PSEUDORELATIVE ACCELERATION SPECTRUM

By M. D. Trifunac[1] and V. K. Gupta[2]

INTRODUCTION

After the introduction of the concept of response spectrum into earthquake engineering by Benioff (1934) and Biot (1941), and following numerous contributions to this subject (Alford et al. 1951; Hudson 1956; Trifunac and Lee 1989), today this technique continues to be one of the key practical vehicles towards earthquake-resistant design. Numerous analyses and applications of spectral method have shown that it is most convenient and sufficiently accurate to work only with the pseudorelative spectral velocity, PSV(T), which is defined in terms of the relative spectral displacement, SD(T), and the natural frequency of the oscillator, ω_n. Though it can be a simple by-product of the relative response calculations, $x(t)$, the relative acceleration spectrum, $|\ddot{x}(t)|_{\max}$, is usually not computed and is used only in specialized applications (Hadjian 1981; Lindley and Yow 1980; Singh and Mehta 1983).

Recently, in our work dealing with generalization of the response spectrum techniques to include the effects of soil-structure interaction (Gupta and Trifunac 1989, 1990), we found that it would be useful to have a simple procedure for the computation of $|\ddot{x}(t)|_{\max}$ from the existing SD(T) spectra, thus avoiding the actual response calculations. In the following, we outline the procedure that has been developed for that purpose.

PSEUDORELATIVE ACCELERATION SPECTRUM AMPLITUDES

For zero initial displacement and velocity, the expression for relative displacement response $x(t)$ of a viscously damped oscillator with frequency ω_n and fraction of critical damping ζ is

$$x(t) = \frac{-1}{\omega_n\sqrt{1-\zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\omega_n\zeta(t-\tau)} \sin \omega_n\sqrt{1-\zeta^2}(t-\tau)\, d\tau \qquad (1)$$

where $\ddot{z}(t)$ = the absolute ground acceleration during strong earthquake shaking. The relative velocity follows from Eq. 1 and is equal to

$$\dot{x}(t) = -\int_0^t \ddot{z}(\tau) e^{-\omega_n\zeta(t-\tau)} \cos \omega_n\sqrt{1-\zeta^2}(t-\tau)\, d\tau$$

$$+ \frac{\zeta}{\sqrt{1-\zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\omega_n\zeta(t-\tau)} \sin \omega_n\sqrt{1-\zeta^2}(t-\tau)\, d\tau \qquad (2)$$

[1]Prof., Civ. Engrg. Dept., Univ. of Southern California, Los Angeles, CA 90089-2531.

[2]Res. Assoc., Civ. Engrg. Dept., Univ. of Southern California, Los Angeles, CA.

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Further differentiation of Eq. 2 gives

$$\ddot{x}(t) = \omega_n \frac{1 - 2\zeta^2}{\sqrt{1 - \zeta^2}} \int_0^t \ddot{z}(\tau) e^{-\omega_n \zeta (t-\tau)} \sin \omega_n \sqrt{1 - \zeta^2}(t - \tau)\, d\tau$$

$$+ 2\omega_n \zeta \int_0^t \ddot{z}(\tau) e^{-\omega_n \zeta (t-\tau)} \cos \omega_n \sqrt{1 - \zeta^2}(t - \tau)\, d\tau - \ddot{z}(t) \quad \text{(3)}$$

In engineering applications, the maxima SD $\equiv |x(t)|_{\max}$, SV $\equiv |\dot{x}(t)|_{\max}$, and SA $\equiv |\ddot{y}(t)|_{\max}$ [where $y(t) = x(t) + z(t)$] are of primary interest, because from these one can evaluate the peak shear forces, bending moments, kinetic energy, and the inertial forces acting in a structure, and thus provide all necessary information for the subsequent design. Further analysis of Eqs. 1–3 results in useful approximations SV $\approx \omega_n$SD $\equiv$ PSV and SA $\approx \omega_n^2$SD $\equiv$ PSA, where PSV and PSA represent pseudorelative velocity and pseudoabsolute acceleration (Trifunac 1972).

In this note, we assume that only SD(T) and $|\ddot{z}(t)|_{\max} \equiv a_{\max}$ are available. We wish to compute PRSA(T), pseudorelative spectral acceleration, an approximation to $|\ddot{x}(t)|_{\max}$. To this end, we define

$$\omega_c = \frac{\int_0^\infty Z(\omega)\omega d\omega}{\int_0^\infty Z(\omega) d\omega} \quad \text{(4)}$$

Physically, ω_c = the coordinate of the "center of gravity" of $Z(\omega)$, the Fourier amplitude spectrum of ground acceleration $\ddot{z}(t)$.

For high frequencies $\omega_n >> \omega_c$ (i.e., short periods $T_n << T_c = 2\pi/\omega_c$), as $\omega_c \to \infty$, ω_n^2SD $\to |\ddot{z}(t)|_{\max}$ and $|\ddot{x}(t)|_{\max} \to 0$. For small ζ, $\ddot{x}(t)$ responds to the main energy in the ground motion, which is centered around ω_c, with approximately zero phase lag, and its amplitudes are essentially equal to $|\ddot{z}(t)|_{\max} = a_{\max}$. The contribution to the relative peak acceleration response near $T_n = 2\pi/\omega_n$ and for $T_n << T_c$ is approximately 90° out of phase from $Z(\omega_n)$. Thus we can write

$$\omega_n^2\text{SD} \approx |\ddot{x}(t)|_{\max}^2 + a_{\max}^2$$

or

$$|\ddot{x}(t)|_{\max}^2 \approx \omega_n^2\text{SD} - a_{\max}^2 \quad \text{(5)}$$

For long-period ground motions, $T_n >> T_c$, and so the oscillator responds to most of the energy in ground motion with phase equal to π, i.e., as $T_n \to \infty$

$$|\ddot{x}(t)|_{\max} \to a_{\max} \quad \text{(6)}$$

and

$$\omega_n^2\text{SD} \to 0 \quad \text{(7)}$$

To frequencies near ω_n and their amplitudes $Z(\omega_n)$, the oscillator responds with near $\pi/2$ phase lag. Then

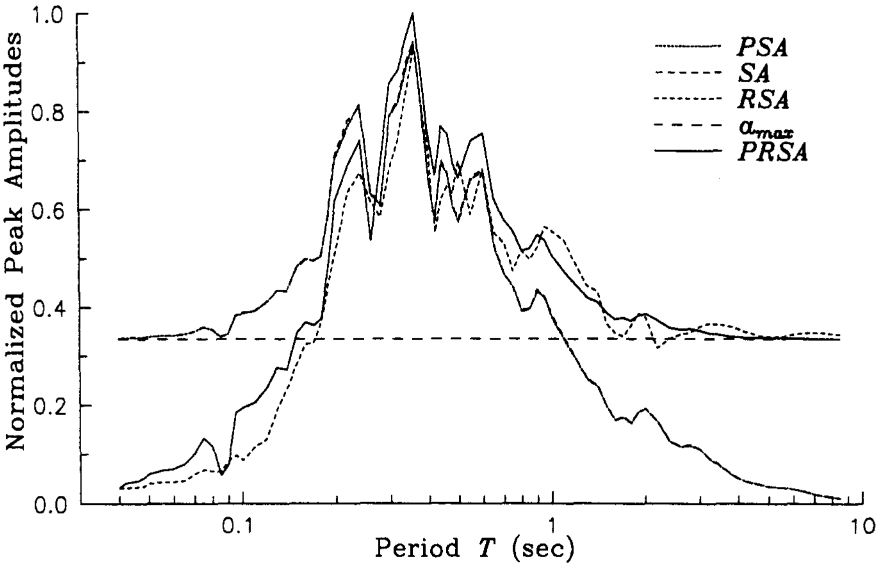

FIG. 1. Comparison of Various Normalized Acceleration Spectra where PSA (= $\omega^2|x(t)|_{max}$) Is Pseudoabsolute Spectral Acceleration; SA (= $|\ddot{y}(t)|_{max}$) Is Absolute Spectral Acceleration; RSA (= $|\ddot{x}(t)|_{max}$) Is Relative Spectral Acceleration, a_{max} (= $|\ddot{z}(t)|_{max}$) Is Peak Ground Acceleration; and PRSA Is Pseudorelative Spectral Acceleration

$$|\ddot{x}(t)|^2_{max} \approx \omega_n^2 \text{SD} + a^2_{max} \quad (8)$$

Thus, we propose an approximate representation of $|\ddot{x}(t)|_{max}$ and a new definition of PRSA(T) by

$$\text{PRSA}(T) \equiv \{[\text{PSA}(T)]^2 - a^2_{max}\}^{1/2}, \qquad (\omega \geq \omega_c),\ T \leq T_c$$

$$\text{PRSA}(T) \equiv \{[\text{PSA}(T)]^2 + a^2_{max}\}^{1/2}, \qquad (\omega < \omega_c),\ T > T_c \quad (9)$$

For $T \sim T_c$, PSA(T) >> a_{max}, and so this expression for PRSA(T) does not experience any noticeable "jumps" there.

To test Eq. 8, we calculated exact amplitudes of $|\ddot{x}(t)|_{max}$ and compared the results for many recorded and artificially generated accelerograms (Lee and Trifunac 1985, 1987). We found that PRSA(T) defined by Eq. 8 results in excellent approximation of RSA(T), the exact amplitudes of the relative acceleration spectra. This is illustrated by Fig. 1, which shows a_{max}, ω^2SD(T) $\equiv$ PSA(T), SA(T), RSA(T), i.e., exact spectra for $|\ddot{x}(t)|_{max}$, and PRSA(T), all plotted versus $\log_{10} T$ and for arbitrarily normalized amplitudes. We conclude that this approximation gives satisfactory results and thus can be used to compute PRSA(T) from SD(T) and a_{max}.

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Appendix. References

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