakes in the long run, adequate preparedness and planning are required. To accomplish this, a tool (consisting of a methodology, computer programs, and a database) is needed that would estimate the possible losses and assist in the decision making. Initial investment in strengthening of existing buildings will reduce future losses. However, the long range financial gain is not a linear function of the initial investment and the optimum investment has to be determined.

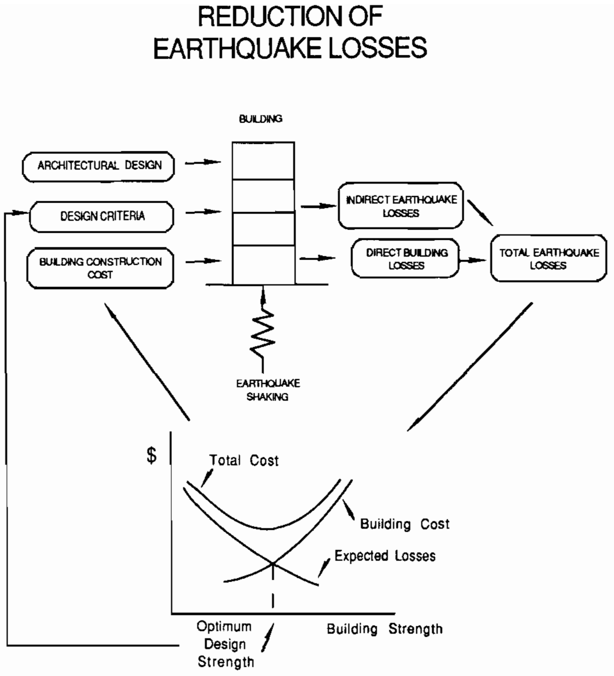

Fig. 5 – A loop of the procedure for determination of the optimum building strength, that minimizes the total cost to the building owner. The total cost is a sum of the building cost and the expected losses.

The decision making tool for prediction of the losses should consists of a user-friendly computer program, a database on the building, probabilistic description of the earthquake hazard at the site, and damage probability distribution functions for given levels of the ground shaking. The computer program should be interactive and easy to use not only by earthquake engineering professionals, but also by the building owner or by an executive, which would secure the confidentiality of the gathered information and of the prediction.

The database on the building should contain information on the structural properties and on the properties of the soil on which the building has been founded (so that the response to earthquake motion can be estimated), the inventory and the various functions of the structure, so that the indirect losses can be predicted. The description of the earthquake hazard consists of the probability of occurrence of ground motion with given intensity (MMI, peak acceleration or uniform risk response or Fourier spectrum) at the site during the service time of the structure. This probability of occurrence can be calculated from geological data and/or from data on the seismic activity in the past. A methodology and a computer program have been developed by the authors to calculate uniform risk spectra and Modified Mercalli Intensity at a site with given confidence that those will not be exceeded during a given exposure time. This methodology has been applied to microzonation of the Los Angeles Metropolitan Area (Lee and Trifunac 1987, Trifunac 1990). The damage probability functions can be determined from post earthquake damage data, by simulation, or from expert opinions. Damage probability matrices have been constructed for structural and non-structural damage of high-rise buildings for given range of MMI at the site, from damage data gathered after the 1971 San Fernando, California, earthquake. Such damage probability matrices are directly applicable to damage assessment in California. However these are incomplete and do not include the indirect losses (no significant empirical data on indirect losses has been gathered so far). Damage probability matrices have been constructed for the direct and indirect losses of different types of buildings, life-lines and other type of structures, based on expert opinion. Those are presently used by the practicing engineers as the most complete set to estimate the losses of given type of buildings (the buildings have been classified according to structural type and size and according to their function). However, these can be used to estimate only roughly the generic losses for given type of buildings and there is a large uncertainty associated with these estimates.

In this paper, a method has been developed to estimate in more detail the total loss of a specific building exposed to given level of hazard. The hazard to which the building is exposed could be an earthquake, fire, tsunami, wind or other natural and man-caused hazards. The unit for which the losses are assessed is called an integral system, and it could be a building, a group of buildings, a whole community, a life-line or any other vulnerable system. The integral system is made of subsystems which consist of elements at risk. The damage probability distribution functions for the physical damage of the elements at risk must be given, and also the input hazard level for the elements. The input hazard level is the level of a response parameter of the system with which the damage of the element is correlated. The input hazard level is a function of the level of shaking at the site. To distinguish between different quality of the elements at risk of a given kind and their susceptibility to damage, resistance classes have been defined. The resistance class of an element may be a function of the level of the forces for which the element has been designed, of the past experience of the element, of its relation to the other elements at risk etc. The indirect losses for the subsystems are calculated from the direct losses, given a proportionality factor. The proportionality factor can be a fixed number or a random variable specified by a probability distribution function. This factor depends on the importance of the function of the subsystem, but also on the overall damage in the region which affects the time required to restore all the functions of the subsystem. The total loss of the integral system is some function of the subsystem losses.

An interactive computer program EQLOSS has been written to estimate the earthquake losses for a community of buildings. This program can be interfaced with the bank of data on all the buildings at an LPO, which can be easily updated by the user. It also allows graphical representation of the damage probability functions for the integral system. Such a computer program can be used by the owner or by an executive as a decision making tool for mitigation of the losses caused by future earthquakes. By executing the program for different scenarios, the optimum steps for future action can be determined. At present the program estimates the losses for given level of shaking at the site. However, it can be easily interfaced with the computer program NEQRISK (Lee and Trifunac 1985) so that, then, the expected value of the losses or the losses that will not be exceeded with a given level of confidence during the service time of the building can be estimated.

What is missing at present are the probability distribution functions (or matrices) of the direct losses associated with damage for the elements at risk, and probability distribution functions for the indirect loss proportionality factors for the subsystems. This task requires at least several years of extensive research and data gathering, and is left for future work.

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