Response of Multistoried Buildings to Ground Translation and Torsion during Earthquakes ***

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SUMMARY - An approach has been presented for the probabilistic estimates of the response peaks of fixed-base, symmetric multistoried buildings which are subjected to the combined excitation of translational and torsional components of earthquake ground motion. An additional case of the rocking component included with this excitation has also been considered. The formulation considers the phase differences between the translational and the rotational components. The proposed model has been illustrated by considering two example buildings and synthetically generated earthquake records.

KEY WORDS: seismic response, torsional components, rocking, ground motion.

1. Introduction

Recently, various investigators /3, 4, 9, 10, 11, 14, 16, 18, 21, 22, 28/ have made attempts to study the structural response excited by the torsional component of earthquake ground motion. This component is caused by the action of obliquely incident plane SH waves and the surface Love waves during passage of seismic disturbance. Due to the presence of this component in the free-field ground motion, torsional deformations of the structure take place even when there exists a symmetry in the structural configuration.

Most of the previous analyses have been based on approximate characterizations of the torsional component and have consistently ignored the dispersion and the presence of different modes in the propagating waves. Gupta and Trifunac /4/ accounted for these factors in their statistical study of the torsional response peaks of the fixed-base, symmetric buildings. For this, they used the results of Trifunac /23/ and Lee and Trifunac /12/ based on the elastic wave propagation in the layered ground. Their approach is more general than the other methods as it can give the response estimates of any order of peak (not just the largest), and for a desired level of confidence. However, they did not consider the phase difference between the translational and torsional components of motion for the study of combined effects of these components. They computed the peak responses separately for these components and then added them algebraically. This approach however appears to be too conservative, particularly in the light of theoretical results /12, 23/ which show, in the ideal conditions, that the two components act orthogonal to each other.

This study is aimed at generalizing the approach of Gupta and Trifunac /4/ by including the effects of phase difference between the ground translation and torsion, and also by accounting for the modal interaction which has so far been ignored in such studies. To do this, formulation has been derived along the similar lines as in Gupta and Trifunac /7, 8/ for the case of combined excitation of translational and rocking components. The formulation has been extended to include effects of rocking component as well. Using the synthetic accelerograms /12, 13, 27/ and related Fourier spectra along with certain example sites and buildings, the proposed model has been shown to provide good estimates of the structural response peaks.

2. Brief Review

Foundations of the approach considered in this paper go back to the works of Rice /19, 20/, Longuet-

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Higgins /15/ and Cartwright and Longuet-Higgins /1/ on the distributions of the maxima of a random function, and to that of Gupta and Trifunac /6/ on the application of order statistics to generalize their results for the various orders of peaks. These are briefly reviewed in the following for the sake of continuity and completeness in this presentation.

For a random response function $f(t)$ having N independent and identically distributed peaks, the n^{th} order peak (in decreasing order of magnitude) with $n \leq N$ is distributed as /6/

$$F_{(\eta)}(\nu) = \sum_{i=n}^{N} \binom{N}{i} (P(\eta))^i (1-P(\eta))^{N-i}, \qquad (2.1)$$

η being the height of the peak normalized by a_{rms}, the root mean square (r.m.s.) amplitude of $f(t)$ and $P(\eta)$ being the probability distribution function of the height of a maximum. $P(\eta)$ is expressed /1/ in terms of η and ε where ε is a measure of the width of the energy spectrum. $E(\omega)$ of $f(t)$ and is defined by

$$\varepsilon = \sqrt{\frac{m_0 m_4 - m_2^2}{m_0 m_4}}. \qquad (2.2)$$

In Eq. (2.2), m_0, m_2, and m_4 are the moments of $E(\omega)$ about the origin defined by

$$m_n = \int_0^\infty \omega^n E(\omega) d\omega, \qquad n = 0, 1, 2, \ldots . \qquad (2.3)$$

The total number of peaks, N in $f(t)$ and the r.m.s. value of $f(t)$, a_{rms} are /1/

$$N = \frac{T}{2\pi} \left(\frac{m_4}{m_2}\right)^{1/2} \qquad (2.4)$$

and

$$a_{rms} = m_0^{1/2}, \qquad (2.5)$$

where T is the total duration of the response, taken same as the duration of input excitation. For a given value of $F_{(n)}(\eta)$ (corresponding to the desired level of confidence), Eq. (2.1) can be used to obtain η which, on being multiplied with a_{rms}, given the peak amplitude of $a_{(n)}$, the n^{th} order peak of $f(t)$. If r.m.s. value $\bar{a}$ of the peaks of $f(t)$ is used in place of a_{rms} for calculation of the peak amplitude by denormalization of η, $\eta/\sqrt{2}$ should be used in place of η as $\bar{a} \simeq \sqrt{2}a_{rms}$ assuming $f(t)$ as a narrow band process /25/. For the expected value of $a_{(n)}$, the approximation proposed by David and Johnson /2/ and illustrated by Gupta and Trifunac /6/ may be used.

For the multi-degree-of-freedom (MDOF) system shown in Fig. 1, modal analysis can be carried out in Fourier-transformed frequency domain for the desired response function and since the energy spectrum $E(\omega)$ is related to $F(\omega)$, the Fourier transform of $f(t)$ as /17, 26/

$$E(\omega) = \frac{1}{\pi T} \mid F(\omega) \mid^2, \qquad (2.6)$$

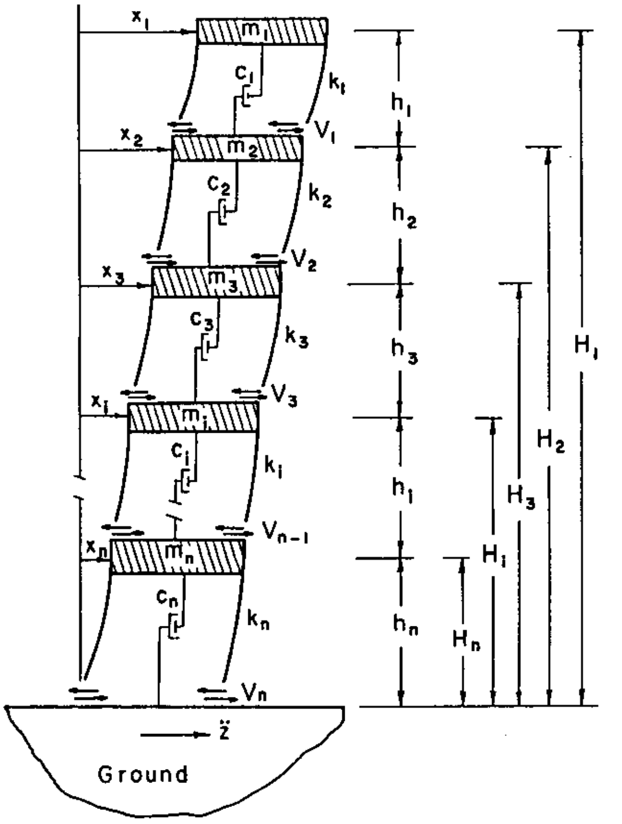

Fig. 1 – Multi-Degree-of-Freedom System for Translational Vibrations.

the amplitude of any peak of the response function can be calculated using the above procedure for obtaining probabilistic estimates. For example, if the n-DOF system in Fig. 1 is excited at the base by the horizontal component of ground acceleration, $\ddot{z}(t)$ alone and the interaction between various (translational) modes is accounted for, it is possible to obtain the following expression for energy spectrum, $ED_i(\omega)$, of displacement at i^{th} floor /7, 8/,

$$ED_i(\omega) = \frac{1}{\pi T} \mid Z(\omega) \mid^2 \left[\sum_{j=1}^{n} A_{ij}^2 \alpha_j^2 \mid H_j(\omega) \mid^2 \left\{ 1 + p_{D,ij} + \left(1 - \frac{\omega^2}{\omega_j^2}\right) q_{D,ij} \right\} \right]. \qquad (2.7)$$

Here, $\mid Z(\omega) \mid$ is the Fourier spectrum of input excitation $\ddot{z}(t)$; $A = [A_{ij}]_{n \times n}$ is the modal matrix; $\alpha_j = \sum_{k=1}^{n} A_{kj} m_k / \sum_{k=1}^{n} A_{kj}^2 m_k$ is the modal participation factor in j^{th} mode for floor masses $m_1, m_2, \ldots m_n$; ω_j and ζ_j are the natural frequency and damping ratio in j^{th} mode; $p_{D,ij}$ and $q_{D,ij}$ represent the interaction of j^{th} translational mode with the other translational modes (for displacement at i^{th} floor) under the excitation of translational component alone; and

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + 2i\zeta_j\omega_j\omega}. \qquad (2.8)$$

To account for the nonstationarity of response (in case of first few peaks), the r.m.s. value of the peaks of response function may be modified as explained in Appendix I (for displacement response at the i^{th} floor) and then be used along with the other statistical parameters (N and ε) to determine the peak amplitudes of response function.

3. Combined Action of Translation and Torsion

When a multistoried building is subjected to the torsional component of ground acceleration, $\ddot{\vartheta}_T(t)$ at its base, the rotational vibrations are set up in each floor about its center of mass. It is convenient to model the building for these vibrations by masses concentrated at the floors and connected with the massless torsional springs and dashpots as shown in Fig. 2. Using modal decoupling of the equations of motion, the transfer function for the response (relative rotation) of the i^{th} floor in the j^{th} torsional mode of vibration can be written as /4/

$$H_{i,jT}(\omega) = A_{i,jT}a_{jT}H_{jT}(\omega), \qquad (3.1)$$

where $H_{jT}(\omega)$ is the transfer function for the relative rotation of equivalent SDOF oscillator for the j^{th} torsional mode, given by

$$H_{jT}(\omega) = \frac{1}{\omega_{jT}^2 - \omega^2 + 2i\zeta_{jT}\omega_{jT}\omega} \qquad (3.2)$$

and a_{jT} is the modal participation factor in j^{th} torsional mode given by

$$a_{jT} = \frac{\sum_{k=1}^{n} A_{k,jT}J_k}{\sum_{k=1}^{n} A_{k,jT}^2 J_k}, \qquad j = 1, 2, \ldots n\,; \qquad (3.3)$$

$A_{k,jT}$ is the k^{th} element of mode shape vector for the j^{th} torsional mode; ω_{jT} is the natural frequency and ζ_{jT} is the fraction of critical damping in the j^{th} torsional mode; J_k is the mass moment of inertia of the k_{th} floor about the vertical axis through its center of mass. Using (3.1), the Fourier transform of relative rotation response at the i^{th} floor becomes

$$\psi_i(\omega) = \sum_{j=1}^{n} A_{i,jT}a_{jT}H_{jT}(\omega)\Theta_T(\omega) \qquad (3.4)$$

where $\Theta_T(\omega)$ is the Fourier transform of $\ddot{\vartheta}_T(t)$. From this rotation follows the translational displacement at the i^{th} floor for the m^{th} column by its multiplication with the transverse distance, b_{mi}, of the column from the center of rotation at the floor. To consider the combined action of translational and torsional components, the so obtained contribution of torsional vibrations to translational displacement may be added to

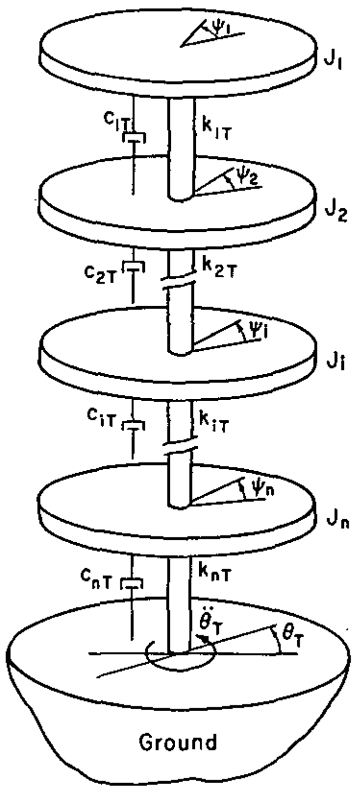

Fig. 2 – Multi-Degree-of-Freedom System for Torsional Vibrations.

that due to the translational component of ground motion alone. Thus, the expression for Fourier transform of the total relative displacement at the i^{th} floor for the m^{th} column becomes

$$X_{mi}(\omega) = \sum_{j=1}^{n} A_{ij}a_jH_j(\omega)Z(\omega) + \\ + \sum_{j=1}^{n} b_{mi}A_{i,jT}a_{jT}H_{jT}(\omega)\Theta_T(\omega) \qquad (3.5)$$

where $Z(w)$ is the Fourier transform of $\ddot{z}(t)$. The corresponding expression for the energy spectrum $ED_{mi}(\omega)$ (using Eq. (2.6)) becomes

$$ED_{mi}(\omega) = \frac{1}{\pi T}\Bigg[\sum_{j=1}^{n}\sum_{k=1}^{n} A_{ij}A_{ik}a_ja_kH_k^*(\omega)H_j(\omega)\,|Z(\omega)|^2 + \\ + \sum_{j=1}^{n}\sum_{k=1}^{n} A_{i,jT}A_{i,kT}a_{jT}a_{kT}b_{mi}^2H_{jT}(\omega)H_{kT}^*(\omega)\,|\Theta_T(\omega)|^2 + \\ + 2b_{mi}\sum_{j=1}^{n}\sum_{k=1}^{n} A_{i,jT}A_{ik}a_{jT}a_k\,|\Theta_T(\omega)|\,|Z(\omega) \\ |Re\{H_{jT}(\omega)H_k^*(\omega)e^{-i\Phi T}\}\Bigg] \qquad (3.6)$$

where $H_k^*(\omega)$ is the complex conjugate of $H_k(\omega)$; ΦT is the phase difference between the translational motion $Z(\omega)$ and torsional motion $\Theta_T(\omega)$ as in

$$\frac{Z(\omega)}{\Theta_T(\omega)} = \left| \frac{Z(\omega)}{\Theta_T(\omega)} \right| e^{i\Phi T} \tag{3.7}$$

Though the phase difference ΦT does depend on frequency ω, for simplicity it has been assumed here to be independent of ω.

Let

$$\mu^T_{k,jT} = \omega_{jT}^{(3-k)} \int_0^\infty |\Theta_T(\omega)|^2 |H_{jT}(\omega)|^2 \omega^k d\omega; \quad k = 0, 1, 2, \ldots \tag{3.8}$$

$$\lambda^T_{k,j} = \omega_j^{(3-k)} \int_0^\infty |Z(\omega)|\,|\Theta_T(\omega)|\,|H_j(\omega)|^2 \omega^k d\omega; \quad k = 0, 1, 2, \ldots \tag{3.9}$$

and

$$\lambda^T_{k,jT} = \omega_{jT}^{(3-k)} \int_0^\infty |Z(\omega)|\,|\Theta_T(\omega)|\,|H_{jT}(\omega)|^2 \omega^k d\omega; \quad k = 0, 1, 2, \ldots \tag{3.10}$$

where $\omega^T_{k,jT}$ is the k^{th} normalized moment of $|\Theta_T(\omega)|^2 |H_{jT}(\omega)|^2$ for an equivalent single-degree-of-freedom (SDOF) oscillator of the j^{th} torsional mode; $\lambda^T_{k,jT}$ and $\lambda^T_{k,jT}$ respectively are the moments of $|Z(\omega)|\,|\Theta_T(\omega)|\,|H_j(\omega)|^2$ and $|Z(\omega)|\,|\Theta_T(\omega)|\,|H_{jT}(\omega)|^2$ for the SDOF oscillators of j^{th} translational and torsional modes, and let $\lambda_{k,j}$ denote the moment of $|Z(\omega)|^2 |H_j(\omega)|^2$ for the oscillator of j^{th} translational mode as defined in Eq. (A1.4). Then, the k^{th} moment of $ED_{mi}(\omega)$ (according to Eq. (2.3)) can be written as /7/

$$\begin{aligned} m_{k,mi} = {} & \frac{1}{\pi T} \sum_{j=1}^{n} A_{ij}^2 a_j^2 \omega_j^{(k-3)} \Big[(1 + p_{D,ij} + q_{D,ij}) \lambda_{k,i} - q_{D,ij} \lambda_{k+2,j} \\ & + b_{mi} \cos\Phi_T \{ (p^T_{D,ij} + q^T_{D,ij}) \lambda^T_{k,j} - q^T_{D,ij} \lambda^T_{k+2,j} \} + \\ & \frac{b_{mi} \sin\Phi_T}{2\zeta_j} \{ (s^T_{D,ij} - (2 - 4\zeta_j^2) p^T_{D,ij}) \lambda^T_{k+1,j} + p^T_{D,ij} \lambda^T_{k+3,j} \} \Big] + \\ & \frac{1}{\pi T} \sum_{j=1}^{n} A^2_{i,jT} a^2_{jT} b^2_{mi} \omega_{jT}^{(k-3)} \Big[(1 + p^T_{D,ijT} + q^T_{D,ijT}) \mu^T_{k,jT} - \\ & - q^T_{D,ijT} \mu^T_{k+2,jT} + \frac{\cos\Phi T}{b_{mi}} \{ (p_{D,ijT} + q_{D,ijT}) \lambda^T_{k,jT} - \\ & q_{D,ijT} \lambda^T_{k+2,jT} \} - \frac{\sin\Phi T}{2 b_{mi} \zeta_{jT}} \{ (s_{D,ijT} - (2 - 4\zeta^2_{jT}) p_{D,ijT}) \\ & \lambda^T_{k+1,jT} + p_{D,ijT} \lambda^T_{k+3,jT} \} \Big] \end{aligned} \tag{3.11}$$

where the coefficients $p_{D,ij}$, $q_{D,ij}$, $p^T_{D,ijT}$, $q^T_{D,ijT}$, $p_{D,ijT}$, $q_{D,ijT}$, $s_{D,ijT}$, $p^T_{D,ij}$, $q^T_{D,ij}$ and $s^T_{D,ij}$ characterize the interaction between various modes. The coefficients $p_{D,ij}$ and $q_{D,ij}$ represent the interaction of j^{th} translational mode with the other translational modes for the displacement at i^{th} floor under the translational excitation; $p^T_{D,ijT}$ and $q^T_{D,ijT}$ represent the interaction of j^{th} torsional mode with other torsional under the excitation of torsional component; $p_{D,ijT}$, $q_{D,ijT}$ and $s_{D,ijT}$ describe the interaction of j^{th} torsional mode with all the translational modes under the simultaneous action of translational and torsional components; coefficients $p^T_{D,ij}$, $q^T_{D,ij}$ and $s^T_{D,ij}$ describe the interaction of j^{th} translational mode with all the torsional modes under the simultaneous action of translational and torsional components. It may be noted in the above expression that the terms in each of the translational and torsional modes are modified due to two types of modal interactions, one due to the interaction between modes of the same type (translational or torsional) and second, due to the interaction between modes of different types (translational and torsional). The extent of interaction in both cases is dependent on the closeness of frequencies of the interacting modes.

It can be shown that for the shear force response, following substitutions are required in the various expression applicable for the displacement response,

i) $a_j \omega_j^2$ for a_j

ii) $(m_1 A_{1j} + \ldots + m_i A_{ij})$ for A_{ij}

iii) $a_{jT} \omega_{jT}^2$ for a_{jT}

iv) $(J_1 A_{1,jT} + \ldots + J_i A_{i,jT})$ for $A_{i,jT}$

v) $M\beta_{mi}$ for b_{mi}. (3.12)

Here, β_{mi} is a quantity, dependent on the geometry of columns and floors, and given the shear force in longitudinal direction in the m^{th} column at the i^{th} floor, by multiplication with the cumulative torque at that floor /7/. M is the total number of columns. If all the M columns have equal stiffnesses in the longitudinal direction as well as in the transverse direction, it can be shown that

$$\beta_{mi} = \frac{b_{mi}}{\sum_{k=1}^{M} a_{ki}^2 k_{yx} + b_{ki}^2}, \tag{3.13}$$

where a_{ki} is the longitudinal distance of the k_{th} column from the center of rotation at the i^{th} floor and k_{yx} is the ratio of stiffness of a column in the transverse direction to that in the longitudinal direction. It may be noted that the torsional vibrations of symmetric buildings do not lead to any story shear and hence, the substitutions as in (3.12) will lead to energy spectra of «fictitious shear force» which varies from column to column unless all the M columns are symmetrically placed in longitudinal and transverse directions. The idea behind obtaining «fictitious shear force» is to form a basis for calculating the relative contribution of the torsional component of ground motion to the column shears in comparison to that of the translational component alone. In calculating the fictitious shear force, longitudinal column shear in the m^{th} column has been multiplied by the total number of columns for the assumption that in the case of translational component, the story shear is equally shared by all the M columns. Further, since torsional vibrations do not lead to any story shear, no additional overturning moment results in the symmetric building due to the torsional component of ground motion.

In the case of the combined action of translational and torsional components, the nonstationary nature of excitation may be accounted for (as in the case of translational component alone) by modifying the r.m.s. peak value first to account for the nonstationarity while neglecting the modal interaction and then by accounting for the interaction. Let the r.m.s. value $\bar{a}_{mi}$ (for displacement response peaks at the i^{th} floor and m^{th} column) be modified to $(\bar{a}_E)_{mi}$ and $(\bar{a}_E)'_{mi}$ respectively. In parallel with Eq. (A1.5), $(\bar{a}_E)_{mi}$ may be expressed as

$$(\bar{a}_E)_{mi} = \left[\sum_{j=1}^{n} (\bar{a}_E)_{ij}^2 + \sum_{k=1}^{n} (\bar{a}_E)_{mi,kT}^2 \right]^{1/2} \qquad (3.14)$$

where $(\bar{a}_E)_{ij}$ is as defined in Appendix I and $(\bar{a}_E)_{mi,kT}$ is the factor for normalizing the maximum value of response function (as calculated from the torsional response spectrum) at the i^{th} floor, in k^{th} torsional mode and for the m^{th} column, to $E[a_{(1)}]_{kT}/\sqrt{2}(E[a_{(1)}]_{kT}$ is the expected value of first order peak in k^{th} torsional mode for the distribution in (2.1). For determining the (normalized) expected value of the first order peak in j^{th} translational mode i.e $E[a_{(1)}]_j$, the same set of equations for ε_j and N_j will hold good as in (A1.3). For determining $E[a_{(1)}]_{kT}$, ε and N values k^{th} torsional mode (i.e. ε_{kT} and N_{kT}) are

$$\varepsilon_{kT} = \left[1 - \frac{(\mu_{2,kT}^T)^2}{\mu_{0,kT}^T \mu_{4,kT}^T} \right]^{1/2}$$

and

$$N_{kT} = \frac{T}{2\pi} \omega_{kT} \sqrt{\frac{\mu_{4,kT}^T}{\mu_{2,kT}^T}} \qquad (3.15)$$

where $\mu_{0,kT}^T$, $\mu_{2,kT}^T$ and $\mu_{4,kT}^T$ are as defined in (3.8). Maximum values of displacement and shear force, D_{ij} and S_{ij} for the i^{th} floor and j^{th} translational mode, are as in (A1.1) and the following expression /5, 7, 8/,

$$S_{ij} = (m_1 A_{1j} + m_2 A_{2j} + \ldots + m_i A_{ij}) a_j \omega_j^2 SD_j \qquad (3.16)$$

where SD_j is as defined in Appendix I. In torsional modes, displacement and shear values are $D_{mi,kT}$ and $S_{mi,kT}$ (i.e. the fictitious story shear) as defined by

$$D_{mi,kT} = A_{i,kT} a_{kT} SD\vartheta_{kT} b_{mi} \qquad (3.17)$$

and

$$S_{mi,kT} = (J_1 A_{1,kT} + J_2 A_{2,kT} + \ldots + J_i A_{i,kT}) a_{kT} \omega_{kT}^2 SD\vartheta_{kT} M \beta_{mi} \qquad (3.18)$$

where $SD\vartheta_{kT}$ is the spectral rotation corresponding to the modal frequency ω_{kT} and damping ratio ζ_{kT}. Once $(\bar{a}_E)_{mi}$ is calculated, it may be modified to $(\bar{a}_E)'_{mi}$ (to account for modal interaction) by using zeroth moments of the energy spectrum (with and without modal interaction) following similar principles as mentioned in Appendix I.

4. Combined Action of Translation, Rocking and Torsion

If the building considered in section 3 is subjected to the rocking acceleration, $\ddot{\vartheta}_R(t)$ at its base in addition to the translational and torsional components of excitation, it is possible to obtain the following expression for $X_{mi}(\omega)$ by including the rocking contribution /7, 8/ in Eq. (3.5),

$$X_{mi}(\omega) = \sum_{j=1}^{n} A_{ij}(a_i Z(\omega) + aH_j \Theta_R(\omega)) H_j(\omega) + $$

$$+ \sum_{j=1}^{n} b_{mi} A_{i,jT} a_{jT} H_{jT}(\omega) \Theta_T(\omega) \qquad (4.1)$$

where $aH_j \left(= \sum_{k=1}^{n} A_{kj} m_k H_k / \sum_{k=1}^{n} A^2{}_{kj} m_k, j = 1, 2, \ldots n \right)$ is the modal participation factor for the response to rocking ground motion and $\ddot{\Theta}_R(\omega)$ is the Fourier transform of $\ddot{\vartheta}_R(t)$. Assuming the phase difference $\Phi_R(\omega)$ between the translational motion $Z(\omega)$ and the rocking motion $\Theta_R(\omega)$ to be a constant value Φ_R,

$$\frac{Z(\omega)}{\Theta_R(\omega)} = \left| \frac{Z(\omega)}{\Theta_R(\omega)} \right| e^{-i\Phi R(\omega)} \qquad (4.2)$$

and $\Phi_{RT} = \Phi_R + \Phi_T$, the expression for $m_{k,mi}$ can be obtained as /7/

$$m_{k,mi} = \frac{1}{\pi T} \left[\sum_{j=1}^{n} A_{ij}^2 a_j^2 \omega_j^{(k-3)} \left((1 + p_{D,ij} + q_{D,ij} \right) \lambda_{k,j} - \right.$$

$$- q_{D,ij} \cdot \lambda_{k+2,j} + b_{mi} \cos\Phi_T \{ (p_{D,ij}^T + q_{D,ij}^T) \lambda_{k,j}^T - q_{D,ij}^T \cdot \lambda_{k+2,j}^T \}$$

$$+ \frac{b_{mi} \sin\Phi_T}{2\zeta_i} \{ (s_{D,ij}^T - (2 - 4\zeta_j^2) p_{D,ij}^T) \lambda_{k+1,j}^T + p_{D,ij}^T \lambda_{k+3,j}^T \} +$$

$$+ \frac{aH_j}{a_j} \cos\Phi_R \{ (1 + p_{D,ij}^{\prime R} + q_{D,ij}^{\prime R}) \lambda_{k,j}^R - q'^R_{D,ij} \lambda_{k+2,j}^R \} +$$

$$+ \frac{aH_j}{a_j} \cdot \frac{\sin\Phi_R}{2\zeta_j} \{ (p_{D,ij}^{\prime\prime R} - (2 - 4\zeta_j^2) q_{0,ij}^{\prime\prime R}) \lambda_{k+1,j}^R +$$

$$\left. + p_{D,ij}^{\prime\prime R} \lambda_{k+3,j}^R \right) \Bigg] + \frac{1}{\pi T} \left[\sum_{j=1}^{n} A_{i,jT}^2 a_{jT}^2 b_{mi}^2 \omega_{jT}^{(k-3)} \left((1 + p_{D,ijT}^T + \right. \right.$$

$$
+ q^T_{D,ijT})\mu^T_{k,jT} - q^T_{D,ijT}\mu^T_{k+2,jT} + \frac{\cos\Phi_T}{b_{mi}}\{(p_{D,ijT} +
$$

$$
+ q_{D,ijT})\lambda^T_{k,jT} - q_{D,ijT}\lambda^T_{k+2,jT}\} - \frac{\sin\Phi_T}{2b_{mi}\zeta_{jT}}\{(s_{D,ijT} -
$$

$$
-(2-4\zeta^2_{jT})p_{D,ijT})\lambda^T_{k+1,jT} + p_{D,ijT}\lambda^T_{k+3,jT}\} +
$$

$$
+ \frac{\cos\Phi_{RT}}{b_{mi}}\{(p^R_{D,ijT} + q^R_{D,ijT})\gamma_{k,ijT} - q^R_{D,ijT}\gamma_{k+2,jT}\} -
$$

$$
- \frac{\sin\Phi_{RT}}{2b_{mi}\zeta_{jT}}\{(s^R_{D,ijT} - (2-4\zeta^2_{jT})p^R_{D,ijT})\gamma_{k+1,jT} +
$$

$$
+ p^R_{D,ijT}\gamma_{k+3,jT}\}\Big)\Big] + \frac{1}{\pi T}
$$

$$
\Big[\sum_{j=1}^{n} A^2_{ij}\alpha H^2_j \omega_j^{(k-3)}\Big((1 + p^R_{D,ij} + q^R_{D,ij})\mu^R_{k,j} - q^R_{D,ij}\mu^R_{k+2,j}
$$

$$
+ \frac{a_j}{\alpha H_j}\cos\Phi_R\{(1 + p'^R_{D,ij} + q'^R_{D,ij})\lambda^R_{k,j} - q'^R_{D,ij}\lambda^R_{k+2,j}\} +
$$

$$
+ \frac{a_j}{\alpha H_j}\frac{\sin\Phi_R}{2\zeta_j}\{(p''^R_{D,ij} - (2-4\zeta_j^2)p''^R_{D,ij})\lambda^R_{k+1,j} +
$$

$$
+ q''^R_{D,ij}\lambda^R_{k+3,j}\} +
$$

$$
+ b_{mi}\cos\Phi_{RT}\{(p^{T,R}_{D,ij} + q^{T,R}_{D,ij})\gamma_{k,j} - q^{T,R}_{D,ij}\gamma_{k+2,j}\} +
$$

$$
+ b_{mi}\frac{\sin\Phi_{RT}}{2\zeta_j}\{(s^T_{D,ij} - (2-4\zeta_j^2)p^{T,R}_{D,ij})\gamma_{k+1,j} +
$$

$$
+ p^{T,R}_{D,ij}\gamma_{k+3,jT}\}\Big)\Big] \tag{4.3}
$$

where $\gamma_{k,j}$ and $\gamma_{k,jT}$ are respectively the k^{th} normalized moments of $|\Theta_R(\omega)||\Theta_T(\omega)||H_j(\omega)|^2$ and $|\Theta_R(\omega)||\Theta_T(\omega)||H_{jT}(\omega)|^2$ for the equivalent SDOF oscillators of j^{th} translational and torsional modes; $\mu^R_{k,j}$ and $\lambda^R_{k,j}$ are the moments of $|\Theta_R(\omega)|^2|H_j(\omega)|^2$ and $|\Theta_R(\omega)||Z(\omega)||H_j(\omega)|^2$ for the oscillator of j^{th} translational mode. Various p's, q's and s's are the interaction coefficients. In the cases of negligible modal interaction, the terms involving these coefficients may be neglected and Eq. (4.3) is then greatly simplified. Details of the above formulation may be found in Gupta and Trifunac /7/. Various expressions for shear force response can be directly obtained by the substitution of certain terms into those for displacement response (as discussed in section 3); but since the effects of torsional vibrations will be included in the resulting expression, the so obtained energy spectra will again be for the fictitious story shear (see section 3).

The statistical parameters required for determining the normalized expected values of the first order response peaks in various translational and torsional modes may be calculated using the expression of ε_j and N_j (for the translational modes) as proposed by Gupta and Trifunac /7/ for the combined case of translational and rocking components and the expressions of ε_{kT} and N_{kT} (for the torsional modes) as in section 3. Similar is the case for the expressions of D_{ij}, S_{ij}, $D_{mi,kT}$ and $S_{mi,kT}$. Various values of $(\bar{a}_E)_{ij}$ and $(\bar{a}_E)_{mi,kT}$ are combined using (3.14) and further, the modification to account for the modal interaction in the nonstationary response may be carried out using the zeroth moments of the energy spectra as in the previous section.

5. Illustrations of the Proposed Model

The approach presented in preceding sections (3 and 4) is illustrated here by comparing the probabilistic estimates of the largest peak values of displacement and shear force with the corresponding results of the time domain analysis involving step-by-step numerical integration. Two fixed-base buildings have been considered, one with 10 stories and the other one with 20 stories. These buildings are the same as those considered by Gupta and Trifunac /8/ in a parallel development of the probabilistic approach for the rocking component. Each building has constant story height of 10 m, four symmetrically arranged columns and 25 m (lungitudinal) × 100 m (transverse) floor dimensions at each story level. The story heights and transverse floor dimensions are taken to be unrealistically large for substantial contributions of the rotational components and thus, for more obvious testing of the results. In both the buildings, floor masses and story stiffnesses vary linearly from top to bottom as illustrated in Fig. 3.

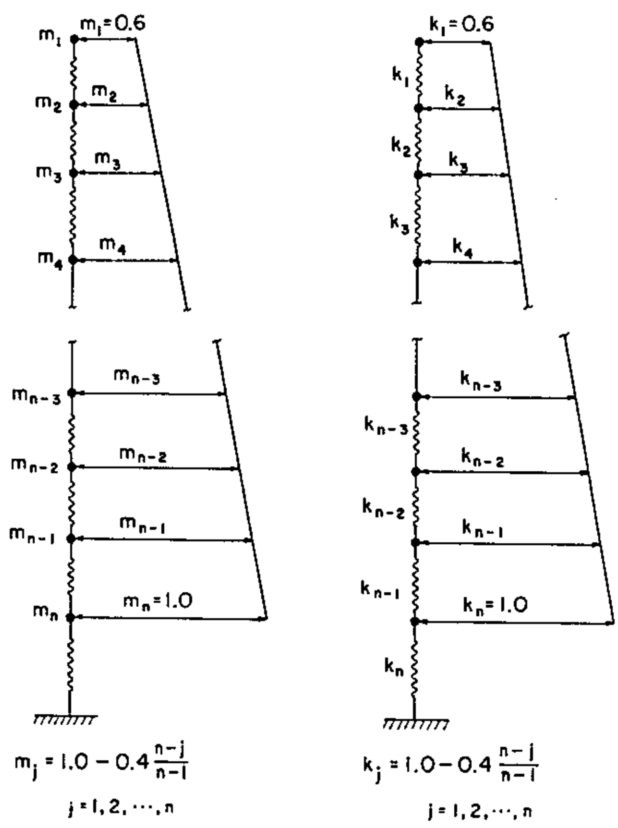

Fig. 3– Story Stiffnesses and Floor Masses in a n-Story Building.

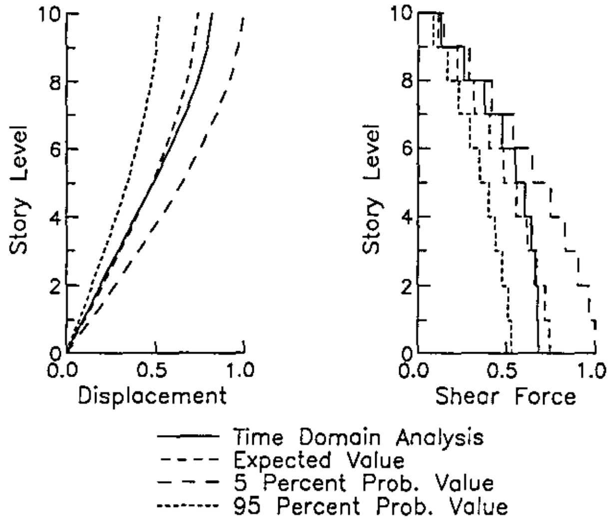

Fig. 4 – Response of 10-Story Building Subjected to Translational + Torsional Components at Imperial Valley Site.

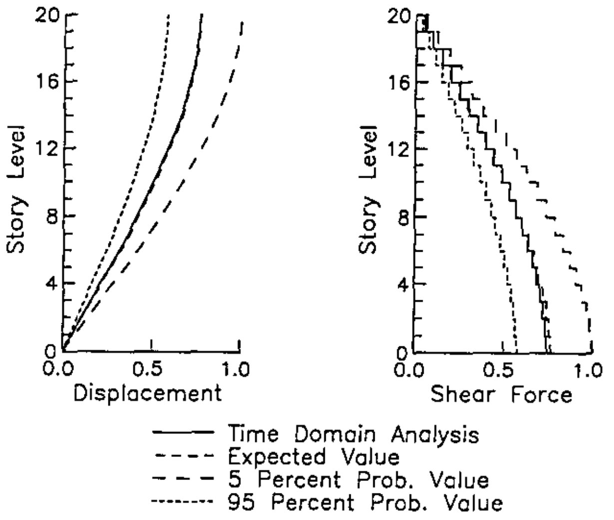

Fig. 5 – Response of 20-Story Building Subjected to Translational + Torsional Components at Mexico City Site.

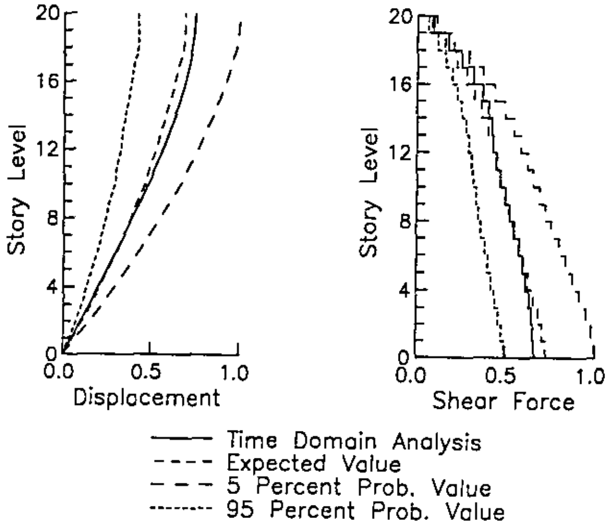

Fig. 6 – Response of 20-Story Building Subjected to Translational + Rocking + Torsional Components at Imperial Valley Site.

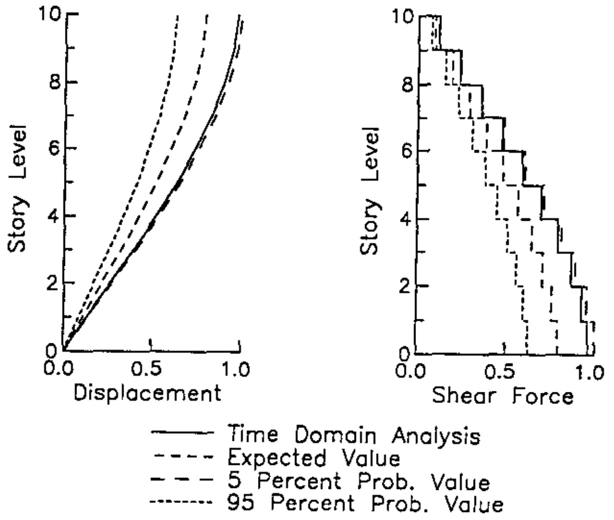

Fig. 7 – Response of 10-Story Building Subjected to Translational + Rocking + Torsional Components at Mexico City Site.

Their relative values are so proportioned that the fundamental period of vibration is 1 and 2 sec respectively for the 10 and 20-story buildings. The critical damping ratio has been assumed uniformly equal to 0.05 in all the modes for both buildings. Further, the column stiffnesses in the transverse and longitudinal directions are assumed to be same (i.e. $k_{yx} = 1$). Torsional stiffnesses of various stories have been computed by neglecting the contributions of the torsional stiffnesses of the columns about their longitudinal axes. Thus, the torsional frequencies are approximately 1.73 times the translational frequencies.

To provide the base excitations to the buildings, synthetic accelerograms and the corresponding Fourier spectra have been generated /12, 13, 27/ for two sites; Westmoreland, Imperial Valley, California, and Mexico City, Mexico. Same earthquake parameters and group and phase velocity curves have been taken for this purpose as in Gupta and Trifunac /7, 8/. The total duration T of the input excitation which corresponds to the stationary part of strong ground motion has been computed as 12.44 and 46.44 sec respectively for these sites based on the results of Trifunac and Brady /24/. Further, in the absence of better estimates, all the results of probabilistic approach have been presented here using the phase differences Φ_R, $\Phi_T = \pi/2$ (as shown analytically by Trifunac /23/ and Lee and Trifunac /12, 13/) for all frequencies.

Figs. 4 through 7 show comparison of the results of the proposed approach with those of time domain analysis. In each figure, envelopes of the maximum peak displacement and shear force have been plotted (for certain combination of the excitation components) by normalizing various response values with respect to the respective overall maximum response values. Probabilistic estimates are represented by the dotted lines, the middle one denoting the expected values and the other two (extreme) lines enclosing the 90% confidence interval. It can be observed in these figures that the time domain results as represented by the solid lines are bounded on either side by the 5% and 95% confidence estimates. They are in good agreement with the expected values depending on how good is the approximation of phase differences to the value of $\pi/2$ in each individual case. The assumption of phase difference to be independent of frequency does not appear to result in noticeable errors. These observations are seen to hold good in many such figures including some representing very large torsional contributions.

Above results do not show explicitly how significant are the contributions of torsional component as compared to the translational component. For that purpose, a separate study which focusses on the comparison of these contributions, will be required while considering several combinations of buildings, sites and earthquakes.

6. Conclusions

It has been seen that the proposed approach can give good estimates of the peak displacement and shear force at all levels of the structure and with the desired level of confidence. Although the testing was carried out here for the first peak values only, this approach can be used to estimate the higher order peak values as well /6/. It is certainly an improvement over the presently available methods as it alone can account for the phase between the translational and rotational components. The assumption of constant phase defference simplifies the analysis considerably without compromising on the accuracy within the reasonable limits.

The presented approach must be used with the correct phase differences. The phase approximation of $\pi/2$ seems to work well but it may not be good enough when the contributions of rotational components are substantially large. Also, greater confidence can be established in the probabilistic estimates if these are based on the phase values obtained from more detailed investigations accounting for the actual ground conditions, source mechanism and the wave propagation from source to site. Additional research efforts should therefore be directed at obtaining more reliable estimates of the phase differences.

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Appendix I

Modification of r.m.s. peak value for nonstationarity

In case of the displacement response at i^{th} floor and the translational excitation, the r.m.s. value $\bar{a}_i$ of peaks may be modified as explained below.

$\bar{a}_i$ is first modified to $(\bar{a}_E)_i$ to take care of the nonstationarity of the response /5, 7/ while neglecting the modal interaction and then, $(\bar{a}_E)_i$ is modified to $(\bar{a}_E)'_i$ to account for this interaction /7/. To calculate $(\bar{a}_E)_i$, all the n modes are considered independently and $(\bar{a}_E)_{ij}$ is computed for the j^{th} mode using D_{ij}, the maximum value of displacement at i^{th} floor and j^{th} mode, calculated from the response spectra and defined as

$$D_{ij} = A_{ij} a_j SD_j \qquad \text{(A1.1.)}$$

where SD_j is the spectral displacement corresponding to the modal frequency ω_j and damping ratio ζ_j. $(\bar{a}_E)_{ij}$ is defined as

$$(\bar{a}_E)_{ij} = \frac{D_{ij}\sqrt{2}}{E[a_{(1)}]_j} \qquad \text{(A1.2)}$$

where $E[a_{(1)}]_j$ is the expected value of the first order peak of the displacement of an equivalent SDOF oscillator of j^{th} mode, as for distribution of Eq. (2.1). The ε and N values in j^{th} mode (i.e. ε_j and N_j) are taken for this calculation as

$$\varepsilon_j = \left[1 - \frac{\lambda_{2,j}^2}{\lambda_{0,j}\,\lambda_{4,j}}\right]^{1/2}$$

and

$$N_j = \frac{T}{2\pi}\omega_j \sqrt{\frac{\lambda_{4,j}}{\lambda_{2,i}}} \qquad \text{(A1.3.)}$$

where, in general,

$$\lambda_{k,j} = \omega_j^{(3-k)} \int_0^\infty |Z(\omega)|^2 |H_j(\omega)|^2 \omega^k d\omega$$

$$k = 0, 1, 2, \ldots \qquad \text{(A1.4.)}$$

is to calculated by the numerical integration. Once $(\bar{a}_E)_{ij}$ is calculated for all j modes, $(\bar{a}_E)_i$ is obtained by

$$(\bar{a}_E)_i = \left[\sum_{j=1}^{n} (\bar{a}_E)_{ij}^2\right]^{1/2} \qquad \text{(A1.5)}$$

For modification of $(\bar{a}_E)_i$ to $(\bar{a}_E)'_i$ (to take care of the interaction between various modes), $m'_{0,i}$ is calculated by neglecting all the modal interaction terms in the expression of $m_{0,i}$, $m_{0,i}$ being the zeroth moment of energy spectrum of the response function at i^{th} floor. Thus in the case of displacement response, $m'_{0,i}$ should have the following form

$$m'_{0,i} = \sum_{j=1}^{n} \frac{A_{ij}^2 \alpha_j^2 \lambda_{0,j}}{\omega_j^3} \quad \text{(A1.6.)}$$

where $\lambda_{0,j}$ is in accordance with Eq. (A1.4). $(\bar{a}_E)'_i$ may then be expressed as

$$(\bar{a}_E)'_i = (\bar{a}_E)_i \left[\frac{m_{0,i}}{m'_{0,i}}\right]^{1/2} \quad \text{(A1.7)}$$

where the contribution of modal interaction terms is negligible, $m'_{0,i} = m_{0,i}$ and then, $(\bar{a}_E)'_i = (\bar{a}_E)_i$.

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