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ORDER STATISTICS OF PEAKS OF THE RESPONSE TO MULTI-COMPONENT SEISMIC EXCITATION

By

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Introduction

The ground motion caused by an earthquake can be considered as the resultant of three orthogonal translational componets along three cartesian coordinates, x, y, and z; and three rotational components about these coordinate axes as shown in Figure 1 (Rosenblueth, 1974). Consequently, the response of a multi-degree-of-freedom structure to such an input excitation will be the result of the corresponding three translational and three rotational responses. In this paper, response of a structure to simultaneoue saction of only the translational components of the earthquake is studied. The effects of rotational excitations will ba considered in other papers.

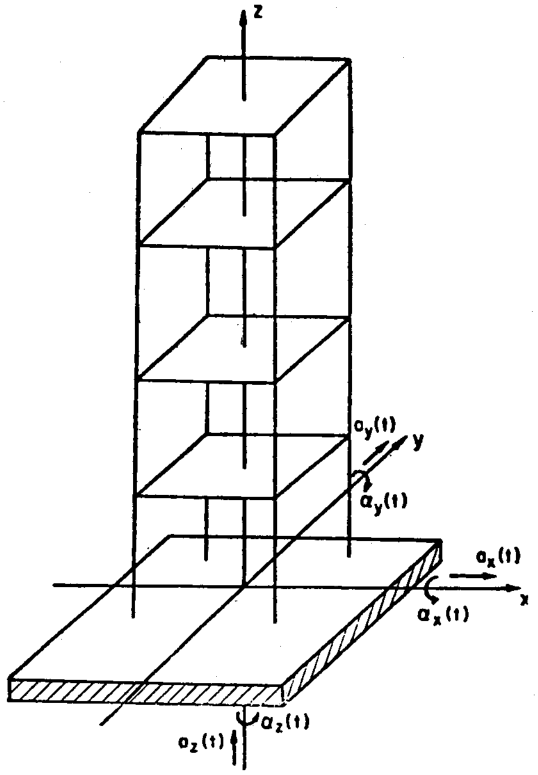

Fig. 1. Multicomponent Seismic Excitation.

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Many investigators have recognized the necessity of finding the response of structures under multi-component earthquake excitation. Bolotin (1963) presented a stochastic theory of structural response to multi-component seismic loading. He expressed the response in terms of generalized coordinates and incorported the nonstationarity effects of the response by correlation functions of the generalized coordinates. Penzien and Watabe (1975) pointed out the importance of finding the response caused by three translational components of earthquake motion and studied the characteristics of three-dimensional ground motion.

Chen (1975) investigated the statistical dependence of the three components of ground motion. Gelman (1974) worked on the combination of the translational component sin the horizontal direction. Rosenblueth and Contreras (1977) developed an approximate procedure for combining the components of response. They expressed the resultant response as a linear combination of the components of interest and found the coefficients of expansion so that the error is minimized. Wilson and Button (1982) presented a method for three-dimensional dynamic analysis for multi-component earthquake spectra.

Other works suggest the methods of combining the maxima of the three components of response, where each of the component maxima is found by the customary modal superposition methods using response spectra of the corresponding ground acceleration component. Chu, Amin and Singh (1972) have suggested the square root of the sum of squares (SRSS) method for finding the resultant of three partial responses. The U S.N.R C. (1976) recommends this method for combining three spatial responses. The (MAX+30%) method suggested by Resenblueth and Contreras (1977) is recommended by ATC-3 (1978) provisions. In this method, the resultant is taken as the maximum of the components plus 30% of the sum of all others components.

Anagnostopoulos (1981) has made a comparative study of different modal and spatial combination methods. In addition to SRSS and (MAX+30%) rules, he has considered the methoda like SUM, SRLS, and their averages with SRSS. He has also used the (MAX+4%) and (MAX+50%) rules. In SUM, the resultant is the sum of absolute values of the three components; whereas in the NRLS method, resultant is defined as the maximum of three components plus the SRSS of the other two. This was suggested as a modal superposition method by O'Hara and Cunfit (1973).

Though it is well recognized that the response of important structures like nuclear reactor components, dams, and long span bridges, may be dependent on all the three translational components of motion, due to complexity of the three dimensional analysis and various uncertainties in spatial combination procedures, most of the studies considers only one component of earthquake motion. The common spatial combination rules like SRSS and (MAX+30%) have been derived on the assumption of statistical independence between components of the response, though such an assumption may not always be justified (Hadjian, 1978). Anagnostopulos (1981) has shown that because of the correlation between components of earthquake motion, additive response effects are created in corner members of a three dimensional structure Moreover, all the above mentioned spatial combination rules combine the maxima of the component responses to get the maximum of the resultant, whereas the component maxima do not all occur at the same time.

The statistical method presented by Gupta and Trifunac (1987b,c) has been extended here to find the resultant response caused by the simultaneous excitation of three transltional components of earthquake motion. This method overcomes some of the uncertainties present in other methods and is based on the fact that the resultant of translational components of response can be found by their vector sum Thus, this method is analogous to combining the component responses by root mean square method and gives the results very similar to the time history solutions. Further, this method is able to give the entire envelope of the resultant response by computing the expected or the most probable values of all peaks. Knowledge of all the maxima is useful to study the time dependent characteristics of the response.

The aim of this paper is only to show how the results of Amini and Trifunac (1981,1985) and Gupta and Trifunac (1987a,b,c) can be generalized to apply to simultaneous multi-component seismic excitation along the three cartesian coordinate directions. The reader should be familiar with the basic theory of order statistics of the peaks of response (Gupta and Trifunac, 1987a,c) and with its generalization to the multi-degree-of-freedom systems (Gupta and Trifunac, 1987,b) before proceding to read the paper.

Statistical Theory for 3-Dimensional Response

Let $a_x(t)$, $a_y(t)$ and $a_z(t)$ be three components of ground acceleration acting simultaneously on a multi-story structure as shown in Figure 1. Assuming that the response of the structure to each of these acceleration components can be found separately, let $f_x(t)$, $f_y(t)$ and $f_z(t)$ be the corresponding responses in x, y and z directions. Since the three components of the response at any time t can be represented by vectors along three orthogonal axes, the resultant is given by the vector sum with magnitude f(t) as follows

$$f^2(t) = f_x^2(t) + f_y^2(t) + f_z^2(t) \tag{1}$$

If T is the total duration of the response, then one can write

$$\frac{1}{T}\int_0^T f^2(t)\,dt = \frac{1}{T}\int_0^T f_x^2(t)\,dt + \frac{1}{T}\int_0^T f_y^2(t)\,dt + \frac{1}{T}\int_0^T f_z^2(t)dt \tag{2}$$

Applying the Parseval's theorm in the above expression there follows

$$\frac{1}{T\pi}\int_0^\infty |F(\omega)|^2 d\omega = \frac{1}{\pi T}\int_0^\infty |F_x(\omega)|^2 d\omega + \frac{1}{\pi T}\int_0^\infty |F_y(\omega)|^2 d\omega +$$

$$\frac{1}{\pi T}\int_0^\infty |F_z(\omega)|^2 d\omega \tag{3}$$

where $F(\omega)$, $F_x(\omega)$, $F_y(\omega)$, $F_z(\omega)$ are the Fourier transforms of the corresponding time series $f(t)$, $f_x(t)$, $f_y(t)$, $f_z(t)$.

The energy density $E(\omega)$ is related to the Fourier transform $F(\omega)$ by the relation,

$$E(\omega) = \frac{1}{\pi T}\,|F(\omega)|^2 .$$

Threfore, equation (3) implies that

$$E(\omega) = E_x(\omega) + E_y(\omega) + E_z(\omega). \tag{4}$$

Thus the energy density of resultant response is given by the sum of energy densities of the component responses. The energy densities for the various component response can be found by using equations (A4 8), (A4 11) and (A4 14) in Amini and Trifunac (1985). These equations give the energy densities for the displacement, shear force and bending moment responses at the i-th floor. Thus the total energy density $E_i(\omega)$ at the i-th floor can be written as

$$E_i(\omega) = E_{xi}(\omega) + E_{yi}(\omega) + E_{zi}(\omega), \tag{5}$$

where $E_{xi}(\omega)$, $E_{yi}(\omega)$ and $E_{zi}(\omega)$ are the energy densities of the response components at the i-th level.

From expression (5), the k-th moment of the energy density spectrum of the resultant response at the i-th floor can be written as

$$m_{ki} = \int_0^\infty \omega^k E_i(\omega)\,d\omega = \int_0^\infty \omega^k E_{xi}(\omega)\,d\omega + \int_0^\infty \omega^k E_{yi}(\omega)\,d\omega + \int_0^\infty \omega^k E_{zi}(\omega)\,d\omega$$

or

$$m_{ki} = (m_{ki})_x + (m_{ki})_y + (m_{ki})_z \tag{6}$$

Thus, the k-th order moment of the energy density of total response is equal to the sum of the k-th order moments of the energy densities of component responses. The moments of interest, m_{0i}, m_{2i} and m_{4i} can be found by adding the respective moments for one-dimensional excitations in x, y, and z directions. Then the parameters (Amini and Trifunac, 1981, 1985) for the statistical distribution of the peaks of the resultant response at the i-th level are given by

$$\varepsilon_i = \frac{m_{0i}\, m_{4i} - m_{2i}^2}{m_{0i}\, m_{4i}} \tag{7}$$

$$N_i = \frac{T}{2\pi} \left(\frac{m_{4i}}{m_{2i}} \right)^{1/2} \tag{8}$$

and

$$\bar{a}_i = \sqrt{2}\, (m_{0i})^{1/2} \tag{9}$$

Equation (9) can also be written as

$$\bar{a}_i = \sqrt{2} \left\{ (m_{0i})_x + (m_{0i})_y + (m_{0i})_z \right\}^{1/2}$$

or

$$\bar{a}_i = \left\{ 2(m_{0i})_x + 2(m_{0i})_y + 2(m_{0i})_z \right\}^{1/2}$$

or

$$\bar{a}_i = \left\{ (a_i)^2_x + (a_i)^2_y + (a_i)^2_z \right\}^{1/2} \tag{10}$$

when ε_i are small (Udwadia and Trifunac, 1974).

From this expression it is clear that by using the modified values of $\bar{a}$ for the component responses, defind by the response spectra for the transient strong motion part of excitation and by the r.m.s. of the response for the ending portion of response (Gupta and Trifunac, 1987), the two modified $\bar{a}$' s for the total response can be defined as follows

$$\bar{a}_{Ei} = \left\{ (\bar{a}_{Ei})^2_x + (\bar{a}_{Ei})^2_y + (\bar{a}_{Ei})^2_z \right\}^{\frac{1}{2}} \tag{11}$$

and

$$\bar{a}_i = \left\{ (\bar{a}_{\mu i})^2_x + (\bar{a}_{\mu i})^2_y + (\bar{a}_{\mu i})^2_z \right\}^{\frac{1}{2}} \tag{12}$$

Thus all the statistical parameters of the result response at the i-th floor can be found from a knowledge of the corresponding parameters for one dimensional responses in x, y and z directions. Then the results of Gupta and Trifunac (1987 a, b) can be applied to find the expected and the most probable values of the peaks of the resultant response.

Example Application

The above procedure to find the probabilistic estmates of peak amplituds of the resultant response has been applied to study the translational response of the three story structure shown in Figure 2.

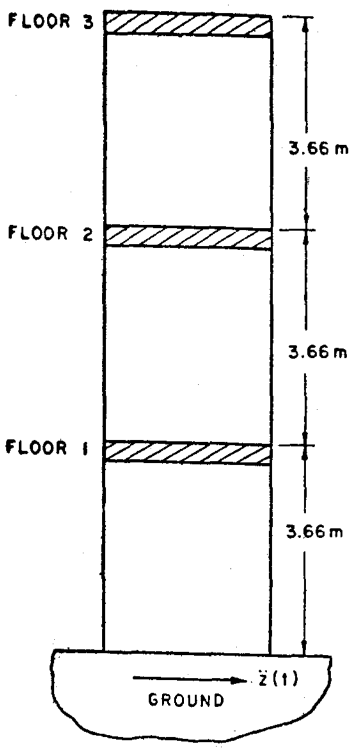

Fig. 2. Elevation of the Example Structure.

The mass metrix of this structure is chosen to be

$$[M] = \begin{bmatrix} 175.13 & 0 & 0 \\ 0 & 262.70 & 0 \\ 0 & 0 & 350.26 \end{bmatrix} \left(\frac{kN\text{-}S^2}{m}\right) \tag{13}$$

The stiffness matrix for x-direction is assumed to be

$$[K]_x = 10^6 \begin{bmatrix} 105 & -105 & 0 \\ -105 & 315 & -210 \\ 0 & -210 & 525 \end{bmatrix} (N/m) \tag{14}$$

The modal freqnency vector is then

$$\{\omega\}_x = \begin{Bmatrix} 14.5 \\ 31.1 \\ 46.1 \end{Bmatrix} (rad/s) \tag{15}$$

The matrix of modal eigenvectors is

$$[A]_x = \begin{bmatrix} 1.0 & 1.0 & 1.0 \\ 0\ 644 & -0.601 & -2.57 \\ 0.300 & -0\ 676 & 2.47 \end{bmatrix} \tag{16}$$

and the mode participation factors are

$$\{\alpha\}_x = \begin{Bmatrix} 1.424 \\ -0.509 \\ 0.020 \end{Bmatrix} \tag{17}$$

For the y-direction, the stiffness matrix has been assumed to be

$$[K]_y = 10^6 \begin{bmatrix} 70 & -70 & 0 \\ -70 & 210 & -140 \\ & -140 & 350 \end{bmatrix} (N\quad) \tag{18}$$

The modal frequency vector and the mode shape matrix are

$$\{\omega\}_y = \begin{Bmatrix} 11.84 \\ 25.40 \\ 37.64 \end{Bmatrix} (rad/s) \tag{19}$$

and

$$[A]_y = \begin{bmatrix} 1.0 & 1.0 & 1.0 \\ .65 & -0.80 & -2.54 \\ .30 & -1.14 & 2.44 \end{bmatrix} \tag{20}$$

and the mode participation factors are

$$\{a\}_y = \begin{Bmatrix} 1.420 \\ -0.544 \\ 0.092 \end{Bmatrix} \tag{21}$$

For vertical exitation, the stiffness matrix is taken as

$$[K]_z = 10^9 \begin{bmatrix} 2.8 & -2.8 & 0 \\ -2.8 & 6.7 & -3.9 \\ 0 & -3.9 & 8.7 \end{bmatrix} (N/m). \tag{22}$$

The modal frequencies and the mode shape matrix are then

$$\{w\}_z = \begin{Bmatrix} 61.16 \\ 143.94 \\ 204.79 \end{Bmatrix} (rad/s) \tag{23}$$

and

$$[A]_z = \begin{bmatrix} 1.0 & 1.0 & 1.0 \\ .766 & -.295 & -1.621 \\ .407 & -.810 & 1.059 \end{bmatrix}, \tag{24}$$

and the participation factors are

$$\{a\}_z = \begin{Bmatrix} 1\ 34 \\ -.425 \\ .096 \end{Bmatrix}, \tag{25}$$

Using the above properties of the structure for exitations along the three coordinate directions and using the three recorded components of the ground motion for Lytle Creek Earthquake of September 12,1979; Parkfleld Earthquake of June 27, 1966; Kern County Earthquake of July 21, 1952; and E1 Centro Earthquake of May 18,1940; the parameters for the statistical distribution functions have been calculated for one dimensional excitations along each of the three directions Then using the results of Gupta and Trifunac (1987a,b) and equations (6) through (12) the statistical parameters for the resultant response were found.

Tables 1 through 4 show these parameters for the resultant displacements, shear forces and bending moments at various floors of fhe structure. Using these parameters, the theoretical values of the amplitudes of the resultant responses have been calculated in terms of the order statistics formulation of Gupta and Trifunac (1987a, b, c). Actual values of the peak amplitudes have also been found by the time solutions for each of the three components and then by combinig the component time services by vector summation. Results of this analysis are discussed in the following section.

Results and Discussion

Figures 3 to 6 show the amplitudes of the first forty peaks of the resultant displacement due to simultaneous action of the three translational components of ground acceleration. The most probable values $\mu(a_{(n)})$ and $\bar{\mu}(a_{(n)})$ corresponding to the parameters $\bar{a}$ and $\bar{a}_\mu$; and the expected values

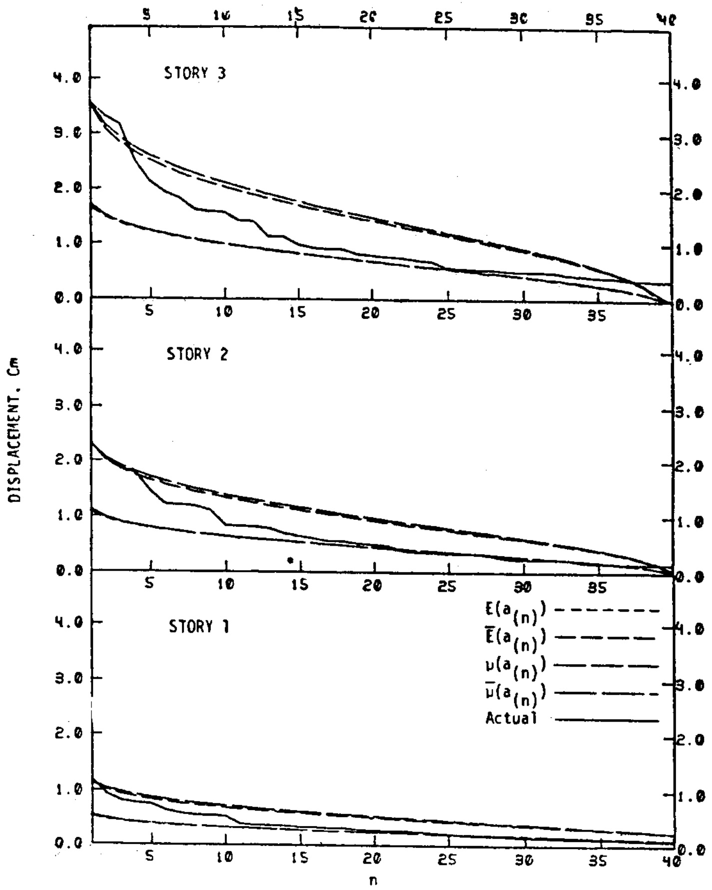

Figure 3 Theoretical and Actual Peaks of Displacement Response to Three Component Seismic Excitation (Lytle Creek Earthquake, Sept. 12, 1979, 11W334).

Figure 3. Thoretical and Actual Peaks of Displacement Response to three Component Seismic Excitation (Lytle Creek Earthquake, Sept 12, 1979, IIW334).

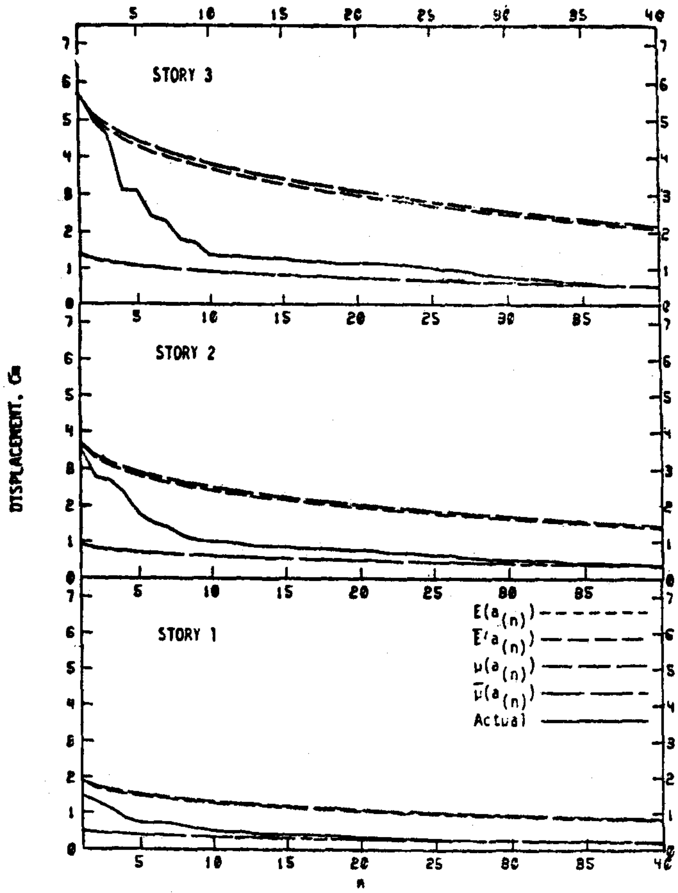

Figure 4. Theoretical and Actual Peaks of Displacement Response to Three Component Seismic Excitation (Parkfield Earthquake, June 27, 1966, IIB037).

$E(a_{(n)})$ and $\bar{E}(a_{(n)})$ corresponding to the parameters a and $\bar{a}_E$, are also plotted along with the actual amplitudes obtained by the time series solution. It is observed that the trend of the theoretical results is similar to that of the time series solution for all four earthquakes and for all the floor levels of the building, However, similar to the case of one dimensional response (Gupta and Trifunac: 1987b,c) the actual results have better agreement with the theoretical values obtained by using the parameters $\bar{a}_\mu$ and $\bar{a}_E$ for the first 5 to 10 peaks. For higher order peaks, the actual results have better agreement with the theoretical results obtained by using the values of $\bar{a}$. As explained by Gupta and Trifunac (1987b) for the case of one dimensional response, the values $\bar{a}_E$ and $\bar{a}_\mu$ are defined from the response spectrum of the ground acceleration and hence

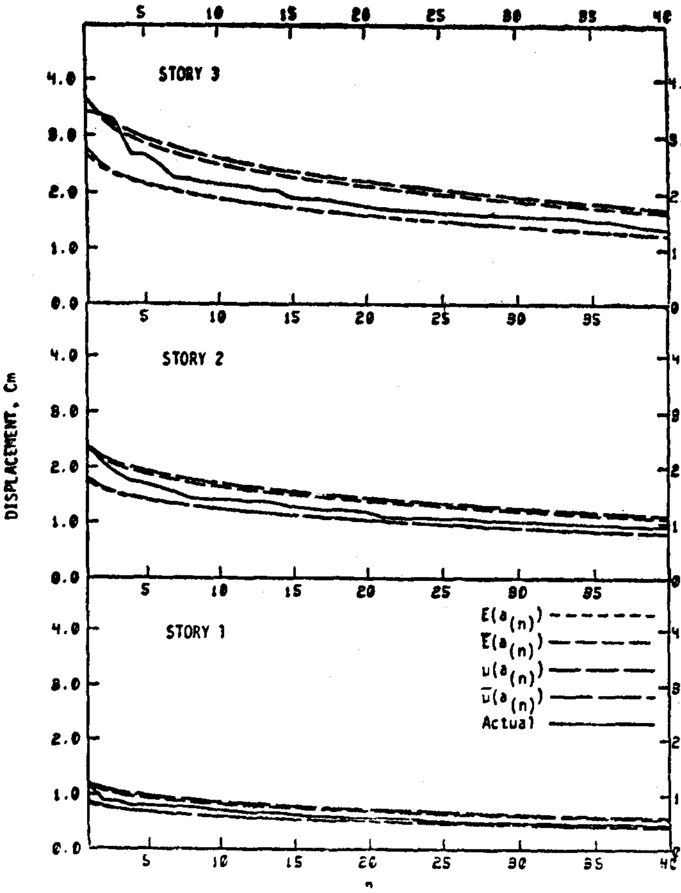

Figure 5. Theoretical and Actual Peaks of Displacement Response to Three Component Seismic Excitation (Kern Country Earthquake, July 21, 1952, IIA004).

account for the effects of nonstationatity only for the strong motion part of the response. As one goes to higher order peaks, the parameter $\overline{a}$ which is based on the r.m.s. value of the total response history, better represents the reality because $\overline{a}$ ϵ nds upon the weaker motion part of the response. Thus it again seems useful that the value of the parameter $\overline{a}$ should be defined in such a way so that it is equal to $\overline{a_E}$ or $\overline{a_\mu}$ for number of peaks corresponding to the strong motion part of the response, and that it smoothly goes to the value $\overline{a}$ for the higher order peaks. This procedure for modifying $\overline{a}$ has been explained in detail by Gupta and Trifunac (1987b,c), and is illustrated in Figure 7 for the case of excitation by the accelerograms recorded during the Parkfield Earthquake of June 27, 1966. Figure 8 illustrates the resulting theoretical estimates and the computed peaks of displacement for the same excitation. Comparison of the results

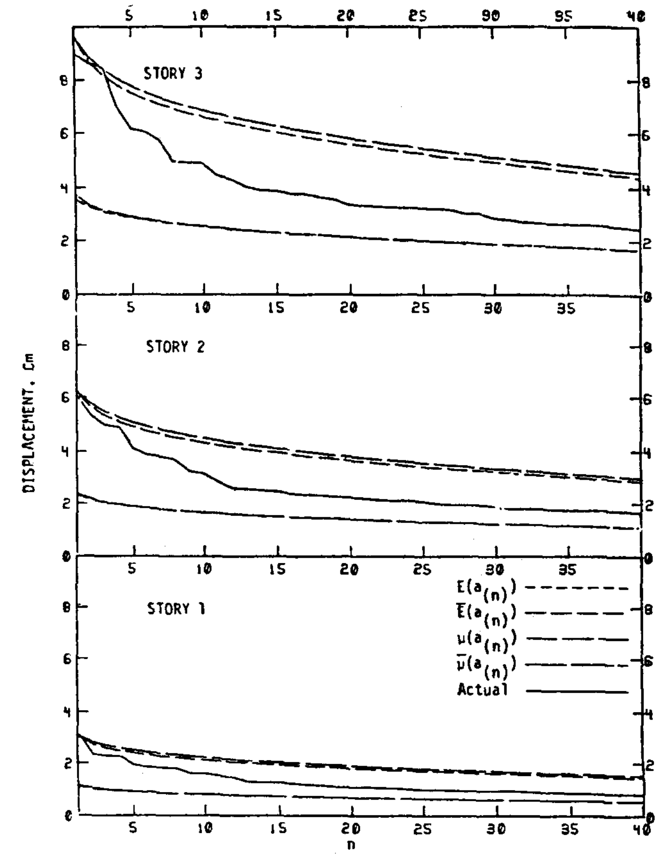

Figure 6. Theoretical and Actual Peaks of Displacement Response to Three Component Seismic Excitation (El Centro Earthquake, May 18, 1940, IIA001).

in Figures 4 and 8 shows the improvement in the theoretical estimates when the modified $\bar{a}$ is employed.

The design engineers are interested in knowing the values of the shear forces at various levels, the bending moments about various floors and the axial forces in the columns at various stories of the structure. The total shear force at certain level can be found by the vector sum of the shear forces due to two horizontal components of motion.

To find the total bending moments about various floors, first the bending moments due to shear forces along the x and y directions are found and then the resultants are found by the vector summation of the two components. The results on the bending moments for the Lytle Creek, 1979 earthquake, for example, are presented in Figures 9 through 11. Figure 9 presents the first 40 peaks of the bending moments in tetms of

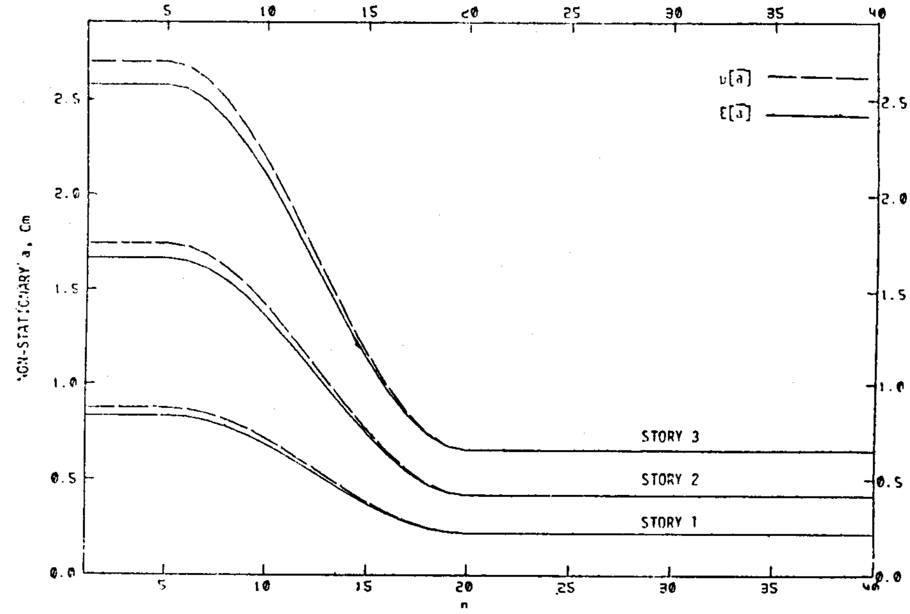

Figure 7. *Expected and Most Probable Non-Stationary rms Parameters for Displacement Response (Case of Figure 4.).*

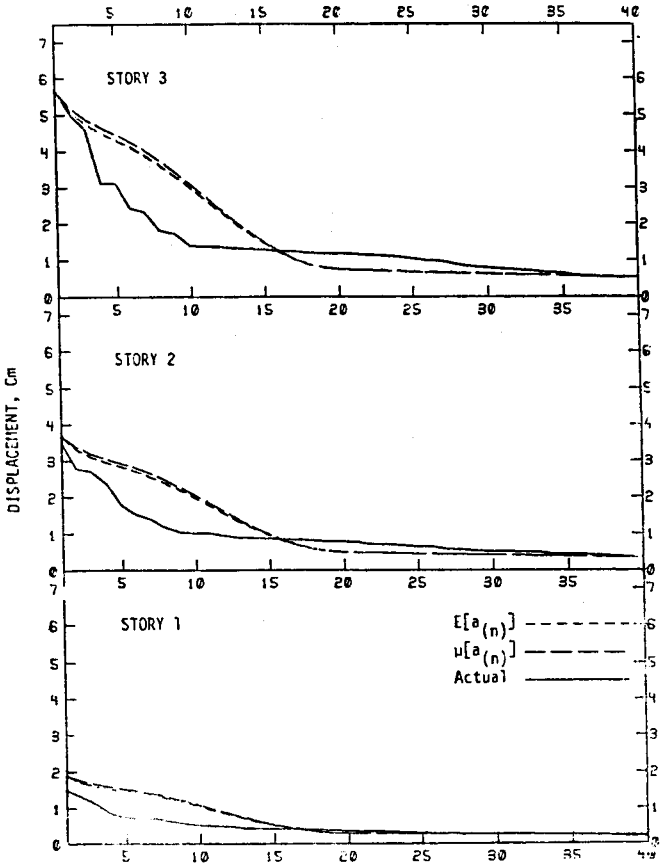

Figure 8. *Theoretical and Actual Peaks of Displacement Response (Case of Figure 4.)*

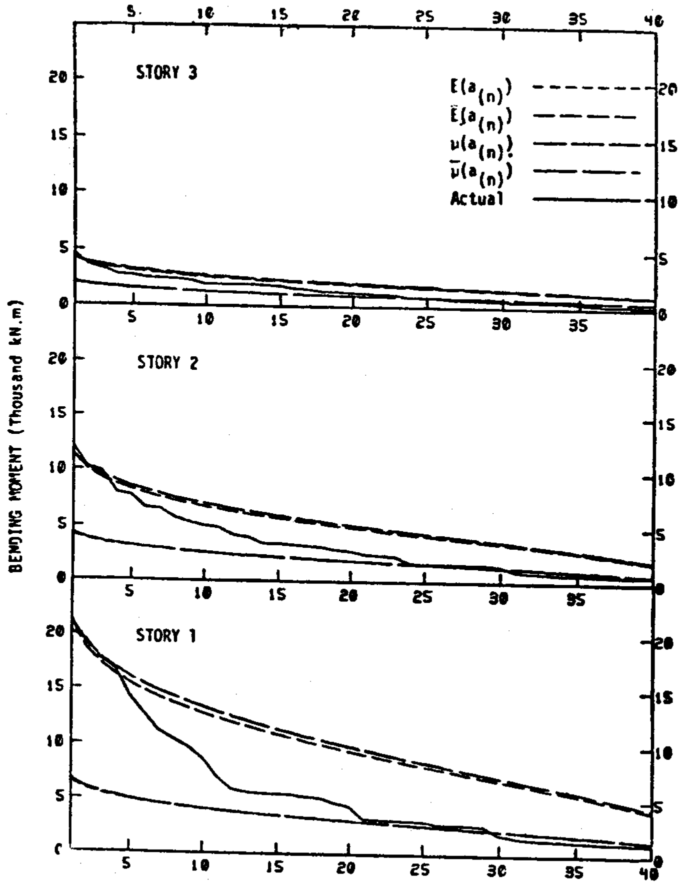

Figure 9. Theoretical and Actual Peaks of Bending moment Response due to Two Horizontal Components of the Earthquake, (Lytle Creek Earthquake, Sept. 12, 1979, IIW334).

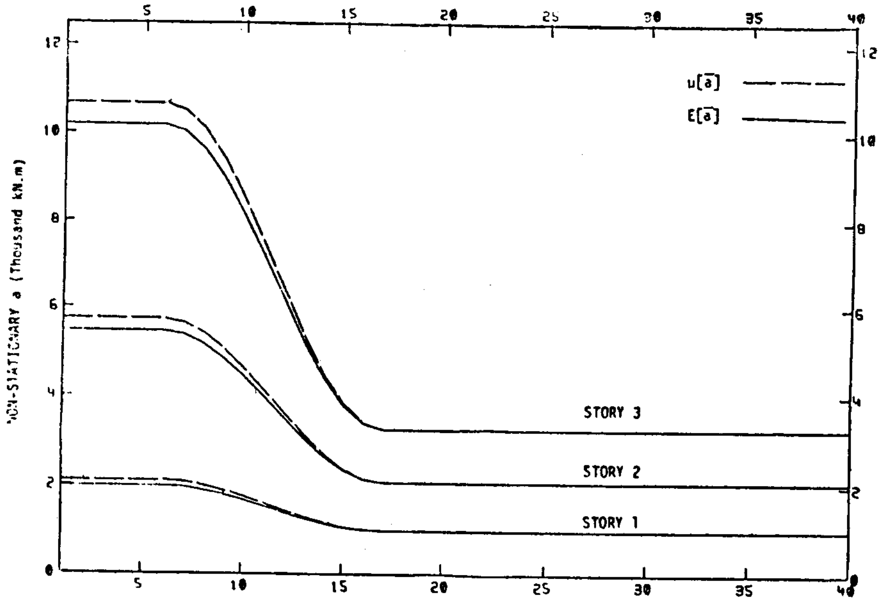

Figure 10. Expected and Most Probable Non-Stationary rms Parameters for Bending Moment Response (Case of Fig. 9).

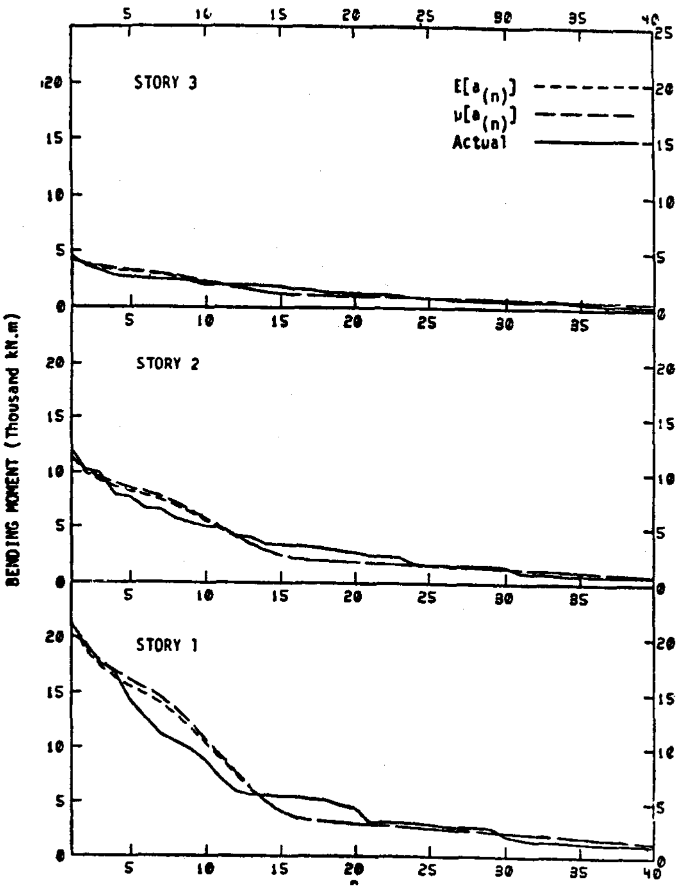

Figure 11. Theoretical and Actual Peaks of Bending Moment Response (Case of Figure 9).

$E(a_{(n)})$, $\overline{E(a_{(n)})}$, $\mu(a_{(n)})$ and $\overline{\mu(a_{(n)})}$. Figure 10 shows the nonstationary changes of $\bar{a}$ versus n. Figure 11 presents the comparison of actual and computed peaks of bending moment in terms of the non-stationary estimate of $\bar{a}$ in Figure 10, and the resulting improvement of theoretical predictions relative to those shown in Figure 9.

To compare the results of the proposed statistical theory for combining various components of the response with various conventional methods in literature, the results for the displacement response were found using SUM SRSS, NRLS and (MAX+30%) methods for spatial combination. These results and the expected and the most probable values of the highest peaks found from the statistical theory developed in this work, along with the time series solutions are presented in Tables 5 through 8. It is observed that the results from the present theory are in good agreement with the time series solutions. The values obtained from the SRSS and the(MAX+30%) methods are also in good agreement, however, as mentioned by Gupta and Trifunac (1987c) these methods are not able to provide the values of other higher orderpeaks of the response.

CONCLUSIONS

In this paper we have generalized the results of Gupta and Trifunac (1987a,b,c)to ayply for the estimation of the peak of the response of Moulti degree-of-freedom system for multi component seismic excitation. In contrast to some of the previous methods, which employ approximations to estimate the resulting peaks of the vector sums of the required responses (displacements, shear forces or bending moments), we have been able to formulate this problem exactly, if it can be assumed that the stationary approximations in time are still valid. Using appropriate corrections for the non-stationary nature of the respone to earthquake excitation and order statistics (Gupta and Trifunac, 1987c) we have shown how all peaks of the responce can be predicated. The method presented here offers important advanteges ralative to the commonly used responce spectrm superposition meteods, because it presents procedure for estimating all peaks, thus introducing the effects of duration of strong shaking into the statistics of the peaks of the response.

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NOTATION

The following symbols have been used in this work :

$\bar{a}$	r.m.s. amplitude of peaks of f(t)
a_{rms}	r.m.s. amplitude of f(t)
a_n	n^{th} local peak of f(t)
$a_{(n)}$	n^{th} order peaks of f(t) when peaks are arranged in decreasing order of amlitudes
a_x, a_y, a_z	components of ground acceleration
$\bar{a}_i$	parameter $\bar{a}$ for i^{th} story response
$\bar{a}^*$	nonstationary form of parameter $\bar{a}$
$\bar{a}_E$	modified r.m.s.amplitude of the expected values of peaks of f(t)
$(\overline{a_{E_i}})$; $(\overline{a_{E_i}})_{x,y,z}$	parameter $\overline{aE}$ for i^{th} story response; in x,y, or z direction
$(\bar{a})_{ij}$	parameter $\overline{aE}$ for i^{th} story rasponse due to j^{th} mode
$\bar{a}^*_E$	nonstationary form of parameter $\overline{a_E}$
$\overline{a_\mu}$	modified r.m.s. amplitude for most probable values of peaks of f(t)
$(\overline{a_{\mu i}})$; $(\overline{a_{\mu i}})_{x,y,z}$	parameter $\bar{a}$ for i^{th} story response; in x,y or z direction
$[A];[A]_{x,y,z}$	modal transformation matrix; for x,y or z direction
$E_{xi}(\omega), E_{yi}(\omega), E_{zi}(\omega)$	functions $E_x(\omega)$, $E_y(\omega)$ and $E_z(\omega)$ for the components of the response of i^{th} story
f(t)	random reaponse function
$f_x(t), f_y(t), f_z(t)$	response functions in x,y and z directions
$F(\omega)$	transform f(t)
$F_x(\omega), F_y(\omega), F_z(\omega)$	transforms of $f_x(t)$, $f_y(t)$ and $f_z(t)$
$[K];[K]_{x,y,z}$	stiffness matrix; for x,y or z direction
m_k	K^{th} order moment of $E(\omega)$
m_{ki}	K^{th} order moment of $E_i(\omega)$

m_{kij}	K^{th} order moment of $E_{ij}(\omega)$
$(m_{ki})_x, (m_{ki})_y, (m_{ki})_z$	K^{th} order moments of $E_{xi}(\omega)$, $E_{yi}(\omega)$ and $E_{zi}(\omega)$
$[M]$	mass matrix
N_i	total number of peaks in the i^{th} story response
T	total duration of the response
$\{\alpha\};\{\alpha\}_{z,y,z}$	mode participation vector; for x,y, or z direction
$\mu[a_{(n)}]$	most probable value of $a_{(n)}$
$\overline{\mu}[a_{(n)}]$	modified most probable value of $a_{(n)}$
ω, ω_n	circular frequency; natural frequency
$\{\omega\};\{\omega\}_{x,y,z}$	modal frequency vector; for x,y or z direction

Table 1

Parameters for Theoretical Distribution of Peaks of Response
Lytle Creak Earthquake, September 12, 1979, IIW334

Floor (1)	N (2)	ε (3)	$\bar{a}$ (4)	$\bar{a}_E$ (5)	$\bar{a}_\mu$ (6)
Resultant Displacement (Cm)					
1	39	.17	.8419	1.7271	1.8122
2	40	.21	.5463	1.1222	1.1779
3	51	.39	.2663	.5522	.5828
Resultant Shear Force (Kilonewton)					
1	57	.41	2.7280×10^2	5.4503×10^2	5.7505×10^2
2	42	.23	4.9879×10^2	9.8987×10^2	1.0405×10^2
3	50	.34	6.8390×10^2	1.3717×10^3	1.4468×10^3
Resultant Bending Moment (Thousand kN.m)					
1	57	.41	0.9978	1.9935	2.1033
2	47	.33	2.0794	5.4777	5.7484
3	49	.34	3.2529	10.1890	10.6900

Table 2
Parameters for Theoretical Distribution of Peaks of Response
Parkfield Earthquake, June 27, 1966, IIB037

Floor (1)	N (2)	ε (3)	$\bar{a}$ (4)	$\bar{a}_E$ (5)	$\bar{a}_\mu$ (6)
Resultant Displacement (Cm)					
1	78	.26	.6507	2 5771	2.6925
2	81	.30	.4237	1.6721	1.7476
3	110	.38	.2280	.8311	.8715
Resultant Shear Force (Kilonewton)					
1	124	.36	$2\ 4971 \times 10^2$	8.9713×10^2	9.3523×10^2
2	84	.33	3.9018×10^2	$1\ 5714 \times 10^3$	$1\ 6442 \times 10^3$
3	109	.36	5.9351×10^2	2.1911×10^3	2.2941×10^3
Resultant Bending Moment (Thousand KN m)					
1	124	.36	0 9133	3 2813	3 4208
2	104	.44	1 6944	8.7482	9 1334
3	108	.39	2.7538	16.1640	16 886

Table 3
Parameters for Theoretical Distribution of peaks of Response
Kern County (Taft) Earthquake, Ju]y 21, 1952, IIA004

Floor (1)	N (2)	ε (3)	$\bar{a}$ (4)	$\bar{a}_E$ (5)	$\bar{a}_\mu$ (6)
Resultant Displacement (Cm)					
1	117	.11	1 2063	1 5939	1 6624
2	119	.14	.7829	1 0353	1.0805
3	147	,34	.3739	5059	5344
Resultant Shear Force (Kilonewton)					
1	171	.41	3.5820×10^2	5.0080×10^2	5.2498×10^2
2	126	.17	6.7202×10^2	$9\ 1198 \times 10^2$	$9\ 5201 \times 10^2$
3	150	.32	$9\ 0549 \times 10^2$	1.2554×10^3	1.3116×10^3
Resultant Bending Moment (Thousand kN m)					
1	171	.41	1.3101	1 8318	1.9202
2	140	.28	2 7853	5 0492	5 2649
3	146	.30	4.3274	9 3829	9 7739

Table 4
Parameters for Theoretical Distribution of Peaks of Response
E1 Centro Earthquake, May 18, 1940, IIA001

Floor (1)	N (2)	ε (3)	$\bar{a}$ (4)	$\bar{a}_E$ (5)	$\bar{a}_\mu$ (6)
Resultant Displacement (Cm)					
1	120	.16	1.6126	4.1783	4.3574
2	123	.20	1.0460	2.7168	2 8344
3	145	.35	.4955	1.3056	1 3763
Resultant Shear Force (Kilonewton)					
1	162	.38	4.8734×10^2	1.1697×10^3	1.2350×10^3
2	125	.16	9.2619×10^3	2.1864×10^3	2.2787×10^3
3	142	.27	1.2352×10^3	2.9783×10^3	3.1379×10^3
Resultant Bending Moment (Thousand kN.m)					
1	162	.38	1.7825	4.2782	4.5170
2	135	.25	3.8279	12.0850	12.6070
3	139	.26	5 9214	22.5330	23.5040

Table 5

Comparison of Various Spatial Combination Methods

Lytle Creek Earthquake, September 12, 1979, IIw334

Floor	Components of Response			SUM	SRSS	NRLS	MAX + 30%	$\overline{E}(a_{(1)})$	$\overline{\mu}(a_{(1)})$	TIME SERIES
	x	y	z							
	Displacement Response (Cm)									
1	2.052	2 967	0.043	5.062	3.606	5.019	3.595	3 523	3.527	3.561
2	1.280	2.027	0.033	3.340	2.397	3.307	2.421	2.294	2.298	2.332
3	0.622	1.077	0.018	1.717	1.244	1 699	1.269	1.154	1.163	1.199

Table 6

Comparison of various Spatial Combination Methods
Parkfield Earthquake, June 27, 1966, IIB037

Floor	Components of Response			SUM	SRSS	NRLS	MAX + 30%	$\overline{E}(a_{(1)})$	$\overline{\mu}(a_{(1)})$	TIME SERIES
	x	y	z							
Displacement Response (Cm)										
1	4.044	4.145	0.063	8.252	5.791	8.189	5 377	5.670	5.674	5.715
2	2.441	2.664	0.051	5.156	3.613	5.105	3.412	3.688	3.694	3.515
3	1.120	1.237	0.028	2.385	1.669	2.357	1.581	1.885	1·896	1.478

Table 7

Comparison of Various Spatial Combination Methods

Kern County (Taft) Earthquake, July 21, 1952, IIA004

Floor	Components of Response			SUM	SRSS	NRLS	MAX + 30%	$\overline{E}(a_{(1)})$	$\overline{\mu}(a_{(1)})$	TIME SERIES
	x	y	z							
Displacement Response (Cm)										
1	2.080	3.287	0 058	5 425	3 890	5.368	3 928	3.663	3.667	3.414
2	1.334	2.128	0.048	3.510	2.512	3.463	2.543	2.382	2.386	2.352
3	0.658	1.056	0.028	1.742	1.244	1.715	1.262	1.182	1.199	1.161

Table 8

Comparison of Various Spatial Combination Methods

E1 Centro Earthquake, May 18, 1940, IIA001

Floor	Components of Response			SUM	SRSS	NRLS	MAX + 30%	$\overline{E}(a_{(1)})$	$\overline{\mu}(a_{(1)})$	TIME SERIES
	x	y	z							
	Displacement Response (Cm)									
1	3.497	8.834	0.264	12.595	9.505	12.341	9.962	9.618	9.628	8.987
2.	2.278	6.037	0.208	8.523	6.456	8.324	6.783	6.262	6.274	6.043
3	1.092	3.175	0.114	4.381	3.359	4.273	3.537	3.045	3.077	3.178