Antiplane Earthquake Waves in Long Structures

By M. I. Todorovska[1] and M. D. Trifunac[2]

Abstract: In this paper, the physical phenomena associated with wave passage under long buildings have been studied. A two-dimensional, continuous model has been used to represent the building vibration. Analytical, closed-form solutions have been obtained for the response to incident monochromatic, plane *SH*-waves. The soil-structure interaction has been neglected. The response of this model abounds with physical phenomena that cannot be seen from the response of one-dimensional building models. Some of these phenomena are important and must be considered in the design practice. For example, the propagating waves excite antisymmetric horizontal modes of vibration even in perfectly symmetrical long buildings. Thus, they contribute with forces and deformations that are not present in the one-dimensional analysis. It is shown in this paper that buildings can vibrate with modes that are exponential functions in the vertical direction, and that the transfer of the wave energy from the ground into the building depends on the phase velocities of the ground waves.

Introduction

In the typical analyses of the response of buildings to strong earthquake ground motion, it is customary to neglect the propagating character of the waves in the ground. Detailed three-dimensional models involving nonlinear analyses are used, but the spatial dependence of excitation is usually oversimplified. It is assumed, for example, that the seismic waves arrive with the same phase delay at various points of the base of the building. This corresponds either to vertical incidence, or to waves with angle of incidence other than zero, but with very large wavelengths compared to the size of the base of the building. In general, however, the seismic waves arrive at the building foundation with incident angles other than vertical, and they may have wavelengths comparable with the horizontal dimensions of the building, resulting in phased excitation at its base. The previously mentioned oversimplification of the assumed character of the excitation may lead to underestimation of the seismic forces that act upon the building during seismic response, and therefore it is important that these effects be studied carefully. The effect of traveling seismic waves on extended structures was studied, for example, for long bridges. Werner et al. (1977) presented a detailed review of the subject up to 1977, and Kashefi and Trifunac (1986) updated this review through 1986.

The effect of traveling seismic waves on buildings has been little studied to date. Tzenov and Boncheva (1979) and Tzenov (1981) noted the need for the two-dimensional models of "long-in-plan" buildings to account for the phase difference between the excitation at different points of the foundation. However, they did not consider the excitation to be propagating waves. The nature of the seismic energy transfer from the ground into the foundation of

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Note. Discussion open until May 1, 1990. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on August 2, 1988. This paper is part of the *Journal of Engineering Mechanics*, Vol. 115, No. 12, December, 1989. ©ASCE, ISSN 0733-9399/89/0012-2687/$1.00 + $.15 per page. Paper No. 24149.

a building, as well as the transport of energy within the building itself, has been discussed recently in the publication by the Soviet Academy of Sciences (1987) on the "Wave Processes in Structures During Seismic Interactions." The authors discuss modeling of buildings and suggest methods for calculating the equivalent P and S wave velocities of the equivalent continuous models.

The purpose of this paper is a qualitative study of the physical phenomena associated with the wave passage under extended buildings on simple models that allow analytical, closed-form solutions. The simplest model suitable for this purpose will be used, a two-dimensional, perfectly elastic plate with prescribed monochromatic SH motion at the base, equal to the free-field displacement on the surface of the halfspace. Homogeneous soil and soil with a vertical discontinuity will be considered. The latter will simulate the displacement patterns in and behind alluvial valleys. This simple model cannot be considered as a real building, but as an equivalent structure whose response to propagating waves will display the same phenomena that are expected to occur in the response of real buildings. Its simplicity eliminates many details in realistic buildings, but abounds in the physical phenomena. More realistic analyses involving more complicated geometries, soil-structure interaction, expansion joints, and realistic excitation are left for future studies.

Continuous Modeling of Buildings

To investigate the effects of traveling seismic waves on extended structures, two- and three-dimensional models are required. In common practice, such structures as buildings, bridges, dams, etc. are modeled by finite elements or by lumped mass models. The advantage in using such methods is that they can be used for structures of arbitrary shape. The disadvantage is that they give only approximate solutions. However, some simple continuous models allow analytical solutions for simple boundary conditions. Further, selected detailed three-dimensional experimental measurements of the deformation of actual buildings (Foutch et al. 1975) suggest that these can be modeled conveniently by an equivalent continuous representation.

The exact analytical solutions of the wave propagation and vibrational problems are desirable because they are convenient to study the physics of the problem. The analytical expressions of the solution directly involve the physical parameters of the system and make it easy to change them and to study their effects. They are advantageous also because they provide a basis for testing the approximate methods.

A two-dimensional rectangular plate or a rectangular solid (Fig. 1) can be used to investigate the effect of traveling seismic waves on long buildings, for example.

Equivalent Physical Constants

A simple method of estimating the equivalent shear wave velocities in buildings may be demonstrated on a building without major discontinuities in the material properties. It is assumed that only the frame of the building (Fig. 2) transmits the wave motion. L and D = the lengths of the building in the x- and y-direction, respectively; H = the total height; A and B = the average distances between the columns in the x- and y-direction, respectively; h = the average story height; and d = the average thickness of the floor panels.

The equivalent shear wave velocity in the z-direction β_z for the frame is

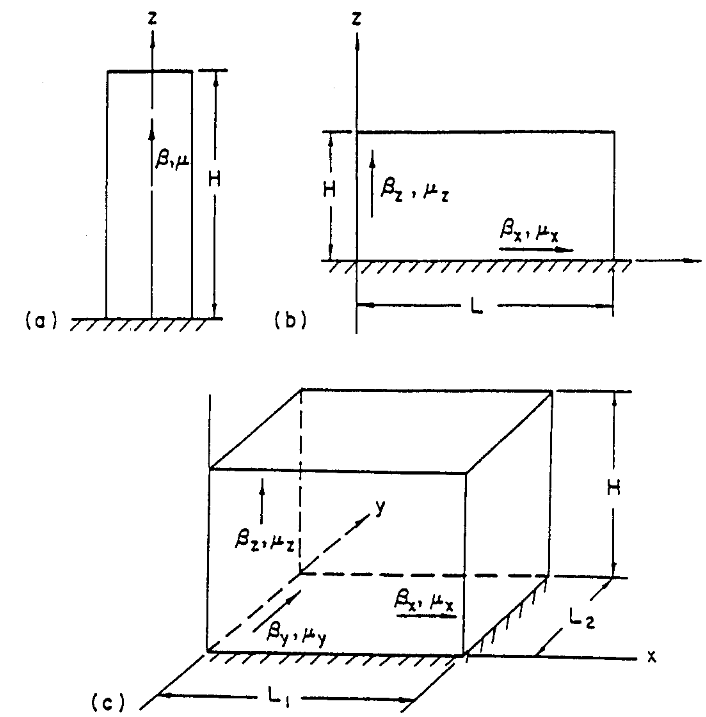


FIG. 1. Continuous Models of Buildings: (*a*) One-Dimensional Model, β and μ = Shear Wave Velocity and Shear Modulus; (*b*) Two-Dimensional Model: β_x and μ_x, and β_z and μ_z = Shear Wave Velocities and Shear Moduli in x- and in z-Directions, Respectively; (*c*) Three-Dimensional Model: β_x and μ_x, β_y and μ_y, and β_z and μ_z = Shear Wave Velocities and Shear Moduli in x, y, and z-Directions, Respectively

equal to the equivalent shear wave velocity of the element of the frame, as shown in Fig. 3(*a*). Assuming that the column of the element deforms in bending only and that the equivalent continuous element deforms in shear only [Fig. 3(*b*)] $\beta_z = \sqrt{\mu_z/\rho_z} = \sqrt{fE_c b^2/h^2 \rho_c}$, where f = a factor depending on the percentage of reinforcement in the cross section of the columns; E_c and ρ_c = the Young's modulus of elasticity and the density of the concrete; and μ_z and ρ_z = the shear modulus and the density of the equivalent continuous model.

The shear wave velocity in the x-direction β_x equals the equivalent shear wave velocity of the element shown in Fig. 4(*a*). Assuming that the floors deform in shear only and that the equivalent continuous model does the same [Fig. 4(*b*)], and also assuming that the floors are made of pure concrete, $\beta_x = \beta_c$, where β_c = the shear wave velocity of the concrete.

When applied to the Imperial County Services Building in El Centro, California, (Kojić et al. 1984), for example, this method gives the values of 420 m/s ≈ 1,400 ft/sec and 1,950 m/s ≈ 6,400 ft/sec for β_z and β_x, respectively. This method is very approximate and probably overestimates β_x, since the floors are not made of a single concrete panel. It also underesti-

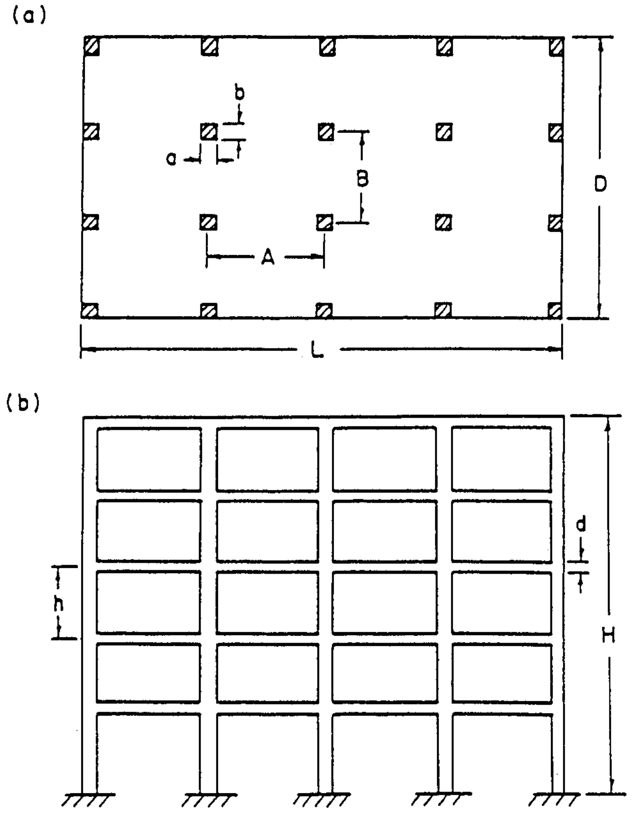


FIG. 2. Moment-Resistant Frame of Building: (*a*) Horizontal Cross Section; (*b*) Vertical Cross Section

mates β_z, since the nonstructural elements also contribute to the transition of the seismic waves vertically through the building. The Soviet engineers (Soviet Academy of Sciences 1987) have found the equivalent shear wave velocities in buildings to be in the range from 300–1,800 m/s (1,000–5,900 ft/sec). Another approach to estimating β_x and β_z consists of using experimentally or empirically determined natural periods of vibration. For a tall building of height H, fixed at its base, the fundamental period of vibration is approximately equal to $4H/\beta_z$. The first natural period corresponds to the time it takes the shear wave to travel four heights of the building. For a long building of length L, its first "free-free" horizontal mode of vibration, if it is associated mainly with shear deformations, would approximately be equal to $2L/\beta_x$.

The anisotropy of the equivalent continuous model is evident for the El Centro building and is expected for most of the buildings, since buildings are more flexible in the vertical than in the horizontal direction. The degree of anisotropy depends on the type of building. For a building with strong masonry nonstructural elements, the degree of anisotropy will be smaller during small linear vibrations, while for a building with light nonstructural

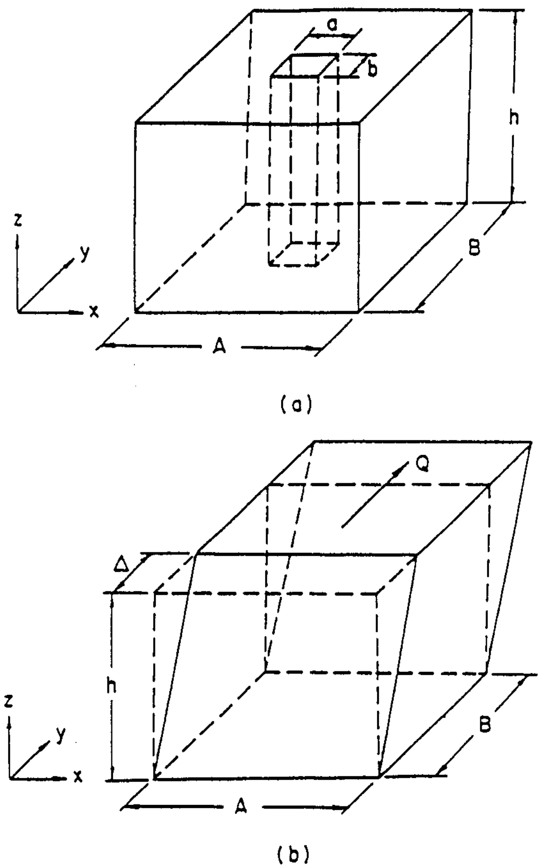


FIG. 3. (*a*) Element of Moment-Resisting Frame in Fig. 2, Used to Calculate Equivalent Shear Wave Velocity in *z*-Direction for Continuous Building Models Shown in Fig. 2; (*b*) Shear Deformation of Corresponding Element of Equivalent Continuous Building Model

elements (e.g., glass or plastic), the degree of anisotropy may be high. However, during strong earthquake motion, the nonstructural elements may fail soon after the shaking has started, and only the structure will transmit the wave motion further.

MODEL

The simplest model to investigate the physical phenomena associated with the wave passage under long buildings is a homogeneous and perfectly elastic two-dimensional model placed on homogeneous soil and excited by monochromatic *SH*-waves, ignoring the soil-structure interaction. This model is shown in Fig. 5, where x, y, and z = the spatial coordinates; L = the length of the building in the x-direction; H = the height; and μ and β = the shear modulus and the shear wave velocity. The material constants may have different values in the x- and the z-direction (for example μ_x and β_x

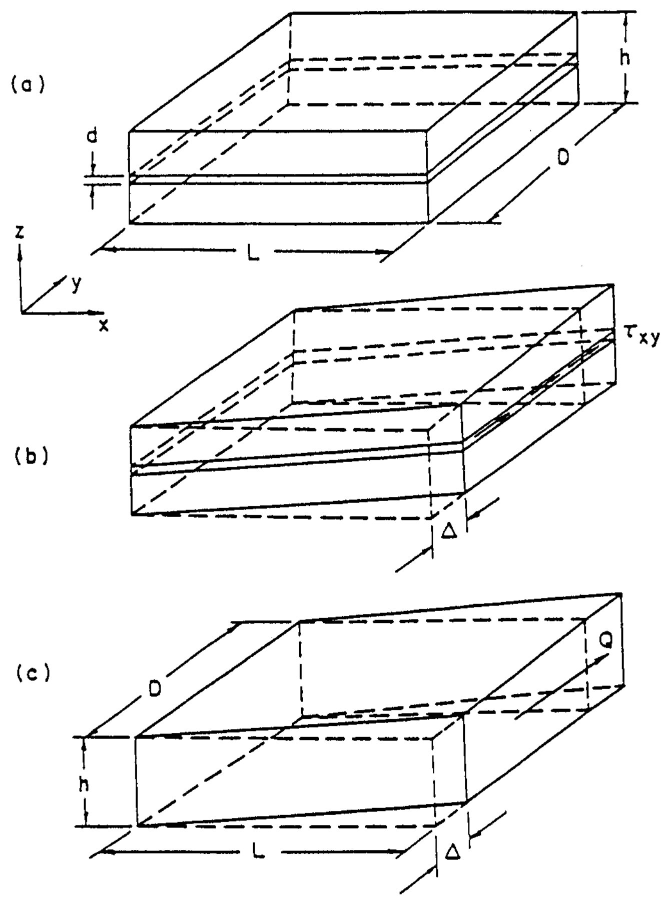


FIG. 4. (*a*) Typical Element of Frame in Fig. 2 Used to Calculate Equivalent Shear Wave Velocity in x-Direction; (*b*) Element Deformed in Shear; (*c*) Continuous Equivalent of Element Deformed in Shear

and μ_z and β_z). The governing equation of motion for the antiplane displacement of this model $\upsilon(x,z,t)$ is the two-dimensional, linear wave equation

$$\beta_x^2 \frac{\partial^2 \upsilon(x,z,t)}{\partial x^2} + \beta_z^2 \frac{\partial^2 \upsilon(x,z,t)}{\partial z^2} = \frac{\partial^2 \upsilon(x,z,t)}{\partial t^2} \quad (1)$$

where x and z = the spatial coordinates; t = the time coordinate; and β = the shear wave velocity of the plate. The conditions that the displacement $\upsilon(x,z,t)$ must satisfy are

$$\tau_{xy} = 0 \quad \text{at } x = 0, \quad 0 \le z < \infty \quad (2a)$$

$$\tau_{xy} = 0 \quad \text{at } x = L, \quad 0 \le z < \infty \quad (2b)$$

$$\tau_{zy} = 0 \quad \text{at } z = 0, \quad 0 \le x \le L \quad (2c)$$

and

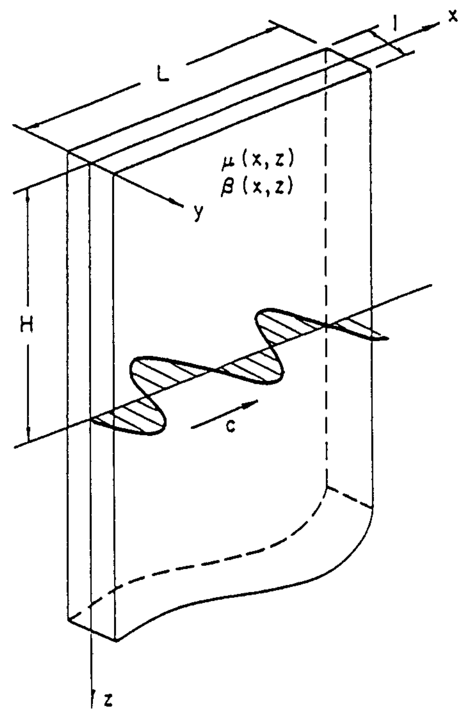


FIG. 5. Homogeneous Model of Building of Length L and Height H, Excited at Base by Wave of Amplitude 1, Frequency ω, and Propagating with Phase Velocity c in Positive x-Direction; Soil-Structure Interaction Is Neglected

$$\upsilon = e^{i\omega(t-x/c)} \quad \text{at } 0 \le x \le L, \quad z = H \quad (2d)$$

where $\tau_{xy} = \mu_x \partial\upsilon/\partial x$ and $\tau_{zy} = \mu_z \partial\upsilon/\partial z$ are the shear stresses in the plate; and ω and c = the circular frequency and the phase velocity in the x-direction of the motion at $z = H$ and $i = \sqrt{-1}$. By definition $c = \beta_s/\sin\gamma$, where β_s = the shear wave velocity in the soil; and γ = the angle between the direction of propagation of the incident wave and the vertical.

The eigenfunction expansion of the displacement solution is the following:

$$\upsilon(x,z,t) = \left[\sum_{n=0}^{N} C_n \cos\frac{n\pi x}{L} \frac{\cos\frac{\beta_x}{\beta_z}\sqrt{\left(\frac{\omega}{\beta_x}\right)^2 - \left(\frac{n\pi}{L}\right)^2}\, z}{\cos\frac{\beta_x}{\beta_z}\sqrt{\left(\frac{\omega}{\beta_x}\right)^2 - \left(\frac{n\pi}{L}\right)^2}\, H} + \sum_{n=N+1}^{\infty} C_n \cos\frac{n\pi x}{L} \frac{\cosh\frac{\beta_x}{\beta_z}\sqrt{\left(\frac{n\pi}{L}\right)^2 - \left(\frac{\omega}{\beta_x}\right)^2}\, z}{\cosh\frac{\beta_x}{\beta_z}\sqrt{\left(\frac{n\pi}{L}\right)^2 - \left(\frac{\omega}{\beta_x}\right)^2}\, H} \right] e^{i\omega t} \quad (3)$$

The coefficients C_n, $n = 0, 1, \ldots$ can be calculated analytically from the displacement condition. Their values are

$$C_0 = \frac{1}{\frac{\omega L}{c}}\left[\sin\frac{\omega L}{c} + i\left(\cos\frac{\omega L}{c} - 1\right)\right] \qquad (4a)$$

$$C_n = \frac{2}{L}\frac{\frac{\omega}{c}}{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n\pi}{L}\right)^2}\left\{(-1)^n \sin\frac{\omega L}{c} + i\left[(-1)^n \cos\frac{\omega L}{c} - 1\right]\right\},$$

$$n \geq 1 \qquad (4b)$$

when $\omega/c \neq m\pi/L$ for any $m = 1, 2, 3, \ldots$, and

$$C_0 = \frac{i}{m\pi}[(-1)^m - 1] = 0, \qquad m \text{ even} \qquad (5a)$$

$$C_0 = \frac{i}{m\pi}[(-1)^m - 1] = \frac{-2i}{m\pi}, \qquad m \text{ odd} \qquad (5b)$$

$$C_n = \frac{2}{L}\frac{i\frac{m\pi}{L}}{\frac{(m^2 - n^2)\pi^2}{L^2}}[(-1)^{n+m} - 1] = 0, \qquad (n + m) \text{ even}, n \neq m \qquad (5c)$$

$$C_n = \frac{2}{L}\frac{i\frac{m\pi}{L}}{\frac{(m^2 - n^2)\pi^2}{L^2}}[(-1)^{n+m} - 1] = \frac{-4im\pi}{\frac{(m^2 - n^2)\pi^2}{L^2}},$$

$$(n + m) \text{ odd}, n \geq 1 \qquad (5d)$$

$$C_m = 1 \qquad (5e)$$

if $\omega/c = m\pi/L$ for some integer m. In the limiting case when $\omega/c \to 0$, the coefficients C_n become

$$C_0 = 1 \qquad (6a)$$

and

$$C_n = 0, \qquad n \geq 1 \qquad (6b)$$

Some properties of $v(x,z,t)$ can be seen by analyzing Eq. 3, without doing any actual calculations. For example, the allowable values of the wave numbers in the z-direction are real only for a finite number of characteristic functions in the x-direction ($n = 0, 1, 2, \ldots, N$), whose index n satisfies the inequality.

$$\frac{\omega}{\beta} \geq \frac{n\pi}{L} \qquad (7)$$

For the rest of the characteristic functions they are purely imaginary. The corresponding shape functions are harmonic functions in x; in z, they are exponentially decaying functions toward the top of the building. The displacement of these characteristic functions in the z-direction can be called quasi-static, and they are not associated with propagation of the wave energy in the z-direction. Resonance happens only for the characteristic functions that are harmonic functions in z and when the following condition is satisfied:

$$\frac{\beta_x}{\beta_z}\sqrt{\left(\frac{\omega}{\beta_x}\right)^2 - \left(\frac{n\pi}{L}\right)^2}\,H = \left(k + \frac{1}{2}\right)\pi,$$

$$k = 0, 1, 2, \ldots \quad \text{and } n = 0, 1, 2, \ldots, N \qquad (8)$$

For the first characteristic function ($n = 0$), the resonant frequences are the same as the resonant frequencies for a cantilevered shear beam. For higher characteristic functions ($n \geq 1$), the resonant frequencies are the same as those of a two-dimensional shear plate, rigidly fixed at one side. When the input wave number ω/β becomes smaller, the number of the harmonic characteristic functions decreases. However, even when the wave number in the plate ω/β is so small that zero is the largest integer satisfying the inequality in Eq. 7, there is a solution that has a harmonic shape function in the vertical direction, through which the energy can be transmitted into the interior of the plate and for which resonance may occur.

The shape functions in the horizontal direction represent standing waves that result from the constructive interference between the waves reflected from the ends at $x = 0$ and $x = L$. For even n, they are symmetric; for odd n, they are antisymmetric functions with respect to $x = L/2$.

The coefficients C_n, $n = 0, 1, 2, \ldots$ have always finite values, even when the input base motion has the same wave number in the horizontal direction as one of the eigenfunctions in the x-direction as can be seen from Eqs. 4–6. These expressions also describe the contribution of the particular characteristic functions to the overall displacement. Eqs. 4 show that, in general, all the Fourier coefficients are nonzero, meaning that all the characteristic functions of vibration are excited.

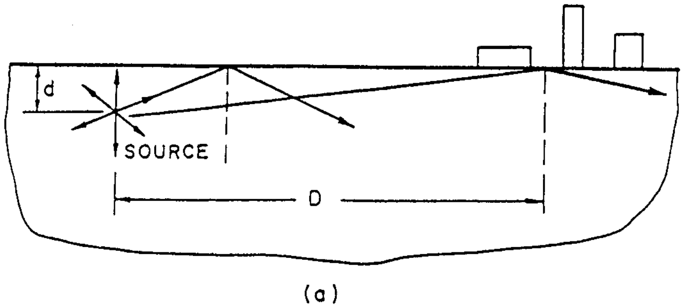
Eqs. 5 imply that in the special case when the wave number in the x-direction of the input motion equals the wave number in the z-direction of one of the higher characteristic functions, i.e., $\omega/c = m\pi/L$ for some $m \geq 1$, the mth coefficient has some value, and the rest of the coefficients are either zero or purely imaginary, with an absolute value less than one. If m is odd, all the other odd coefficients are zero, while the even ones are nonzero. Even in this special case, both symmetric and antisymmetric characteristic functions are excited.

In the one-dimensional case, i.e., when the wave number of the input motion $\omega/c = 0$, the input motion at $z = H$, which is $v(x,H,t) = e^{i\omega t}$, becomes a function of time only. Then all the coefficients C_n are zero except C_0, which is equal to one, meaning that only the first symmetric characteristic function contributes to the total displacement $v(x,z,t)$, and that no antisymmetric characteristic functions can be excited.

RESULTS

Nature of Strong Ground Motion

Investigations have shown that most earthquakes in California are shallow, with the earthquake source lying not deeper than about 25 km. The region



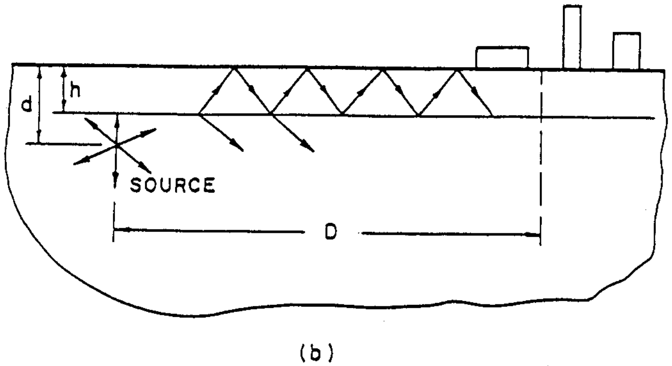


FIG. 6. Relative Positions of Earthquake Source and Building Sites for Typical Earthquakes in California Considering: (*a*) Homogeneous Half-Space; (*b*) Layered Half-Space

around the epicenter where the buildings are most threatened has a radius up to about 100–150 km for large damaging earthquakes. This means that if there are no major discontinuities in the earth's crust in the region between the source and the building, the earthquake energy can arrive at the site of the building in the form of body waves at angles γ (γ = the angle between the direction of propagation of the incident wave and the vertical) varying from zero, which happens when the building is directly above the source, up to very large angles, when the source is shallow and the building is far from the epicenter [Fig. 6(*a*)]. If there are soft layers near the earth's surface, and the building is not situated directly above the source, the seismic energy will be transmitted to the building site mainly in the form of surface waves [Fig. 6(*b*)]. Trifunac (1971) has shown that from 70–90% of the seismic energy arriving at the building sites in California can come through the surface waves. In the figures mentioned, D = the distance between the epicenter and the building; d = the depth of the source; and h = the thickness of the soft layer in the layered half-space.

The phase velocity in the horizontal direction c can become infinite only in two cases: (1) When the earthquake waves arrive nearly vertically at the building site, which is possible if the source is deep under the building; or (2) when it is far from the building so that the first body waves arrive almost vertically because of the progressive bending of the rays up toward the vertical. This can result from the presence of low velocity surface layers. In both given cases, the earthquake waves arrive at all points of the base of the building with the same phase. In all other instances, the phase velocities of the earthquake motion under the building will be finite, and there will be phase differences in the motion at different points of the base. This fact calls for the investigation of the various phenomena associated with the response of buildings to phased excitation at the base.

To make the problem more general, it is convenient to describe the input motion in terms of the dimensionless parameters $\eta = L/cT$ and c/β_x, where $T = 2\pi/\omega$ is the period of the input motion.

The phase velocity in the horizontal direction c was defined as the ratio between the shear wave velocity of the soil β_s and the sine of the incident angle γ. The minimum value of the shear wave velocity in the soil is about $\beta_{s,\min}$ = 50 m/s and the maximum value of $\sin\gamma = 1$. This gives the minimum value of the phase velocity in the soil to be $c_{\min}$ = 50 m/s. The maximum value is $c_{\max} = \infty$, and corresponds to the vertical incidence of seismic waves. The equivalent shear wave velocity in the building is in the range 500 m/s $\le \beta_x \le$ 1,800 m/s. An analysis of 57 modern tall buildings in the Los Angeles area (Moslem and Trifunac 1987), for example, shows that the maximum length for most buildings in that area is $L_{\max}$ = 80–100 m.

Considering all the given and taking the value of 40 Hz to be the maximum frequency of interest in the spectrum of the earthquake waves, the range of the dimensionless length η becomes $0 \le \eta \le 60$. In the calculations considered in this paper, the maximum value of η is four. No higher values of η were needed, because all physical phenomena associated with the wave passage under the building were evident even for $\eta = 2$. The values of the dimensionless phase velocity c/β_x are in the range $0.03 \le c/\beta < \infty$. The values c/β_x = 0.05, 1, and 20 were used in the calculations. The range of the height-to-length ratio for long buildings that are of interest in this investigation can be roughly estimated to be $0.25 \le H/L \le 3$. The values H/L = 0.25, 1, and 2 were used in the calculations.

Transfer of Energy of Ground Motion into Continuous Structural Systems

The local transfer of energy from the ground into buildings can be considered qualitatively on the two-dimensional continuous model placed over the homogeneous half-space, as shown in Fig. 7. When an incident *SH*-wave hits the interface between the two different media, it will be partially reflected back into the half-space and partially transmitted into the other medium. The incident angle γ and the refracted angle α (Fig. 7) must satisfy Snell's law

$$\frac{\sin\alpha}{\sin\gamma} = \frac{\beta_x}{\beta_s} \quad \text{(9a)}$$

where β_x and β_s = the shear wave velocities of the plate (in the x-direction) and of the half-space, respectively. Since the ratio $\beta_s/\sin\gamma$ is equal to the phase velocity in the horizontal direction, Snell's law can be written in the following form:

$$\sin\alpha = \frac{\beta_x}{c} \quad \text{(9b)}$$

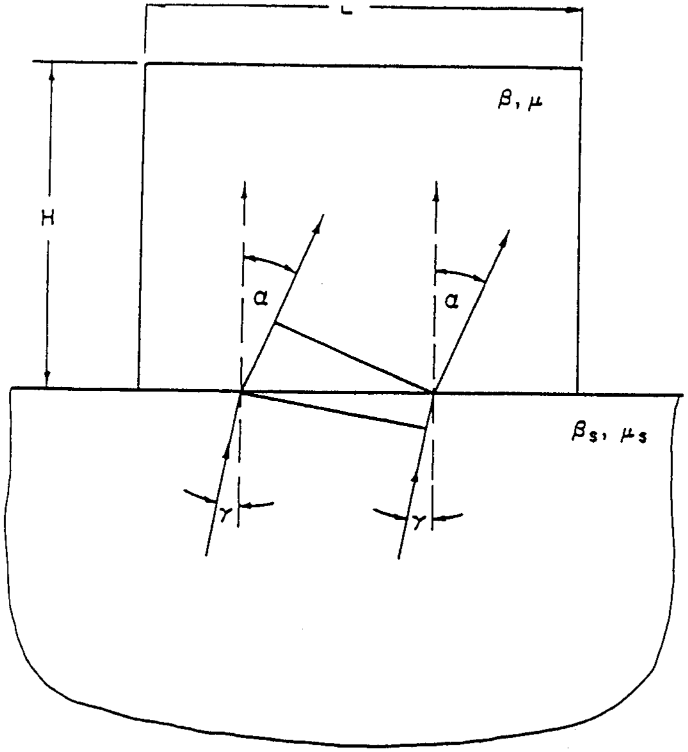


FIG. 7. Refraction of Plane Earthquake Wave at Interface between Half-Space and Building, Modeled as Homogeneous, Anisotropic, Two-Dimensional Elastic Plate

From Eq. 9*a* it can be seen that the refracted angle α is real only if $\beta_x \leq c$. If $\beta_x > c$, α takes on imaginary values, and the wave in the plate will be inhomogeneous instead of progressive, having exponentially decreasing amplitude toward the top of the plate, implying that no energy will be transferred into the plate. Hudson (1961) has proved for the case of Love waves in a layer that the energy transmitted from the layer into the half-space, during one half period, goes back into the layer during the other half period, and that the resultant energy that enters the half-space during the time of one period of motion is zero. A similar situation occurs in our case too. The preceding is a qualitative discussion because it applies only to two infinite media in contact. The finite dimensions of the building (H and L) and the boundary conditions (2*d*) limit such inferences asymptotically only to those cases in which $C_n \to 0$ for $n \leq N$, C_n is large for $n > N$, and n is near but different from $\omega L/\pi c$. This occurs for small wavelengths of the incident waves, i.e., when $c \to 0$ and for $\omega \neq \omega_r$, where ω_r = the resonant frequencies satisfying Eq. 8.

Figs. 8–10 show this dependence in terms of the ratio c/β_x. They represent the displacements of a vertical cross section of a long isotropic build-

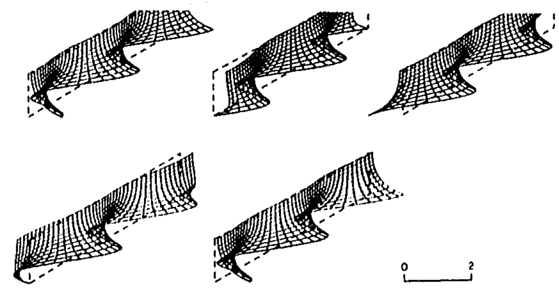


FIG. 8. Displacement Response of Long Building (H/L = 0.25), Represented by Homogeneous and Isotropic Model, for Propagating *SH*-Waves (η = 2 and c/β = 0.05) at Times t = 0, $T/4$, $T/2$, $3T/4$, and T; Wave Energy Does Not "Enter" into Building, As Can Be Seen from Exponential Nature of Displacements along Vertical Lines

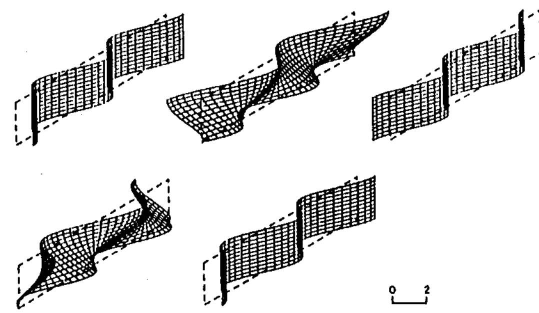


FIG. 9. Displacement Response of Long Building (H/L = 0.25), Represented by Homogeneous and Isotropic Model, for Propagating *SH*-Waves at Its Base (η = 2 and c/β = 1) at Times t = 0, $T/4T/2$, $3T/4$, and T; Wave Energy "Enters" into Building

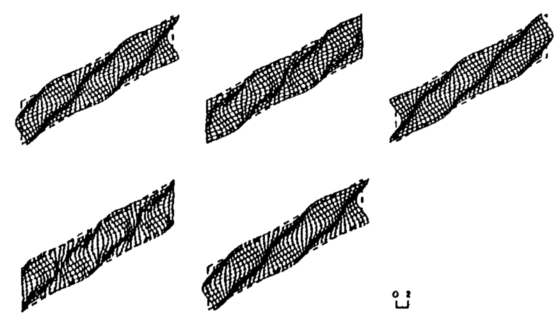


FIG. 10. Displacement Response of Long Building (H/L = 0.25), Represented by Homogeneous and Isotropic Model, for Propagating *SH*-Waves at Its Base (η = 2 and c/β = 20), at Times t = 0, $T/4$, $T/2$, $3T/4$, and T; Wave Energy "Enters" into Building

ing (H/L = 0.25 and $\beta_x = \beta_z = \beta$) at times equal to 0, $T/4$, $T/2$, $3T/4$, and T, where T = the period of the incident wave motion. The amplitude of the incident wave is 0.5, (i.e., the amplitude of surface displacement is 1.0), and the scale in this and in all the subsequent figures is in the same units as the displacement of the incident wave. The figures show that when c/β = 0.05, the "hyperbolic" characteristic functions are dominant in the displacement. The displacement is the largest at the base and exponentially decays towards the top of the building (Fig. 8). When c/β = 1, it can be seen from the figures that the harmonic characteristic functions are dominant in the displacement and that energy is entering the building. When c/β = 20, even the direction of propagation of the transmitted wave can be recognized from the displacement pattern.

Excitation of Symmetric and Antisymmetric Characteristic Functions of Vibration

Varieties of characteristic functions, symmetric as well as antisymmetric with respect to the center of the building, are used to represent the displacement response for almost any base excitation. Thus, for seismic design of large buildings, it is important to understand how the passage of seismic waves excites different characteristic functions of response.

In the discussion of the analytical expressions for the displacement response of the homogeneous model, it was shown that, in general, all the characteristic functions of vibration are excited. Both symmetric and antisymmetric characteristic functions (with respect to $x = L/2$) contribute to the overall displacement, even when the wave number ω/c of the input motion equals the wave number k_x of one of the characteristic functions. However, when the waves arrive vertically at the base of the building, i.e., ω/c = 0, only the first symmetric characteristic function is excited, and the problem becomes one-dimensional. This means that the "traditional" analysis of the response of buildings to strong ground motion neglects the fact that the higher x characteristic functions of vibration participate in the response of the building.

In the discussion of the displacement solution, the resonant frequencies ω_{nk} were defined as frequencies at which the displacement of the homogeneous plate becomes infinite. Realistic buildings possess damping, and therefore the more realistic continuous model would be made of a viscoelastic medium. The effect of the damping can be added to the theory of the undamped model as a perturbation that will change, but not significantly, the resonant frequencies and the Fourier coefficients of the expansion of the displacement. At the resonant frequencies, such a model will experience finite, but still large, displacements that can lead to large forces in the structure.

The significant frequency content of the earthquake waves is continuous and extends from zero up to 30 or 40 Hz. High frequencies are present in the Fourier spectrum of the earthquake source, but they will have small amplitudes by the time they reach a building. In Fig. 11, the resonant frequencies corresponding to the first six characteristic functions in the x-direction and the first three characteristic functions in the y-direction (n = 0, 1, ..., 5 and k = 0, 1, and 2), in the range 0–60 Hz, have been shown for a low building (L = 100 m, H = 25 m) and for a high building (L = 50

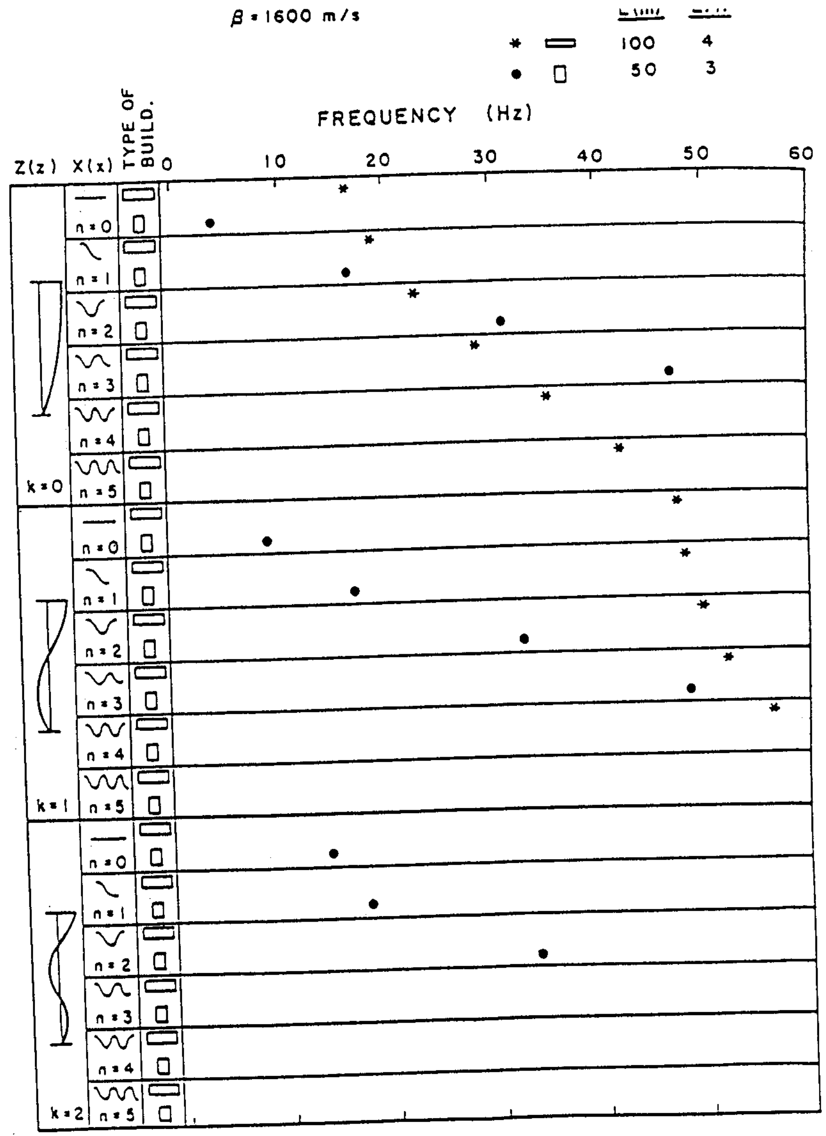


FIG. 11. Resonant Frequencies for First Few Modes of Long and Short, and of Long and High Building (H/L = 0.25 and 3, Respectively), Represented by Homogeneous and Isotropic Model, in Range between 0–60 Hz

m, H = 150 m). For the low building, the resonant frequencies corresponding to k = 0 and n = 0, ..., 5 and k = 1 and n = 0, ..., 4 fall in this range. For the high building the cases for k = 0 and n = 0, ..., 3, k = 1 and n = 0, ..., 3, and k = 2 and n = 0, ..., 2 are shown.

The building will vibrate with all frequencies that are in the range of sigresonant frequencies. The contribution of the resonant frequencies to the displacement will depend on the amplitude of the Fourier spectrum of the excitation at that frequency and on the coefficients of the expansion C_n. The displacements and the stresses at the resonant frequencies corresponding to the higher characteristic functions in x ($n \geq 1$) can be very large, even larger than the ones corresponding to the first characteristic function in x ($n = 0$) and that are expected by one-dimensional analysis. For design purposes, however, one-dimensional models of buildings are commonly used. This means that the buildings may not be designed for some loads that may occur during their life.

Figs. 12–14 show the displacement of an isotropic building ($\beta_x = \beta_z =$

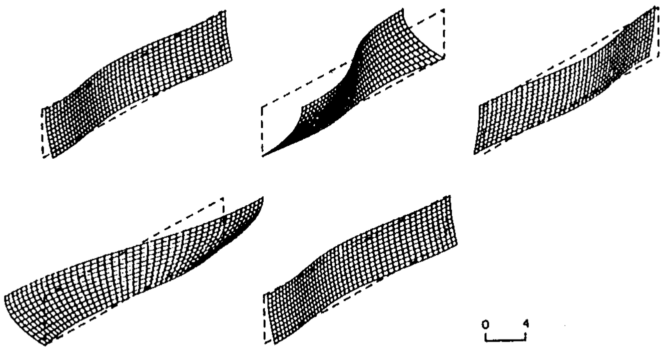


FIG. 12. Displacement Response of Long Building (H/L = 0.25), Represented by Homogeneous and Isotropic Model, for Propagating S/H-Waves (η = 1 and c/β = 1). At Times t = 0, $T/4$, $T/2$, $3T/4$, and T; Antisymmetric Modes of Vibration Are Seen to Contribute to Overall Response

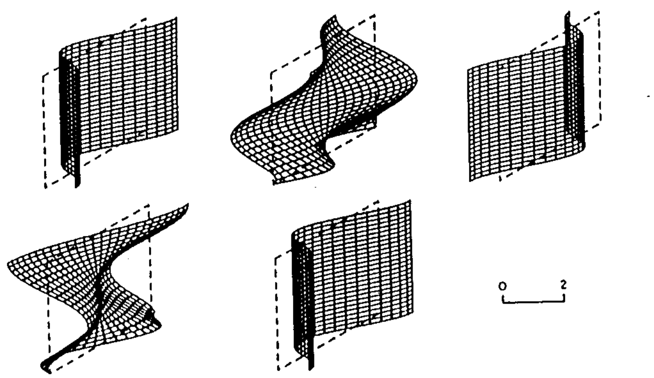


FIG. 13. Displacement Response of Building with Comparable Length and Height (H/L = 1), Represented by Homogeneous and Isotropic Model, for Propagating SH-Waves (η = 1 and c/β = 1) at Times t = 0, $T/4$, $T/2$, $3T/4$; and T; Antisymmetric Modes of Vibration Contribute to Overall Response

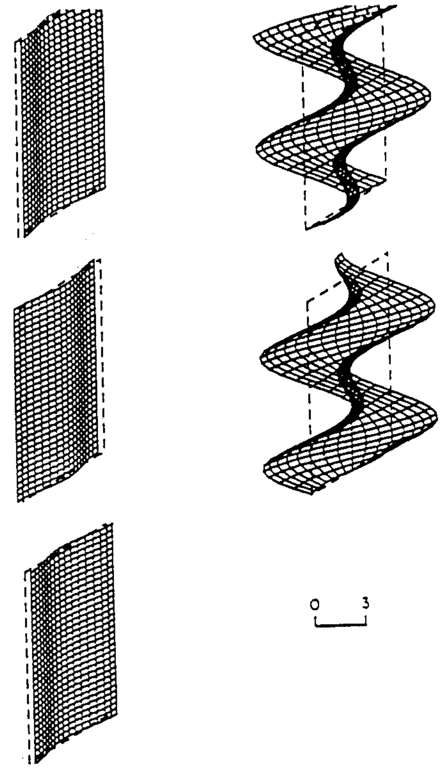


FIG. 14. Displacement Response of Long and Tall Building (H/L = 2), Represented by Homogeneous and Isotropic Model for Propagating SH-Waves (η = 1 and c/β = 1) at Times t = 0, $T/4$, $T/2$, $3T/4$ and T; Antisymmetric Modes of Vibration are Excited

β) while $\eta = 1$ and $c/\beta = 1$ and when the ratio $H/L = 0.25$, 1, and 2, respectively. It can be seen from these figures how the contribution of the different characteristic functions changes as the wave passes under the building. At time $t = 0$, $T/2$, and T, only one of the symmetric characteristic functions in x ($n = 2$) contributes to the displacement; the one that has the wave number equal to ω/c. At time $t = T/4$ and $3T/4$, only antisymmetric characteristic functions in x contribute to the displacement, and, moreover, the displacement at these two moments is larger than the displacement during $t = 0$, $T/2$, and T. The buildings experience large torsional deformations that give rise to horizontal stresses. These large torsional deformations may be responsible for some failure mechanisms of the long buildings.

Comparing the displacement patterns of the three models in Figs. 12–14, it can be concluded that for a larger H/L ratio, the displacement patterns contain more wavelengths in the vertical direction.

Effect of Anisotropy on Response

The anisotropy changes the wave numbers of the characteristic functions and therefore the displacement pattern. Its effect is equivalent to changing

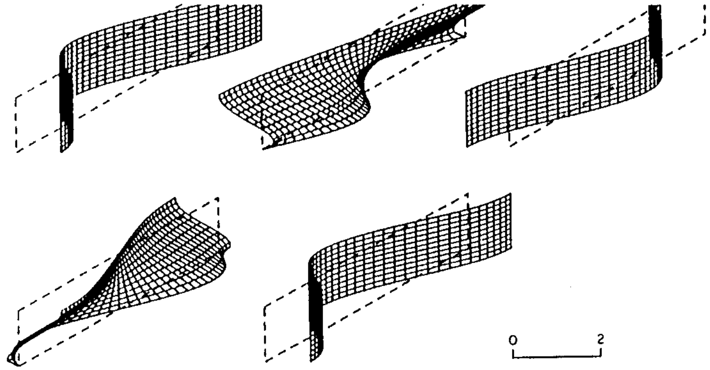


FIG. 15. Displacement Response of Long and Low Anisotropic Building (H/L = 0.25 and β_x/β_z = 2), Represented by Homogeneous and Isotropic Model, for Propagating SH-Waves (η = 1 and c/β = 1) at Times t = 0, $T/4$, $T/2$, $3T/4$, and T; Antisymmetric Modes of Vibration Are Seen to Contribute to Overall Response

the scale of the z-coordinate. For example, where $\beta_x/\beta_2 = 2$, the z-wave numbers and the displacement pattern of the building are the same as those of a building with an H/L ratio twice as large. An anisotropic building that is "softer" in the vertical direction contains wave forms with higher frequencies than if it were isotropic, as can be seen by comparing the displacement of the building in Fig. 12 with the displacement of the corresponding anisotropic building, with $\beta_x/\beta_z = 2$, in Fig. 15. The anisotropy does not illuminate any new physical phenomena that cannot be seen from the response of the isotropic model. Therefore, the isotropic model is satisfactory for the purposes of this study.

Response of Building to General Ground Motion

The response of the building model to monochromatic wave motion at the base can be used as a transfer function in calculating the response to the general excitation. Todorovska et al. (1988) discusses the Fourier synthesis of the response for nondispersed and dispersed base motion, as well as the response to a monochromatic wave of random amplitude coming from a random direction.

To investigate the effects of the wave passage on the response of a building placed on inhomogeneous soil, two quarter-spaces of different material properties, perfectly bonded to each other to represent the soil, are used (Fig. 16). In this figure, μ_L, β_L and μ_R, β_R = the shear moduli and the shear wave velocities of the medium on the left and on the right, respectively, of the discontinuity at $x = d$; and x' and z' = the spatial coordinates in the soil. μ and β and x and z are the material properties and the spatial coordinates in the building. If the soil on the left is softer than the soil on the right and if an incident monochromatic wave propagates from left to right, the steady-state displacement in the soft soil will consist of standing and propagating waves, thus having the main features of the displacements in the alluvium valleys (Moeen-Vasiri and Trifunac 1986). The displacement behind the discontinuity may be a propagating wave or an exponentially decaying function of x', depending on the material properties of the soils and on the x-wave number of the incident wave ω/c_L. Displacement response of the homogeneous building model (Fig. 5) placed in front, onto, or behind the soil discontinuity can be calculated analytically, if only the part of the displacement in the soil that satisfies the "Sato condition" is considered (Sato 1961).

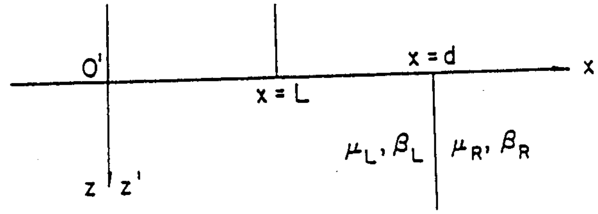


FIG. 16. Half-Space with Vertical Discontinuity and Building Located In Front of Discontinuity

The results show that if the building is situated entirely on the soft medium there will be points on the base with displacement amplitude close to zero. These are the points of the nodes of the nearly standing waves on the ground surface and also the sources of torsional deformations. This is not the case when the motion at the base is a propagating wave and each point of it passes through all the different phases of the wave motion. This is shown in Fig. 17, where $\eta = L/c_LT = 0.5$, $c_L/\beta = 5$, $\beta_L/\beta_R = 0.025$, $d/L = 1.2$, and the wave hitting the discontinuity in the soil is described by $e^{i\omega(t-x'/c_L)}$ cos $\omega z'/c_z$, where c_L and c_z = the phase velocities in the soft medium in the x'- and in the z'-direction, respectively.

If the building is partly situated on soft and partly on hard soil, e.g., 18, where $d/L = 0.7$ and all the other parameters are the same as in Fig. 17,

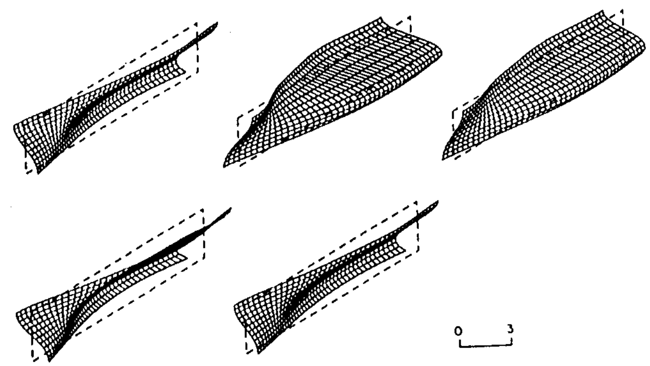


FIG. 17. Displacement Response of Long and Low Isotropic Building (H/L = 0.25) Placed over Soft Soil In Front of Vertical Discontinuity In Soil (β/β_L = 0.5, β_L/β_R = 0.025 and d/L = 1.2) for Incident SH-Waves In Soft Medium (η = 0.5 and c_L/β = 5) at Times t = 0, $T/4$, $T/2$, $3T/4$, and T; Building Is Practically "Sitting" on Standing Wave with Wave Length Equal to Twice Length of Building

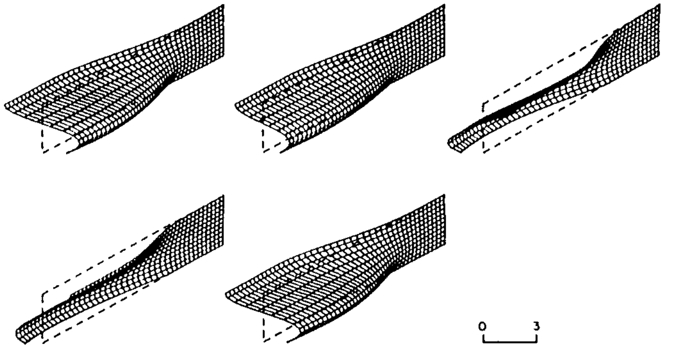


FIG. 18. Displacement Response of Long and Low Isotropic Building (H/L = 0.25) Placed over Vertical Interface between "Soft" and "Hard" Soils (β/β_L = 0.5, β_L/β_R = 0.025, and d/L = 0.7) for Incident SH-Wave in Soft Medium (η = 0.5 and c_L/β = 5) at Times t = 0, $T/4$, $T/2$, $3T/4$ and T; Wave Energy Propagates Nearly Vertically into Building through Interface with "Soft" Soil Only; Part of Building that Is Sitting over "Hard" Soil Has Displacement Amplitudes Practically Equal to Zero

it may happen that the displacements of the soft soil have large amplitudes, opposite to the displacement amplitudes in the hard soil, which may be negligible. Then if $c_L/\beta \geq 1$, the wave energy will propagate into the building only through the contact with the soft soil and not with the hard soil (either because c_R is imaginary, or if c_L is real, because the amplitude of the motion in the hard medium is practically equal to zero). Further, if the transmitted wave is propagating nearly vertically (as in Fig. 18), only the part of the building that is sitting on the soft soil will vibrate. Thus, the consideration of the propagating character of the seismic waves may be used to explain the extensive damage in certain areas of a building, in contrast to practically no damage in the remaining part of the same building.

Conclusions

The principal observations of this analysis can be summarized as follows.

Two-dimensional models of buildings are more representative than one-dimensional models because of the possibility they give: (1) To apply a more realistic excitation to the building model; and (2) to investigate and understand the variety of physical phenomena in their response. The models used in this paper abound in new phenomena that cannot be seen from the one-dimensional models. At the same time, they are simple enough to allow an analytical form of the solution.

It has been shown that a building will vibrate not only with harmonic characteristic functions, but also with hyperbolic characteristic functions in the vertical direction, having exponentially decaying amplitude toward the top of the building. The number of harmonic characteristic functions with which a building can vibrate is finite. The hyperbolic characteristic functions are not associated with propagation of the wave energy into the building, and the phenomenon of resonance occurs only for the harmonic characteristic functions. One-dimensional models can vibrate only with harmonic characteristic functions.

The two-dimensional analysis shows that the transfer of energy from the ground into the building depends on phase velocities with which the ground

motion propagates. Energy will propagate into the building efficiently when $c/\beta_x \geq 1$, where c = the phase velocity in the horizontal direction of the ground motion; and β_x = the equivalent shear wave velocity of the building in the x-direction. In one-dimensional models of buildings, the ground motion representation always has infinite phase velocities in the horizontal direction. This corresponds to vertically incident waves, and under those conditions, all incident energy always propagates into the building. The fact that the wave energy is not always transmitted into the building is of considerable practical importance. A soft layer under the building will reduce the phase velocities of the incident ground motion and could eventually make the ratio $c/\beta_x << 1$. Another way of reducing c is by channeling the ground motion to arrive at the building site nearly horizontally. This way much of the wave energy may be prevented from propagating into the building.

The anisotropy changes the wave numbers of the characteristic functions of vibration. Typically, the buildings are more flexible in the vertical direction, which will cause the displacement pattern to have shorter wave lengths.

The waves that are propagating in the horizontal direction excite the building to vibrate with a variety of symmetric and antisymmetric characteristic functions of vibration, even when the building is perfectly symmetric. The one-dimensional theory neglects all the higher characteristic functions, and the one-dimensional model vibrates only with the first symmetric characteristic function that has constant displacement in the horizontal direction. Current typical design practices consider 5–10% accidental torsion of the building and thus torsional response due to the eccentricity of the building only. Typically, the buildings are not designed for the rotational excitation that is associated with strong ground motion. Gupta and Trifunac (1987), using a probabilistic approach, investigated the contributions of this torsional excitation to the earthquake response of simple symmetric buildings and concluded that the rotation of the ground should be considered in the design of buildings. Such rotational excitation is a good respresentation of the rotational characteristics of the ground motion only when the wavelength of the seismic waves is long compared to the in-plane dimensions of the building. The two-dimensional model used in this paper does not put any limitations on the wavelengths of the input wave motion.

If the building is long and on inhomogeneous soil and near a vertical discontinuity in the material properties of the soil, which partially reflects the incoming waves, the building will be excited in part by the standing waves. In this case, the points of the base of the building that are standing on the nodes of the standing wave will experience large torsional excitation. This does not happen if the displacement of the base is a propagating wave. If the building is long and partly sitting on soft soil and partly on hard soil, it may happen that the building vibrates asymmetrically because of the large ground displacements in the soft soil and very small displacements in the hard soil. When the wave that has been transmitted into the building through the contact with the soft soil propagates nearly vertically through the building, it may happen that only the part of the building on the soft soil vibrates and that the other part of the building is relatively quiet. Only the two-dimensional analysis can meaningfully be used to understand the response of such buildings.

Acknowledgments

We are indebted to an anonymous reviewer who found an error in the

conclusions of the original manuscript and thus contributed to the improvement of this paper.

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