Seismic response of multistoried buildings including the effects of soil-structure interaction

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A simplified response spectrum superposition method has been generalized for the dynamic analysis of the multistoried building-soil response to earthquake ground motions via Fourier-transformed frequency domain. It involves the "scaling" of the Fourier amplitudes of the free-field translational and rocking motions to account for the soil-structure interaction effects, and then analyzing the building as fixed at the base. Envelopes of peak displacements, shear forces and overturning moments in the building are illustrated in terms of the order statistics of the response peaks.

1. INTRODUCTION

In the series of recent papers[6–13] we investigated an application of the order statistics of peaks in the dynamic response of structures[10] and found that it is possible to predict amplitudes of n-th peak with good accuracy[12]. The representation of random input motion via energy spectra permitted generalization of this work to the three-dimensional response estimates for three simultaneous translations[7,8] and later to more general base excitation including also torsion[6] and rocking ground motions, all caused by the passage of seismic waves[29]. These formulations were based on the order statistics and employ the eigenvalue expansion to describe the response. Their accuracy could be tested by direct comparison with computed time response of simple structural models, fixed at the base and excited by three translations and two rotations[18], when incident earthquake waves are long relative to the representative dimension of the foundation[29].

The purpose of this paper is to show how our recent results[6–13] can be generalized to include the effects of soil structure interaction, and at the same time retain the overall approach and simplicity of the response spectrum superposition in the development of shear and overturning moment envelopes for use in engineering design[7,8]. The results of this formulation can be tested by comparison with actually computed time response of simplified structural models. Since our aim here is only to show how the proposed order statistics approach can be generalized to include the effects of soil structure interaction, we will use only the simplest soil-foundation-structure models. One can generalize this to more complex systems via specific and detailed transfer function representation.

The dynamic response of buildings is modified, depending on the structural and soil properties, by the translation and rotation of the foundation, relative to the soil, during dynamic structure-soil interaction. These additional degrees of freedom in the building-soil system lead to alteration of the apparent response frequencies and the system damping[1,3,5,12,16,19–23,25–28,33,36]. In many studies[4,14,27,32], the structure is idealized as a one-dimensional system, with the base as a rigid plate bonded to the surface of the soil (half-space). Analyses associated with the dependence of the soil stiffness and radiation damping on the excitation frequency, and the absence of a set of classical normal modes for the complete building-foundation system have led to analyses via Fourier-transformed frequency domain[3,5,12,20,28,33], and to transformation of equations of motion using normal modes of the building on rigid foundation[3,12]. Few studies[2,12,15] have also considered the rocking effects in the input ground motion, associated with non-vertically incident waves. Gupta and Trifunac[12] presented a stochastic approach to examine the response of flexible-base buildings, subjected to translational and rocking components of the free-field motion. Their approach has been generalized in this paper by explicitly accounting for the phase difference between the base translation and rocking of the structure (relative to the soil). Further, the computational procedure has been simplified by avoiding the calculations of time histories of the motion of foundation relative to the soil.

2. INTERACTION ACCELERATIONS

We consider the lumped mass model of a typical multistoried building, shown in Fig. 1, subjected to horizontal component $\ddot{z}(t)$ and rocking components $\ddot{\theta}(t)$ of ground acceleration at its base. The base of the building is assumed to be a rigid rectangular slab footing of negligible thickness bonded to the surface of a uniform and visco-elastic half space. Under the influence of base shear $V_s(t)$ and moment $M_s(t)$ exerted by the superstructure, and the tractions caused by the incident waves, the footing translates and rotates by $z_0(t)$ and $\theta_0(t)$ relative to the underlying half space.

If the rocking component of the input ground motion, $\ddot{\theta}(t)$, is ignored and the motions are small, the equations of dynamic equilibrium of the n floor masses and the equilibrium of the building as a whole in translation and rotation can be written as[3]

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[m]\{1\}(\ddot{z}_0 + \ddot{z}) - [m]\{H\}\ddot{\theta}_0 \tag{1a}$$

$$V_s(t) + \sum_{i=1}^{n} m_i\ddot{x}_i + \sum_{i=1}^{n} m_i H_i \ddot{\theta}_0 + \sum_{i=0}^{n} m_i(\ddot{z} + \ddot{z}_0) = 0 \tag{1b}$$

$$M_s(t) + \sum_{i=1}^{n} m_i H_i(\ddot{x}_i + \ddot{z}_0 + \ddot{z}) + [I_0 + \sum_{i=1}^{n} (I_i + m_i H_i^2)]\ddot{\theta}_0 = 0 \tag{1c}$$

where for the simple model in Fig. 1, deforming only in shear, $[m]$ is the diagonal matrix of n floor masses; $[k]$ is the tri-diagonal stiffness matrix of the fixed-base building in terms of the story stiffnesses; $[c]$ is the tri-diagonal damping matrix of the fixed-base building (in terms of the inter-story damping); $\{1\}$ is the unit vector; I_i is the mass moment of inertia of the i^{th} mass, m_i, about a horizontal axis through its mass center; and other quantities are as indicated in Fig. 1. The contributions of gravitational forces have been ignored. Expressing $\{x\} = [A]\{\xi\}$, where $[A]$ is the modal matrix for the fixed base structure and $\{\xi\}$ is the vector of normal coordinates, and including the contribution of the input rocking excitation $\ddot{\theta}(t)$, it is possible to rewrite Equations (1) as[12],

$$[A]^T[m][A]\{\ddot{\xi}\} + [A]^T[c][A]\{\dot{\xi}\} + [A]^T[k][A]\{\xi\} = -[A]^T[m]\{1\}(\ddot{z}_0 + \ddot{z}) - [A]^T[m]\{H\}(\ddot{\theta}_0 + \ddot{\theta}) \tag{2a}$$

$$V_s(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} m_i A_{ij}\ddot{\xi}_j + m_{HT}(\ddot{\theta} + \ddot{\theta}_0) + m_T(\ddot{z} + \ddot{z}_0) = 0 \tag{2b}$$

$$M_s(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} m_i A_{ij} H_i \ddot{\xi}_j + I_T(\ddot{\theta} + \ddot{\theta}_0) m_{HT}(\ddot{z} + \ddot{z}_0) = 0 \tag{2c}$$

where $m_T = \Sigma_{i=0}^{n} m_i$ is the total mass of structure and the foundation; $m_{HT} = \Sigma_{i=1}^{n} m_i H_i$ and $I_T = I_0 + \Sigma_{i=1}^{n} (I_i + m_i H_i^2)$. Assuming the viscous damping $[c]$ to be of such a form that the building on a rigid foundation

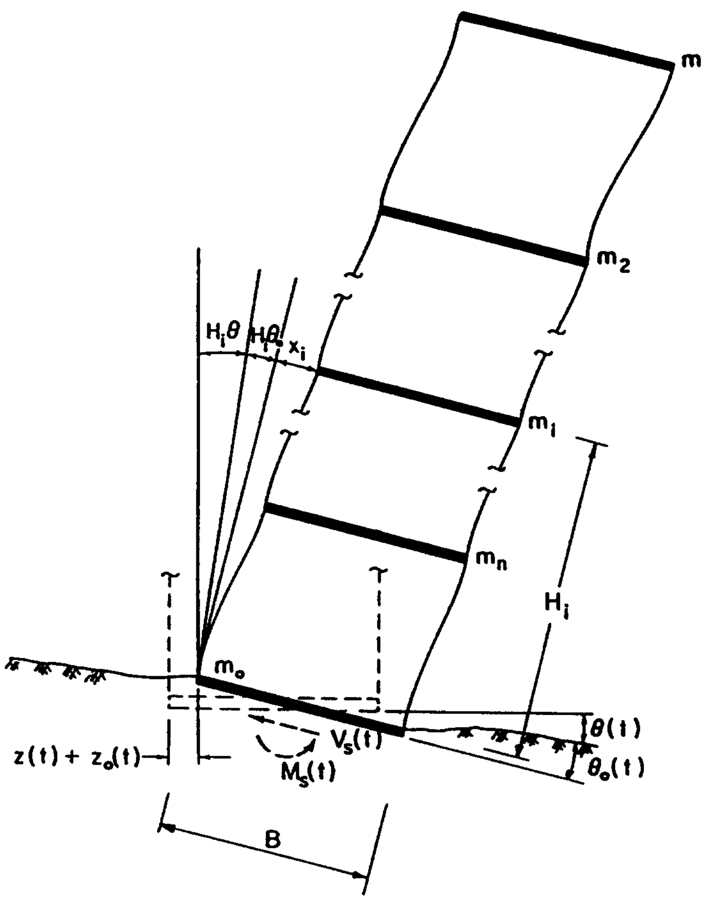

Fig. 1. Idealized Building-Foundation System of a n-Storey Building

admits decomposition into classical normal modes, $[A]^T[c][A]$ can be written as a diagonal matrix, j^{th} diagonal element being $2\zeta_j\omega_j m_j$ where m_j, ω_j and ζ_j respectively are the modal mass, natural frequency and the damping ratio in j^{th} fixed based mode. Since the matrices $[A]^T[m][A]$ and $[A]^T[k][A]$ are diagonal and j^{th} diagonal elements respectively are m_j and $\omega_j^2 m_j$, Equation (2a) can be written as a set of n decoupled equations, each describing the motion in a specific fixed base mode. These equations, along with the equilibrium equations (2b) and (2c) can be Fourier-transformed, with the time dependent variables $V_s(t)$, $M_s(t)$, $\ddot{\theta}(t)$, $\theta_0(t)$, $\ddot{z}(t)$ and $z_0(t)$ being represented as $\bar{V}_s(\omega)$, $\bar{M}_s(\omega)$, $\Theta(\omega)$, $\Theta_0(\omega)$, $Z(\omega)$ and $Z_0(\omega)$ respectively in the frequency domain. For a massless foundation, it is possible to express the interaction forces $\bar{V}_s(\omega)$ and $\bar{M}_s(\omega)$ in terms of the footing displacements $Z_0(\omega)$ and $\Theta_0(\omega)$ by using the impedance functions[34] as

$$\begin{Bmatrix} \bar{V}_s \\ \bar{M}_s/L \end{Bmatrix} = \begin{bmatrix} K_{VV} & K_{VM} \\ K_{MV} & K_{MM} \end{bmatrix} \begin{Bmatrix} Z_0 \\ L\Theta_0 \end{Bmatrix} \tag{3}$$

where $K_{VV}(\omega)$, $K_{VM}(\omega)(=K_{MV}(\omega))$ and $K_{MM}(\omega)$ are the complex valued impedance functions having the dimension of force per unit length. These functions are proportional to the shear modulus of rigidity G in the soil and the length of reference L which is defined as the radius of a circular foundation of area equal to that of the rectangular foundation. For rectangular slab footings, the impedance functions also depend on the aspect ratio of the slab. Furthermore, impedance functions depend on Poisson's ratio ν, hysteretic damping ratio ξ of the soil and the dimensionless frequency $\bar{\omega} = \omega L/\beta$ where $\beta = \sqrt{G/\rho}$ and ρ are the shear wave velocity and the mass density of the soil[34]. Using Equation (3), it is possible to solve the equilibrium equations (in fequency domain) for the interaction displacements $Z_0(\omega)$ and $\Theta_0(\omega)$ which, after multiplication with $-\omega^2$, give[11] the interaction accelerations, $\hat{Z}_0(\omega)$ and $\hat{\Theta}_0(\omega)$

$$\begin{Bmatrix} \hat{Z}(\omega) \\ L\hat{\Theta}_0(\omega) \end{Bmatrix} = \begin{bmatrix} \chi_{zz}(\bar{\omega}) & \chi_{z\theta}(\bar{\omega}) \\ \chi_{\theta z}(\bar{\omega}) & \chi_{\theta\theta}(\bar{\omega}) \end{bmatrix} \begin{Bmatrix} Z(\omega) \\ L\Theta(\omega) \end{Bmatrix}. \tag{4}$$

The quantities χ_{zz}, $\chi_{z\theta}$, $\chi_{\theta z}$ and $\chi_{\theta\theta}$ are expressed in terms of the dimensionless quantities obtained by the normalization of all mass, height and frequency terms (including ω) respectively by ρL^3, L and β/L. If various quantities, in dimensionless form, are denoted by bars, χ's become

$$\begin{aligned}
\chi_{zz} &= \frac{1}{\Delta}\,[\bar{\omega}^2(\bar{m}_T(\bar{\omega})\kappa_{MM} - \bar{m}_{HT}(\bar{\omega})\kappa_{MV}) + \bar{\omega}^4(\bar{m}_{HT}^2(\bar{\omega}) \\
&\quad - \bar{m}_T(\bar{\omega})\bar{I}_T(\bar{\omega}))]\,, \\
\chi_{z\theta} &= \frac{1}{\Delta}\,[\bar{\omega}^2(\bar{m}_{HT}(\bar{\omega})\kappa_{MM} - \bar{I}_T(\bar{\omega})\kappa_{MV})]\,, \\
\chi_{\theta z} &= \frac{1}{\Delta}\,[\bar{\omega}^2(\bar{m}_{HT}(\bar{\omega})\kappa_{VV} - \bar{m}_T(\bar{\omega})\kappa_{MV})]\,, \\
\chi_{\theta\theta} &= \frac{1}{\Delta}\,[\bar{\omega}(\bar{I}_T(\bar{\omega})\kappa_{VV} - \bar{m}_{HT}(\bar{\omega})\kappa_{MV}) \\
&\quad + \bar{\omega}^4(\bar{m}_{HT}^2(\bar{\omega}) - \bar{m}_T(\bar{\omega})\bar{I}_T(\bar{\omega}))]
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\Delta = {} & (\kappa_{VV}\kappa_{MM} - \kappa_{MV}^2) + \bar{\omega}^2(2\bar{m}_{HT}(\bar{\omega})\kappa_{MV} - \bar{I}_T(\bar{\omega})\kappa_{VV} \\
& - \bar{m}_T(\bar{\omega})\kappa_{MM}) + \bar{\omega}^4(\bar{m}_T(\bar{\omega})\bar{I}_T(\bar{\omega}) - \bar{m}_{HT}^2(\bar{\omega}))
\end{aligned}$$

where κ_{VV}, κ_{MV} and κ_{MM} are the "coefficients" (functions of $\bar{\omega}$), obtained from K_{VV}, K_{MV} and K_{MM} by dividing with GL, and $\bar{m}_T(\bar{\omega})$, $\bar{m}_{HT}(\bar{\omega})$ and $\bar{I}_T(\bar{\omega})$ are the dimensionless forms of $m_T(\bar{\omega})$, $m_{HT}(\omega)$ and $I_T(\omega)$ respectively. Functions $m_T(\bar{\omega})$, $m_{HT}(\omega)$ and $I_T(\omega)$ are[11],

$$\begin{aligned}
m_T(\omega) &= m_T + \omega^2 \sum_{i=1}^{n}\sum_{j=1}^{n} m_i A_{ij}\alpha_j H_j(\omega) \\
m_{HT}(\omega) &= m_{HT} + \omega^2 \sum_{i=1}^{n}\sum_{j=1}^{n} m_i A_{ij}\alpha_j H_j H_j(\omega) \\
I_T(\omega) &= I_T + \omega^2 \sum_{i=1}^{n}\sum_{j=1}^{n} m_i A_{ij}\alpha_j H_j H_i H_j(\omega)
\end{aligned} \tag{6}$$

where $\alpha_j(=\Sigma_{k=1}^n A_{kj}m_k/\Sigma_{k=1}^n A_{kj}^2 m_k)$ and $\alpha H_j(=\Sigma_{k=1}^n A_{kj}m_k/\Sigma_{k=1}^n A_{kj}^2 m_k)$ are the participation factors for translation and working motions respectively in the jth mode, and $H_j(\omega)$ is the transfer function for the displacement of a single-degree-of-freedom (SDOF) oscillator having unit mass, ω_j natural frequency and ζ_j critical damping ratio.

3. MODIFICATIONS IN INPUT MOTION

i) Formulation of Interaction Functions

As the interaction accelerations at the base, $\hat{Z}_0(\omega)$ and $\hat{\Theta}_0(\omega)$ are expressed in terms of the input excitation according to Equation (4), it is possible to write the excitation for the oscillator in jth fixed base mode, in case of soil-structure interaction, as (in replacement of $-\alpha_j Z(\omega) - \alpha H_j\Theta(\omega)$ which is applicable in case of negligible interaction)

$$\begin{aligned}
& -\alpha_j\,[Z(\omega) + \chi_{zz}(\bar{\omega})Z(\omega) + L\chi_{z\theta}(\bar{\omega})\Theta(\omega)] \\
& \quad -\alpha H_j\,[\Theta(\omega) + \chi_{\theta z}(\bar{\omega})Z(\omega)/L + \chi_{\theta\theta}(\bar{\omega})\Theta(\omega)]\,.
\end{aligned}$$

If one represents this excitation as $-\alpha_j Z'(\omega) - \alpha H_j\Theta'(\omega)$ where $Z'(\omega) = Z(\omega) + \chi_{zz}Z(\omega) + L\chi_{z\theta}\Theta(\omega)$ is the modified input translation excitation and $\Theta'(\omega) = \Theta(\omega) + \chi_{\theta z}Z(\omega)/L + \chi_{\theta\theta}\Theta(\omega)$ is the modified input rocking excitation, then the approach of Gupta and Trifunac[11,13] for the fixed base buildings can be used here.

Expressing,

$$Z(\omega) = |Z(\omega)|\,e^{i\phi_z(\omega)}$$

and

$$\Theta(\omega) = |\Theta(\omega)|\,e^{i\phi_\theta(\omega)} \tag{7}$$

and taking $\phi_R(\omega) = \phi_\theta(\omega) - \phi_z(\omega)$, it follows that

$$Z'(\omega) = |Z(\omega)|\,e^{i\phi_z}\left[1 + \chi_{zz} + L\chi_{z\theta}\,\frac{|\Theta(\omega)|}{|Z(\omega)|}\,e^{i\phi_R(\omega)}\right]$$

and

$$\Theta'(\omega) = |\Theta(\omega)|\,e^{i\phi_z}\left[e^{i\phi_R} + \frac{|Z(\omega)|}{L|\Theta(\omega)|}\,\chi_{\theta z} + \chi_{\theta\theta}e^{i\phi_R(\omega)}\right]. \tag{8}$$

The above expressions for $Z'(\omega)$ and $\Theta'(\omega)$ suggest the following modifications in $|Z(\omega)|$, $|\Theta(\omega)|$ and $\phi_R(\omega)$ to account for the interaction:

$|Z(\omega)|$ becomes $|Z'(\omega)| = F_Z(\omega)|Z(\omega)|$

$|\Theta(\omega)|$ becomes $|\Theta'(\omega)| = F_\theta(\omega)|\Theta(\omega)|$

and

$\phi_R(\omega)$ becomes $\phi_R'(\omega)$

where

$$F_Z(\omega) = [\{1 + \mathrm{Re}(\chi_{zz}) + \cos\phi_R \mathrm{Re}(\chi_{z\theta})/\sigma - \sin\phi_R \mathrm{Im}(\chi_{z\theta})/\sigma\}^2 + \{\mathrm{Im}(\chi_{zz}) + \sin\phi_R \mathrm{Re}(\chi_{z\theta})/\sigma + \cos\phi_R \mathrm{Im}(\chi_{z\theta})/\sigma\}^2]^{1/2} \tag{9}$$

$$F_\Theta(\omega) = [\{\cos\phi_R + \sigma \mathrm{Re}(\chi_{\theta z}) + \cos\phi_R \mathrm{Re}(\chi_{\theta\theta}) - \sin\phi_R \mathrm{Im}(\chi_{\theta\theta})\}^2 + \{\sin\phi_R + \sigma \mathrm{Im}(\chi_{\theta z}) + \sin\phi_R \mathrm{Re}(\chi_{\theta\theta}) + \cos\phi_R \mathrm{Im}(\chi_{\theta\theta})\}^2]^{1/2} \tag{10}$$

and

$$\phi_R'(\omega) = \tan^{-1}\left[\frac{\sin\phi_r + \sigma \mathrm{Im}(\chi_{\theta z}) + \sin\phi_R \mathrm{Re}(\chi_{\theta\theta}) + \cos\phi_R \mathrm{Im}(\chi_{\theta\theta})}{\cos\phi_R + \sigma \mathrm{Re}(\chi_{\theta z}) + \cos\phi_R \mathrm{Re}(\chi_{\theta\theta}) - \sin\phi_R \mathrm{Im}(\chi_{\theta\theta})}\right] - \tan^{-1}\left[\frac{\mathrm{Im}(\chi_{zz}) + \sin\phi_R \mathrm{Re}(\chi_{z\theta})/\sigma + \cos\phi_R \mathrm{Im}(\chi_{z\theta})/\sigma}{1 + \mathrm{Re}(\chi_{zz}) + \cos\phi_R \mathrm{Re}(\chi_{z\theta})/\sigma - \sin\phi_R \mathrm{Im}(\chi_{z\theta})/\sigma}\right] \tag{11}$$

with $|Z(\omega)|/(L|\Theta(\omega)|)$ being denoted by σ. $\phi_R'(\omega)$ is the (frequency dependent) phase difference between the modified motions $Z'(\omega)$ and $\Theta'(\omega)$. It is seen that as χ's tend to zero in case of rigid soil, $F_z(\omega)$ and $F_\theta(\omega)$ tend to 1 and $\phi_R'(\omega)$ tends to $\phi_R(\omega)$. Further, several examples have suggested interaction functions F_Z, F_Θ and ϕ_R' to be insensitive to the fluctuations in ϕ_R with ω, especially at low frequencies and hence, the input phase difference $\phi_R(\omega)$ is assumed here as independent of frequency.

ii) Illustration of Interaction Functions

To illustrate the behavior of interaction functions F_Z, F_Θ and ϕ_R', a simplified 5-story symmetric building has been considered with the following values for floor masses and story stiffnesses (with $n = 5$):

$$m_i = 1487 r_i \text{ tonnes}$$

and

$$k_i = k r_i$$

where

$$r_i = 1.0 - 0.4\left(\frac{n-i}{n-1}\right), \quad i = 1, 2, \ldots n. \tag{12}$$

The stiffness parameter k has been chosen so that the fundamental period of vibration of the building is $0.1n = 0.1 * 5 = 0.5$ sec. The natural frequencies of the building are 12.22, 31.71, 49.37, 63.22 and 72.07 rad/sec. The critical damping ratio in all the fixed base modes of vibration has been uniformly taken as 0.05. Length of reference L and the Poisson's ratio ν for the foundation slab have been chosen as 10.2 m and 0.3. The foundation slab has been assumed to be of neglibile mass in comparison with the floor masses. Hysteretic damping ratio ξ and mass density ρ of the half space[21] have been respectively taken as 0.02 and 1000 kg/m^3. Further, the shear wave velocity β in the supporting medium has been assumed to be uniform and equal to β_{min} which corresponds to the topmost layer of the soil stratum at the site. Aspect ratio B/C of the foundation slab has been assumed to be 1 where B and C are the slab dimensions in longitudinal and transverse directions[34]. Phase difference $\phi_R(\omega)$ between the input translational and rocking motions has been approximated[18,30] as $\pi/2$. The building is assumed to be situated in Pasadena, California ($\beta = 185$ m/sec) and the base excitation is assumed to correspond to a hypothetical earthquake of 6.5 magnitude occurring along the San Andreas fault with the epicentral distance of 40 km. Fourier spectra for this excitation have been generated synthetically[11,18,35]. It has been assumed that the building is subjected to the free field motion without any modification due to scattering of waves by the embedded foundation[17,24].

Figures 2, 3 and 4 with the plots of $F_Z(\omega)$, $F_\Theta(\omega)$ and $\phi_R'(\omega)$ (for $\omega < 80$ rad/sec) illustrate the "modifications" introduced into the "input excitation" by the interaction between the example building and its foundation. The plot of $F_Z(\omega)$ reflects the response of the system in terms of the combined base translation, for unit translational input. In the vicinity of the first peak corresponding to the first system frequency $\tilde{\omega}_1$, a minimum is observed at $\omega = \tilde{\omega}_1^*$. This is the frequency of the system if the foundation is constrained against translation relative to the half space[21]. It may be expressed by

$$\frac{1}{\tilde{\omega}_1^{*2}} = \frac{1}{\omega_1^2} + \frac{1}{\omega_R^2} \tag{13}$$

where ω_R corresponds to the rocking frequency of the system if the superstructure is rigid and the foundation is prevented from translation. The experimental system identification techniques are based on the measurements of the translational motion at the foundation level and at the top of the structure. The transfer function obtained

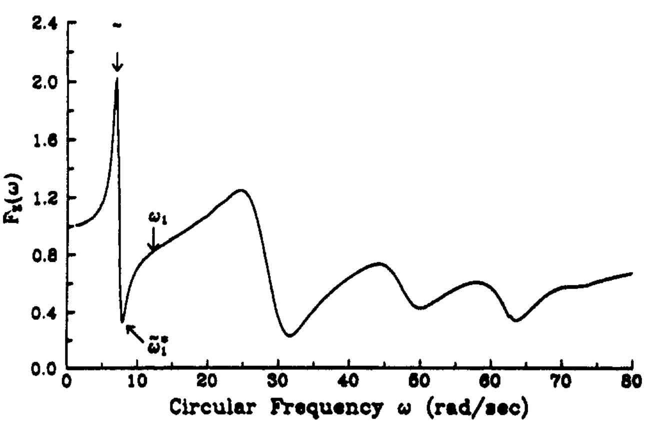

Fig. 2. $F_Z(\omega)$ versus ω for Pasadena Site

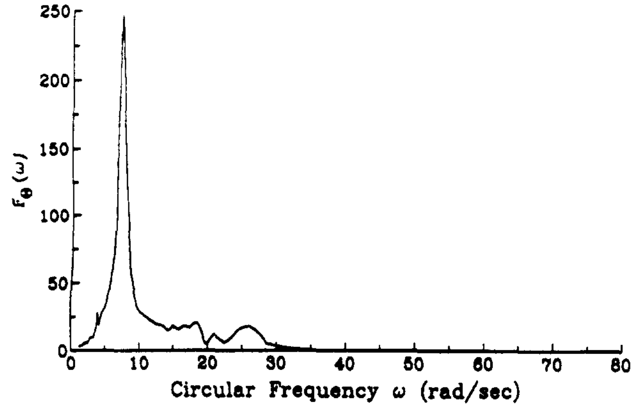

Fig. 3. $F_\Theta(\omega)$ versus ω for Pasadena Site

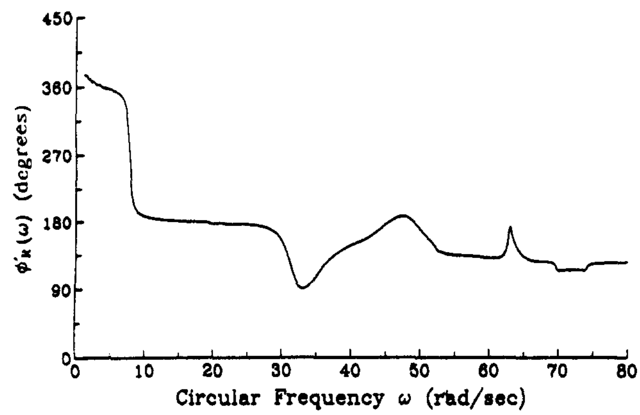

Fig. 4. $\phi'_R(\omega)$ versus ω for Pasadena Site

from the ratio of these measurements reflects $\tilde{\omega}_1^*$ as the characteristic frequency and therefore, $\tilde{\omega}_1^*$ is also called *the apparent system frequency*[21]. If there is no constraint on the translation of the foundation, then $\tilde{\omega}_1^*$ is further reduced to $\tilde{\omega}_1$,

$$\frac{1}{\tilde{\omega}_1^2} = \frac{1}{\tilde{\omega}_1^{*2}} + \frac{1}{\omega_H^2} \tag{14}$$

where ω_H is to the frequency of the system if the superstructure is rigid and the foundation is prevented from rocking. The frequencies ω_1, $\tilde{\omega}_1^*$ and $\tilde{\omega}_1$ have been indicated in Fig. 2.

Variation of the function $F_\Theta(\omega)$ with frequency (see Fig. 3) exhibits different behavior. Here, only one peak corresponding to $\tilde{\omega}_1$ dominates. No minimum is observed in the vicinity of $\omega = \tilde{\omega}_1^*$ because even though, this plot reflects the steady state response in terms of the combined base rotation (for unit rocking input), it behaves like the transfer function of foundation rocking for the input translational component (i.e. $\chi_{\theta z}$). This follows from the dominance of the combined base rocking (i.e. $|\Theta(\omega) + \Theta_0(\omega)|$) by the foundation rocking (i.e. $|\Theta_0(\omega)|$), and from the fact that the latter receives most of its contributions from the translational part of the input motion.

The peak amplification of the rocking amplitudes at the system natural frequency $\tilde{\omega}_1$, depends on the ratio of translational to rocking amplitudes (in the input motion) at $\tilde{\omega}_1$, and the system damping in the fundamental mode. The first factor, in mean, depends on the phase velocity of waves at a given frequency ω; however, local fluctuations in its value may be governed by the energy distribution in the incoming waves. System damping may be higher or lower than the structure damping[21]. Further, it also depends on the fundamental frequency ratio, $\bar{\omega}_1(=\omega_1 L/\beta)$, mass ratio, $\bar{m}_T(=m_T/\rho L^3)$, slenderness ratio, $\bar{H}_1(=H_1/L)$ and the Poisson's ratio, ν for the soil medium.

Input phase angle of $\pi/2$ between the translational and rocking components is modified by interaction, as shown by the plot of $\phi'_R(\omega)$ in Fig. 4. Since the changes in translational phase due to interaction are small and the combined rocking excitation is dominated by the interaction rocking at low frequencies, the behaviour of $\phi'_R(\omega)$ near $\tilde{\omega}_1$, reflects the tendency of response to get more and more out of phase with the forcing function, as the frequency of the forcing function is increased. At relatively small excitation frequencies, the system behaves "stiffer" and the response is nearly in phase. With increasing frequencies, the system becomes "more flexible" and thus exhibits delayed response.

4. PROPOSED CHANGES FOR INTERACTION

It has been noted above that with certain modifications in the Fourier spectra of input translational and rocking excitations and the phase difference between them, it is possible to account for the soil-structure interaction effects while using the response spectrum superposition approach presented by Gupta and Trifunac[11-13] for the fixed-base buildings. The use of $|Z'(\omega)|(=F_z(\omega)|Z(\omega)|)$, $|\Theta'(\omega)|(=F_\Theta(\omega)|\Theta(\omega)|)$ and $\phi'_R(\omega)$ in place of $|Z(\omega)|$, $|\Theta(\omega)|$ and $\phi_R(\omega)$ (while computing the response energy spectra) is an essential part of the proposed changes. In addition to this, it will be worthwhile to see if $\phi'_R(\omega)$ can be approximated by a constant value (independent of frequency) for a simpler computational algorithm.

It may be recalled that the modification of rocking amplitudes represented by $F_\Theta(\omega)$, is considerable near the system frequency, $\tilde{\omega}_1$,in case of significant interaction. This peak behavior closely resembles a "delta

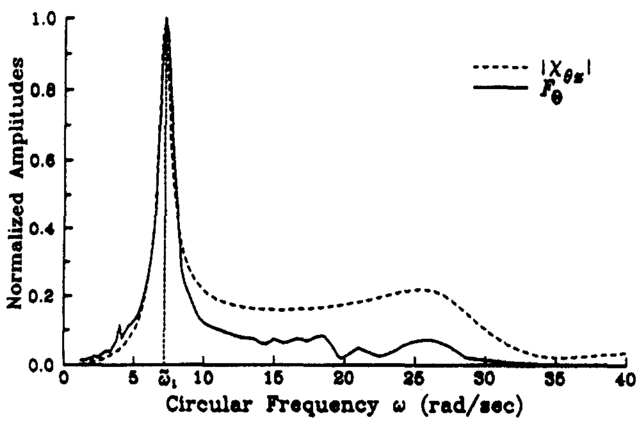

Fig. 5. Comparison of Normalized Amplitudes of F_Θ and $|\chi_{\theta z}|$.

function" at $\tilde{\omega}_1$ and so does the behavior of $|H_j(\omega)|^2$ near the frequency ω_j. Gupta and Trifunac[11] have reasoned that due to the simultaneous presence of two "delta functions", an effective constant value of $\phi'_R(\omega)$ which should lead to the same root-mean-square (r.m.s.) value of the response function (say, a_{rms}) as $\phi'_R(\omega)$, is likely to lie somewhere in between $\phi'_R(\tilde{\omega}_1)$ and $\phi'_R(\omega_j)$. Based on this, they have suggested a simple approximation of $\phi'_R(\omega)$ by $(\phi'_R(\tilde{\omega}_1)+\phi'_R(\omega_1))/2$ for all the frequencies. Determination of $\tilde{\omega}_1$ may be based on the first peak in the F_Z plot as pointed out earlier, and $\phi'_R(\omega)$ can be calculated using Equation (11). This approximation of $\phi'_R(\omega)$ is, however, valid only when the Fourier spectra of the input motion are reasonably flat over significant range of frequencies, and the imput energy is not concentrated in a narrow band of frequencies. In this paper, we will consider a few example cases in which these conditions are not met. Therefore, our calculations of a_{rms} here will be based on the actual frequency-dependent value of $\phi'_R(\omega)$.

The approach presented by Gupta and Trifunac[11-13] for the fixed-base case involves, for first few peaks of the response function, the modification of the r.m.s. value of the response peaks, since the value computed from the energy spectrum does not account for the nonstationarity in response. This is done by using the response spectra of the input excitations. A parallel approach in the present case will lead to the lengthy process of getting acceleration time histories corresponding to $\ddot{z}_0(t)$ and $\ddot{\theta}_0(t)$. Assuming that the building response in case of soil-structure interaction has similar nonstationary characteristics as in the building response for fixed base case, we proposed[11] that the r.m.s. value of peaks in case of interaction (calculated by using $|Z'(\omega)|$, $|\Theta'(\omega)|$ and $\phi'_R(\omega)$) be magnified by a factor η. This involves the use of standard response spectra for the input motion, and thus obviates the need for calculating spectra for the modified motion. This, however, over-estimates the effects of nonstationarity when the peak in the interaction function $F_\Theta(\omega)$ coincides with the peak in the input translation spectrum $|Z(\omega)|$ for a narrow-band excitation. This has motivated the following alternative approach for an approximate modeling of the nonstationarity.

Considering that the modified exictation is primarily dominated by the input translation $Z(\omega)$ and interaction rocking $\hat{\Theta}_0(\omega)$, it is necessary to calculate the degree of nonstationarity associated with the building response to $\hat{\Theta}_0(\omega)$ $(=\chi_{\theta z}(\tilde{\omega})Z(\omega)/L)$ alone. The behavior of transfer function $|\chi_{\theta z}(\tilde{\omega})|$ closely resembles the transfer function of relative acceleration response of a SDOF oscillator, and since it is possible to conveniently obtain the peak value of this response[31] from the peak value of ground acceleration i.e. $|\ddot{z}(t)|_{max}$ and the pseudo spectral acceleration (available from the standard response spectra), $|\ddot{\theta}_0(t)|_{max}$ i.e. the peak value of interaction rocking can be calculated. This requires the calculation of natural frequency and damping of the equivalent SDOF oscillator from the peak in $|\chi_{\theta z}(\tilde{\omega}|$ (or alternatively, from the peak in $F_\Theta(\omega)$ without significant loss of accuracy). Statistical calculations of the "expected" peak interaction rocking acceleration, say $|\overline{\ddot{\theta}_0(t)}|_{max}$, can now be made as $\Theta_0(\omega)$ is a derived process from $Z(\omega)$. Thus, we can calculate the ratio $\eta_\theta = |\ddot{\theta}_0(t)|_{max}/|\overline{\ddot{\theta}_0(t)}|_{max}$ which is the measure of nonstationarity in $\ddot{\theta}_0(t)$. Further, statistical calculations of the "expected" peak ground translation, say $|\overline{\ddot{z}(t)}|_{max}$ can give $\eta_z = |\ddot{z}(t)|_{max}/|\overline{\ddot{z}(t)}|_{max}$, the measure of nonstationarity in $\ddot{z}(t)$. The ratio $\eta_{\theta z} = \eta_\theta/\eta_z$ thus indicates the nonstationarity associated with the process of deriving $\ddot{\theta}_0(t)$ from $\ddot{z}(t)$. In cases of significant interaction, this ratio is less than unity. It is possible to simplify the calculation of $\eta_{\theta z}$ by assuming the process $\ddot{z}(t)$ and $\ddot{\theta}_0(t)$ to have same statistical characteristics. This corresponds to same ratio of "expected" value of peak amplitude to the r.m.s. value of response function in these processes. $\eta_{\theta z}$ is then approximated by $(|\ddot{\theta}_0(t)|_{max}/|\ddot{z}(t)|_{max})\sqrt{m_{0z}/m_{0\theta}}$ where m_{0z} and m_{0z} respectively are the areas of the energy spectra of $Z(\omega)$ and $\Theta_0(\omega)$. The degree of nonstationarity associated with the building response to $\hat{\Theta}_0(\omega)$ can now be approximated by $\eta_{\theta z}\bar{\eta}$ where $\bar{\eta}$ is the degree of nonstationarity in the building response to $Z(\omega)$ alone while the effects of modal interaction have been ignored. Same degree of nonstationarity i.e. $\eta_{\theta z}\bar{\eta}$ can now be assumed for the response to combined rocking excitation $\Theta'(\omega)$ also since $\Theta'(\omega)$ is mainly contributed to by $\hat{\Theta}_0(\omega)$ only.

With the knowledge of the degrees of nonstationarity in the responses to $Z(\omega)$ and $\Theta'(\omega)$, it is possible to estimate the effect of the nonstationarity in peak response to these two excitations acting simultaneously while ignoring the effects of modal interaction and those of interaction between the individual responses to these excitations. For this, let $\bar{a}_z$ and $\bar{a}_\theta$ denote the r.m.s. values of the response peaks respectively when the building is respectively subjected to $\ddot{z}(t)$ and $(\ddot{\theta}(t)+\ddot{\theta}_0(t))$, and the effects of nonstationarity are not accounted for. $\bar{a}_z$ and $\bar{a}_\theta$ are calculated from the zeroth moments of the respective energy spectra. Similarly, let $\bar{a}'_z$ $(=\bar{\eta}\bar{a}_z)$ denote the r.m.s. value for the response to $\ddot{z}(t)$ excitation when the effects of nonstationarity are accounted for. $\bar{a}'_z$ is calculated from the standard response spectra. Then, the ratio $\hat{\eta} = \bar{\eta}(\bar{a}^2_{\theta z} + \eta^2_{\theta z}\bar{a}^2_\theta)^{1/2}/(\bar{a}^2_z + \bar{a}^2_\theta)^{1/2}$ can be a working approximation to the nonstationarity in response to combined excitation $Z'(\omega)$ and $\Theta'(\omega)$. In those cases, where the effects of interaction between the individual responses to the excitations $Z'(\omega)$ and $\Theta'(\omega)$ dominate, the above may not be a valid approximation. The examples presented in this paper will be based on the values of $\hat{\eta}$ only.

5. PROPOSED ALGORITHM

The preceding discussion leads to the following simplified algorithm for estimating the seismic response of buildings:

1) Calculate the interaction functions $F_Z(\omega)$ and $F_\Theta(\omega)$ (Equations 9 and 10), and multiply those with the input translational and rocking spectra $|Z(\omega)|$ and $|\Theta(\omega)|$, to obtain the modified spectra $|Z'(\omega)|$ and $|\Theta'(\omega)|$. Similarly obtain the modified phase $\phi'_R(\omega)$ using Equation (11).
2) Determine the energy spectrum of the desired response function by considering $|Z'(\omega)|$, $|\Theta'(\omega)|$ and $\phi'_R(\omega)$ as the input excitation data. Calculate various moments of this spectrum and thus obtain all the statistical parameters related to the response function.
3) Modify the r.m.s. value of the response peaks (as calculated in step 2) to account for the nonstationarity in

response. The modification factor $\tilde{\eta}$ is calculated as follows:

a) Determine the natural frequency, $\tilde{\omega}_1$, and damping, ζ_{eq} of the equivalent SDOF oscillator from the peak in $F_\Theta(\omega)$. Obtain[31] the "pseudo relative spectrum acceleration" for $\tilde{\omega}_1$ and ζ_{eq}. Scale it by the factor $2\zeta_{eq}|\chi_{\theta z}(\tilde{\omega}_1)|/L$ (so that the heights of peaks in $|\chi_{\theta z}(\tilde{\omega})|$ and the transfer function of relative acceleration of SDOF oscillator are same) to give $|\ddot{\theta}_0(t)|_{max}$.
b) Calculate $\eta_{\theta z}$ as described in section 4 from the values of $|\ddot{z}(t)|_{max}$, $|\ddot{\theta}_0(t)|_{max}$ and the energy spectra of $\ddot{z}(t)$ and $\ddot{\theta}_0(t)$.
c) Consider the response function for the excitation by $\ddot{z}(t)$ and calculate the degree of nonstationarity $\bar{\eta}$ while ignoring the effects of modal interaction. Also evaluate $\bar{a}_z$ from the energy spectra of the response function.
d) Repeat step (c) for the excitation by $(\ddot{\theta}(t) + \ddot{\theta}_0(t))$ and evaluate $\bar{a}_\theta$.
e) Calculate $\tilde{\eta}(=\bar{\eta}(\bar{a}_z^2 + \eta_{\theta z}^2\bar{a}_\theta^2)^{1/2}/(\bar{a}_z^2 + \bar{a}_\theta^2)^{1/2})$.

4) Obtain the desired order of response peak amplitude with the specified probability of exceedance[11-13].

6. ILLUSTRATION OF PROPOSED MODEL

For illustration and testing of the above approach, the peak values of various response functions have been estimated using the time domain analysis, and compared with the estimates of the proposed approach. For this, the Fast Fourier Transform technique has been used to obtain the acceleration time histories of the relative motion of foundation.

Four example "buildings" with 5, 10, 15 and 20 stories, each having constant storey height of 5 m, have been considered with the values of floor masses and storey stiffnesses as in (12). Length of reference, L, has been respectively chosen as 10.2 m, 14.4 m. 17.7 m and 20.4 m. For modification of the Fourier spectra at high frequencies ($\tilde{\omega} > 10$), the transfer functions (χ's) have been assumed at these frequencies to be equal to their respective values at $\tilde{\omega} = 10$. Synthetic records[18,35] for three example excitations have been used: i) a hypothetical earthquake of 6.5 magnitude with 15 km epicentral distance from a site at Westmoreland, Imperial Valley, California, ii) Mexico Earthquake, 1985 with the recording site and Mexico City, and iii) Borrego Mountain Earthquake, 1968 as recorded at Hollywood, California. These example cases correspond to different degrees of interaction since the shear wave velocity in the top layer at the corresponding sites is respectively 1000 m/sec, 40 m/sec and 230 m/sec.

Figures 6 through 9 show some of the results obtained in the case of example buildings for the considered earthquake excitations. In each figure, the probabilistic estimates have been compared with the time domain analysis results by plotting the envelopes of maximum peak displacement, shear force and overturning moment. In each figure, various response values have been normalized with respect to the respective overall maximum response values. The dotted lines represent

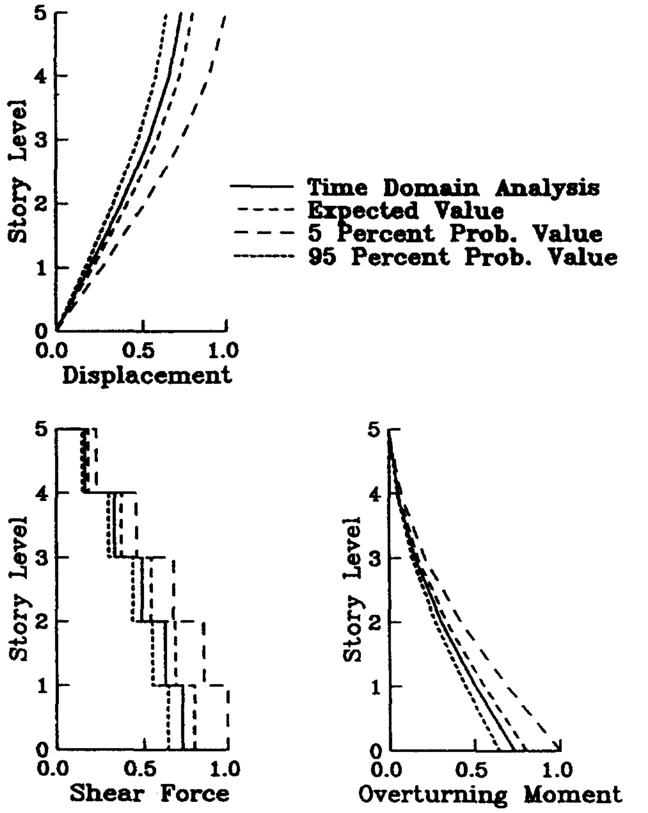

Fig. 6. Response of 5-Storey Building at Hollywood Site

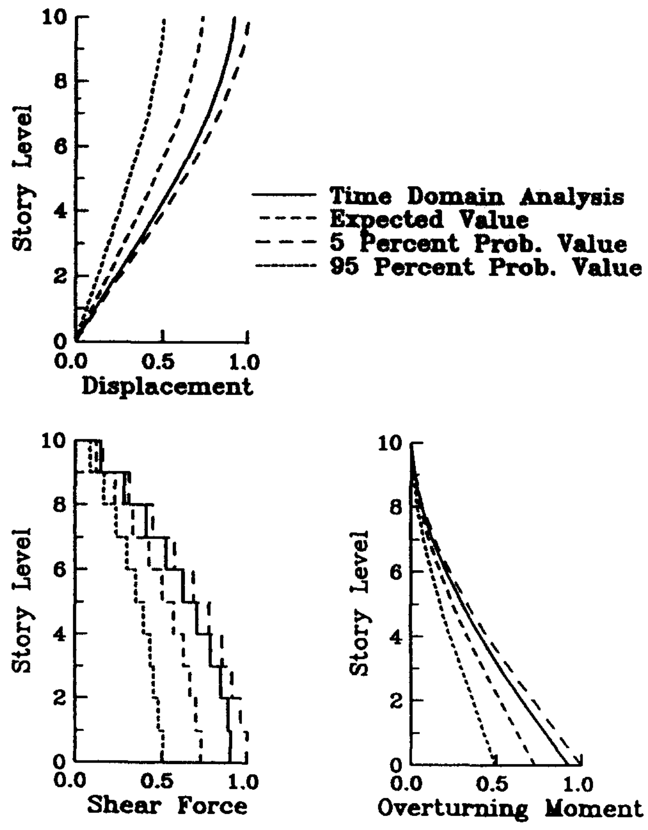

Fig. 7. Response of 10-Storey Building at Mexico City Site

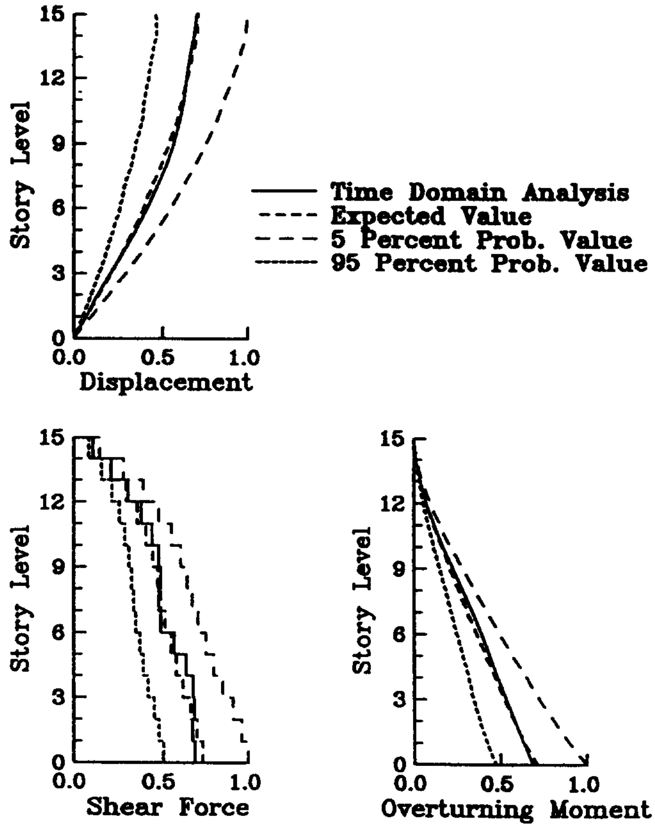

Fig. 8. Response of 15-Storey Building at Imperial Valley Site

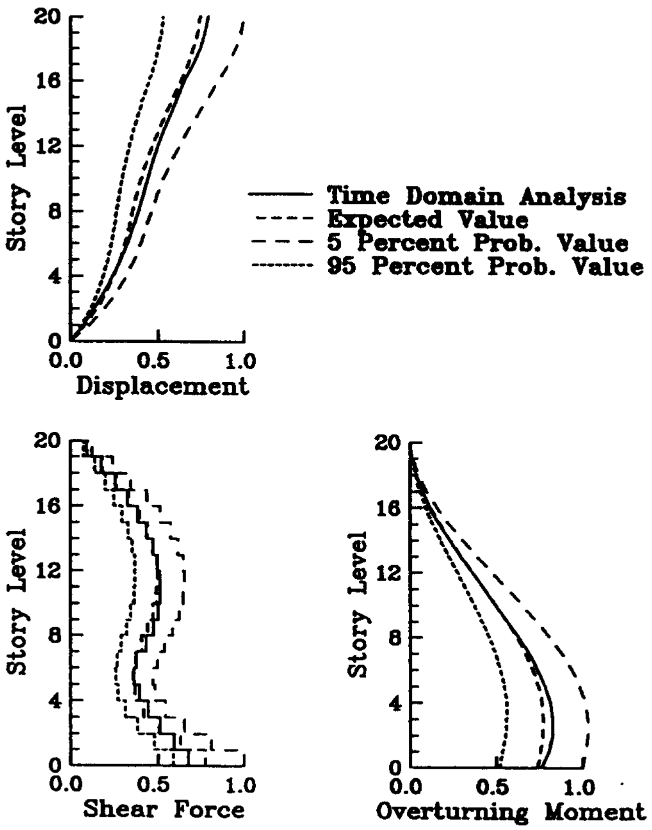

Fig. 9. Response of 20-Storey Building at Mexico City Site

the probabilistic estimates. The middle line represents the expected values while the two extreme lines represent the values with probability of exceedance equal to 5% and 95% and thus, enclose the 90% confidence interval. It is observed in these figures that the time domain results are bounded on either side by the 5% and 95% confidence estimates for all the four buildings. Further, they are in good agreement with the expected values in almost all such cases. Many such figures in several other example cases suggest that the proposed approach can give good estimates of the building response while accounting for the soil-structure interaction.

7. CONCLUSIONS

This study has shown that it is possible to include the soil-structure interaction effects in the analysis of multistoried building response via response spectrum superposition method, by incorporating a few modifications in the input excitation. These modifications include the phase differences between the input and interaction accelerations, and thus the proposed approach is more general than the existing response superposition methods.

Significant simplifications have been introduced in this approach by accounting for the nonstationarity in building response by using the standard response spectra for the input motion as for the fixed-based case. In contrast, other approaches of this kind are based on lengthy calculations involved in obtaining acceleration time histories of the modified motion.

It may be noted that the proposed approach is computationally more convenient that the time domain analysis. Moreover, it has the advantage that it can give the estimates of all the peak values of any response function with the desired level of confidence. It can be easily extended to more practical cases of embedded foundations and actual ground conditions of layered half-space by using appropriate impedance functions.

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