Propagation of Earthquake Waves in Buildings with Soft First Floor

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Abstract: Simplified response of buildings with a soft first floor and excited by propagating wave motion at the base are studied, so that the physical phenomena associated with phased input excitation can be characterized. Analytical solutions are obtained for two-dimensional continuous building models, neglecting the soil-structure interaction, and for monochromatic antiplane excitation. It is shown that the wave energy does not always propagate from the ground into the building, depending on the value of the horizontal phase velocity of the ground motion. There is a possibility that it propagates only into the "soft" first floor, which then acts as an "isolation layer" for the upper floors. However, this is at the expense of very large deformations of the columns of the first floor, that are not considered in the conventional analysis of buildings. Also, the out-of-phase motion of the base may excite torsional vibrations with large amplitudes. It is concluded that the design for P-delta effects in the soft first floor should not ignore those additional wave passage effects.

Introduction

Soviet Academy of Sciences (Krivelev 1986) suggested several continuous models to study the seismic action on long buildings and presented the shear-wave velocities for the equivalent continuous building models (in the range of 300–1,800 m/s). Todorovska et al. (1988) studied two-dimensional homogeneous continuous building models excited by propagating horizontally polarized shear waves (*SH*) waves along the base. They showed that the wave energy of ground motion is transmitted into the building when the horizontal phase velocity of incident waves is greater than the equivalent shear-wave velocity of the building. They also showed that the base motion, which is a propagating wave, excites the building to vibrate with a variety of symmetric and antisymmetric modes of vibration. The conventional dynamic analysis essentially eliminates the excitation of antisymmetric modes by the assumption of synchronous base excitation.

For architectural reasons, many buildings have the ground or the ground and the first several floors with stiffness smaller than the stiffness of the upper floors. These are the buildings that typically house stores or various passages and open spaces. Thus, one finds there are many "walls" and partitions made of glass rather than of concrete, steel, or masonry.

The method discussed in this paper and other similar analyses (Todorovska et al. 1988; Kojić et al. 1984) may then be used to understand additional causes of frequent failures of the columns of the "soft" first floors and very little damage of the upper parts of such buildings during strong earthquake shaking (Kojić et al. 1984). More important, the simple model presented here can demonstrate the relationships between the velocity of waves in the soil, the angles of wave incidence, transmission of energy into the building, and the case when the earthquake waves just deform the building at its base. Better understanding of these basic phenomena should go a long way in contributing to better and more rational, simplified design criteria.

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The Model

A refinement of the homogeneous model used by Todorovska et al. (1988) may be used to represent a building with a soft first floor (Fig. 1). It is a two-dimensional, semiinfinite, elastic plate with horizontal discontinuity in the material properties and prescribed displacement at level $z = H$. L and H represent the length and the height of the building; h_1 is the sum of heights of the upper floors; h_2 is the height of the first floor; and x, y and z are the spatial coordinates. μ_1 and β_1 and μ_2 and β_2 are the shear moduli and the shear-wave velocities of the upper floors and of the first floor, respectively. Those may be anisotropic, i.e., having different values in the x- and in the z-direction, for example, $\mu_{i,x}$, $\beta_{i,x}$, and $\mu_{i,z}$ and $\beta_{i,z}$, $i = 1, 2$. Neglecting the soil-structure interaction, such a model allows closed-form solutions for the displacement response to antiplane monochromatic motion at the base.

The model in Fig. 1 is a special case of a plate made up of M layers (Todorovska et al. 1988) when $M = 2$. The displacements in the layers, $v^{(i)}(x,z,t)$ $i = 1, 2$, have to satisfy the two-dimensional wave equations

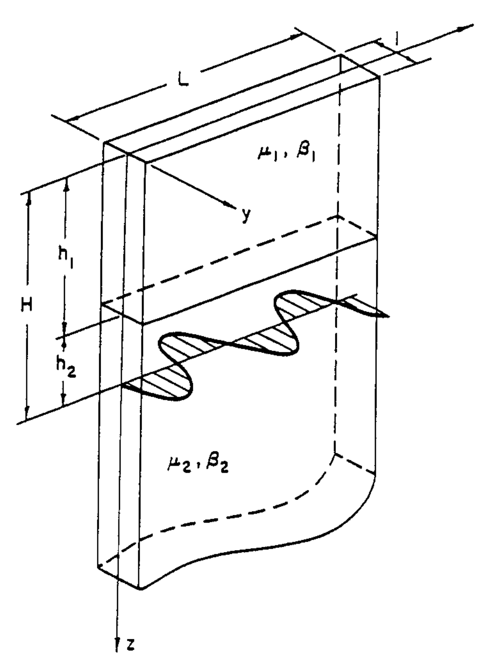

FIG. 1. Two-Dimensional, Continuous Model of Building with Soft First Floor. Soil-Structure Interaction Is Neglected and Model Is Excited to Vibrate by Imposed Displacement $e^{i\omega(t-x/c)}$ at $z = H$

$$\beta_{i,x}^2 \frac{\partial^2 v^{(i)}(x,z,t)}{\partial x^2} + \beta_{i,z}^2 \frac{\partial^2 v^{(i)}(x,z,t)}{\partial z^2} = \frac{\partial^2 v^{(i)}(x,z,t)}{\partial t^2} \qquad i = 1, 2 \tag{1}$$

The boundary conditions to be satisfied are

$$\tau_{xy}^{(i)} = 0 \quad \text{at} \quad x = 0, \qquad 0 \le z \le H, \qquad i = 1, 2 \tag{2a}$$

$$\tau_{xy}^{(i)} = 0 \quad \text{at} \quad x = L, \qquad 0 \le z \le H, \qquad i = 1, 2 \tag{2b}$$

and

$$\tau_{zy}^{(1)} = 0 \quad \text{at} \quad z = 0, \qquad 0 \le x \le L \tag{2c}$$

where $\tau_{xy}^{(i)} = \mu_{i,x}\partial v^{(i)}/\partial x$ and $\tau_{zy}^{(i)} = \mu_{i,z}\partial v^{(i)}/\partial z$ are the shear stresses in the ith layer. At the interface between the two media, the following continuity conditions have to be satisfied

$$v^{(1)}(x, h_1, t) = v^{(2)}(x, h_1, t) \tag{3a}$$

$$\tau_{xz}^{(1)}(x, h_1, t) = \tau_{xz}^{(2)}(x, h_1, t) \tag{3b}$$

In addition, the following displacement condition has to be satisfied

$$v^{(2)} = e^{i\omega[t-(x/c)]} \quad \text{at} \quad 0 \le x \le L, \qquad z = H \tag{3c}$$

This is the free-field displacement on the surface of the half-space when the incident wave has frequency ω and amplitude 1/2 and propagates with phase velocity c in the positive x-direction. ($c = v_s/\sin\theta$, where v_s is the shear-wave velocity of the soil and θ is the angle between the direction of propagation of the incident wave and the vertical direction).

The eigenfunction expansion of the displacement in the plate, v, is

$$v(x,z,t) = \sum_n C_n \cos\frac{n\pi x}{L} \frac{Z_n(z)}{Z_n(H)} e^{i\omega t} \tag{4}$$

where

$$Z_n(z) = \cos k_{z,n}^{(1)} z \qquad 0 \le z \le h_1 \tag{5a}$$

$$Z_n(z) = A_n^{(2)} \cos k_{z,n}^{(2)} z + B_n^{(2)} \sin k_{z,n}^{(2)} z \qquad h_1 \le z \le H \tag{5b}$$

is the nth shape function in the z-direction and

$$k_{z,n}^{(i)} = \frac{\beta_{i,x}}{\beta_{i,z}} \sqrt{\left(\frac{\omega}{\beta_{i,x}}\right)^2 - \left(\frac{n\pi}{L}\right)^2} \tag{6}$$

is its wave number in the ith layer. The coefficients $A_n^{(2)}$ and $B_n^{(2)}$ of the nth mode shape can be calculated from the following

$$A_n^{(2)} = \cos k_{z,n}^{(1)} h_1 \cos k_{z,n}^{(2)} h_1 + \frac{\mu_{1,z}}{\mu_{2,z}} \frac{k_{z,n}^{(1)}}{k_{z,n}^{(2)}} \sin k_{z,n}^{(1)} h_1 \sin k_{z,n}^{(2)} h_1 \tag{7a}$$

$$B_n^{(2)} = \cos k_{z,n}^{(1)} h_1 \sin k_{z,n}^{(2)} h_1 + \frac{\mu_{1,z}}{\mu_{2,z}} \frac{k_{z,n}^{(1)}}{k_{z,n}^{(2)}} \sin k_{z,n}^{(1)} h_1 \cos k_{z,n}^{(2)} h_1 \tag{7b}$$

The coefficients of the expansion C_n, $n = 0, 1, 2, \ldots$ derived from the displacement condition (Eq. 3c) are:

$$C_0 = \frac{1}{\frac{\omega L}{c}} \left[\sin\frac{\omega L}{c} + i\left(\cos\frac{\omega L}{c} - 1\right)\right] \tag{8a}$$

$$C_n = \frac{2}{L} \frac{\frac{\omega}{c}}{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n\pi}{L}\right)^2} \left[(-1)^n \sin\frac{\omega L}{c} + i\left((-1)^n \cos\frac{\omega L}{c} - 1\right)\right],$$

$$n \ge 1 \tag{8b}$$

when $\omega/c \ne m\pi/L$, $m = 1, 2, 3, \ldots$, and

$$C_0 = \frac{i}{m\pi}[(-1)^m - 1] = 0, \qquad m \text{ even} \tag{9a}$$

$$C_0 = \frac{i}{m\pi}[(-1)^m - 1] = \frac{-2i}{m\pi}, \qquad m \text{ odd} \tag{9b}$$

$$C_n = \frac{2}{L} \frac{i\frac{m\pi}{L}}{\frac{(m^2 - n^2)\pi^2}{L^2}} [(-1)^{n+m} - 1] = 0, \qquad (n + m) \text{ even}, n \ne m \tag{9c}$$

$$C_n = \frac{2}{L} \frac{i\frac{m\pi}{L}}{\frac{(m^2 - n^2)\pi^2}{L^2}} [(-1)^{n+m} - 1]$$

$$= \frac{-4im\pi}{\frac{(m^2 - n^2)\pi^2}{L^2}}, \qquad (n + m) \text{ odd}, n \ge 1 \tag{9d}$$

$$C_m = 1 \tag{9e}$$

if $\omega/c = m\pi/L$ for some integer m. In the limit when $\omega/c \to 0$, the coefficient $C_0 = 1$ and $C_n = 0$, $n \ge 1$. In the Eqs. 8 and 9 $i = \sqrt{-1}$.

Applied to the model of a "stiff" building with "soft" first floor, Eq. 6 implies that the wave numbers $k_{z,n}^{(1)}$ and $k_{z,n}^{(2)}$ will be real only for finite number of modes. For example, $k_{z,n}^{(1)}$ will be real for $n = 1, \ldots N_1$ and $k_{z,n}^{(2)}$ will be real for $n = 1, \ldots N_2$, ($N_1 \le N_2$), and imaginary for the rest of the modes. Consequently, the shape functions $Z_n(z)$ in the layers may be harmonic or exponential functions of z. Thus, there may exist modes that are harmonic functions of z in the soft layer and exponential functions in the upper part of the "building." Via such modes, the wave energy from the ground will be "carried" only into the soft first floor.

The building will be in resonance with excitation when the denominator in Eq. 4 is

$$A_n^{(2)} \cos k_{x,n}^{(2)} H + B_n^{(2)} \sin k_{x,n}^{(2)} H \to 0 \tag{10}$$

When this happens, the displacements of the model become unbounded.

Results and Analysis

For convenience, the input motion can be expressed in terms of the dimensionless parameter $\eta = L/cT$ and the ratio $c/\beta_{2,x}$. η is the dimensionless frequency of excitation, or the number of apparent wave lengths in the x-direction of the base motion contained in one length of the building. The ratio $c/\beta_{2,x}$ is associated with the transfer of the wave energy from the ground into the building.

Using the analogy with ray theory of seismic waves, the transfer of transcient energy of *SH* waves from the ground into the building can be illustrated for the layered building model as in Fig. 2. In this figure, β_1, β_2, and β_s are the shear-wave velocities of the upper part of the building, of the first floor and of the soil, respectively. As noted previously, β_1 and β_2 may have different values in the x- and in the z-direction. γ is the incident angle, and α and β are the angles of refraction in the "soft" and in the "hard" layer. Then, from the Snell's law

$$c = \frac{\beta_s}{\sin\gamma} = \frac{\beta_{2,x}}{\sin\alpha} = \frac{\beta_{1,x}}{\sin\beta} \qquad (11)$$

α will be real, i.e., the wave energy will be "propagated" into the soft layer if $c/\beta_{2,x} \geq 1$. If $c/\beta_{2,x} < 1$, then α will be pure imaginary and thus β will also be pure imaginary (since $\beta_{1,x} > \beta_{2,x}$) and the wave energy will not be "propagated" into the building. If $c/\beta_{2,x} \geq 1$, whether the wave energy will or will not be propagated into the upper part of the building depends on the value of the ratio $\beta_{2,x}/\beta_{1,x}$. The energy will propagate into the upper part of the building only if $c/\beta_{1,x} = (c/\beta_{2,x})(\beta_{2,x}/\beta_{1,x}) \geq 1$. When $c/\beta_{1,x} < 1$, the displacements in the "hard" layer will be exponentially decaying towards

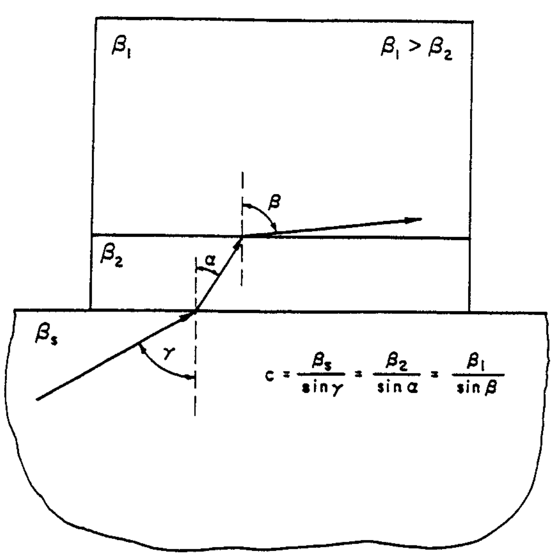

FIG. 2. Illustration of Transient Transfer of Wave Energy from Ground into Building with Soft First Floor

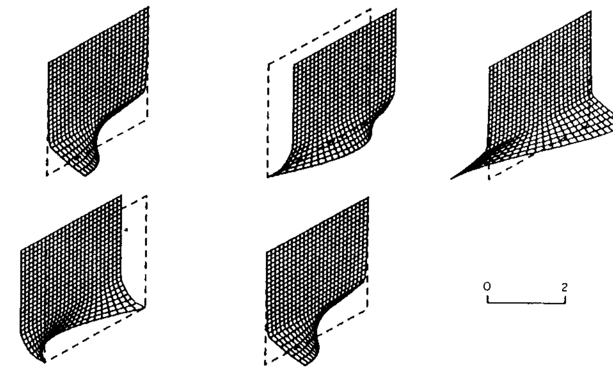

FIG. 3. (*a*) Displacement Response of Long Building, with Height Comparable to Its Length (H/L = 1) and with Soft First Floor (h_2/H = 0.25 and β_2/β_1 = 0.25), for Propagating *SH*-Wave (η =0.5 and c/β_2 = 0.05), at Times t = 0, $T/4$, $T/2$, $3T/4$, and T. Wave Energy Does not "Propagate" into Building, as Can be Seen from Exponential Displacements along Vertical

the top of the building, and it will vibrate as a "rigid box" welded to the "soft" first floor.

Examples of different situations of energy transfer that may occur during the passage of a monochromatic wave under the buildings with a soft first floor are shown in Figs. 3(*a* and *b*) and 4(*a* and *b*). Figs. 3(*a*) and 4(*a*) illustrate the displacements of the model with "soft" first floor, while Figs. 3(*b*) and 4(*b*) illustrate the displacements of the homogeneous model having the same shear-wave velocities, β_x and β_z, as the soft first floor. The solid lines represent the displacements of a vertical cross section of the building models at times t = 0, $T/4$, $T/2$, $3T/4$, and T, where $T = 2\pi/\omega$ is the period of the ground motion. The dashed lines represent the undeformed

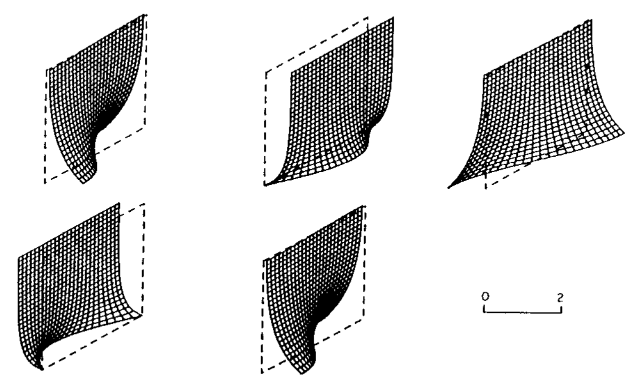

FIG. 3. (*b*) Displacement Response of "Homogeneous" Building of Same Size as Building in Fig. 3(*a*), with Shear Wave Velocity Same as Shear Wave Velocity of First Floor of Building in Fig. 3(*a*) and Subjected to Same Ground Motion

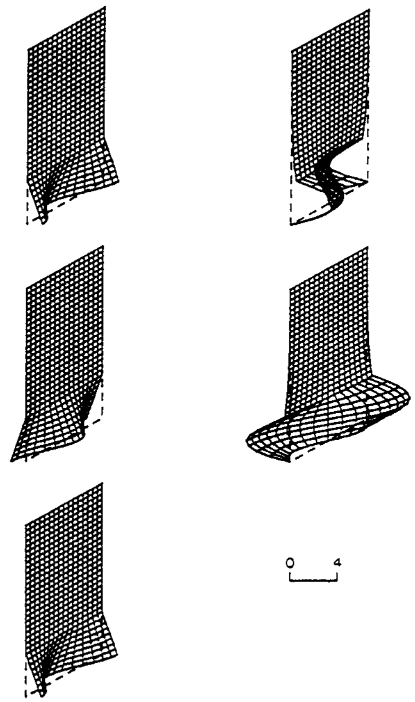

FIG. 4. (*a*) Displacement Response of Long and Tall Building (H/L = 2), with Soft First Floor (h_2/H = 0.25 and β_2/β_1 = 0.25), for Propagating Wave (η = 1 and c/β_2 = 1), at Times t = 0, $T/4$, $T/2$, $3T/4$, and T. Wave Energy "Propagates" Only into First Floor, but not into Upper Floors of Building

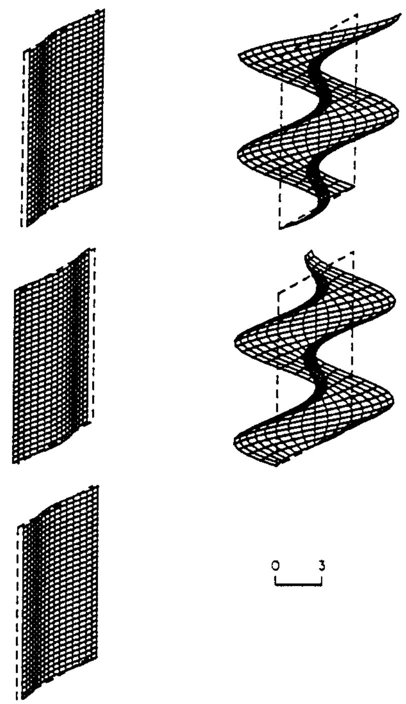

FIG. 4. (*b*) Displacement Response of "Homogeneous" Building, of Same Size as Building in Figs. 4(*a*) with Same Shear Wave Velocity as in First Floor of Building in Fig. 4(*a*) and Subjected to Same Ground Motion. Wave Energy Propagates Through Whole Building

cross section. The motion at the base of the model is of unit amplitude and the scale in the right is in the same units as the input base motion. In these examples the height of the first floor is 1/4 of the total height of the building. Both layers are assumed to be isotropic, i.e., $\beta_{i,x} = \beta_{i,z} = \beta_i$, i = 1, 2.

Fig. 3(*a*) illustrates the situation when the wave energy is propagating neither into the "soft" nor in the "hard" part of the building. The upper part of the building has four times larger shear-wave velocity than the "soft" first floor. The motion at the base has apparent wave length in the x-direction twice longer than the length of the building (i.e., η = 0.5) and c/β_2 = 0.05. For the homogeneous model in Fig. 3(*b*) also, c/β = 0.05. The comparison between Eqs. 3*a* and 3*b* shows that increasing the stiffness of the upper floors decreases the relative displacements of the upper part of the building, but increases the relative displacements of the first floor.

Fig. 4(*a*) illustrates the case when the energy propagates through the first

floor, but does not propagate into the upper floors of the building. The building in this figure vibrates mostly with modes that are harmonic functions in the first floor and hyperbolic functions at upper floors. The base motion is a propagating wave in the positive x-direction with c/β_2 = 1. The value of η is 1 and the value of H/L is 2. The ratio β_1/β_2 = 4. There is a significant difference between the displacement patterns in Figs. 3(*a*) and (*b*). The relative displacements of the upper part of the buildings with the soft floor are small, smaller than the displacements of the corresponding "homogeneous" building. However, the "soft" first floor experiences large relative displacements and its columns and shear walls become candidates for where the failure of the building may occur. Fig. 4(*a*) shows large "torsional" deformations of the first floor, while the upper part of the building hardly moves. The first floor vibrates as if it were fixed at its upper edge. For higher value of η and for "long" buildings, the whole upper part of the building would rotate almost as a rigid body. This causes (Todorovska et al. 1988) additional large overall torsional deformations and stresses in the first floor.

The "soft" first floor does not always act as an "isolator" for the upper part of the building and for horizontally propagating seismic excitation the wave energy can propagate into the whole building. Such buildings will vibrate with modes that are harmonic functions both in the first floor and in the upper parts of the buildings (Todorovska et al. 1988). Real buildings are more flexible in the vertical direction than in the horizontal direction. Taking the upper part of the building to be anisotropic will increase the number of the apparent wave lengths in the vertical direction contained in the displacement pattern of the upper part of the building.

The analysis presented in this paper may suggest, for example, some additional features of the response of the former Imperial County Services Building in El Centro (Kojić et al. 1984). Above the second floor, this building had shear walls at the east and west ends, while at the first floor it had only four concrete panels. This reduced the resistance of the first floor in the North-South direction. During the Imperial Valley California earthquake of 1979, the columns of the first floor experienced large displacements in the East-West direction and were badly damaged. The upper part of the building experienced much smaller deformations. The building was instrumented and according to the recorded motions during this earthquake, the upper part of the building vibrated like a "rigid" box placed over flexible columns.

Conclusions

If the *SH* ground motion under long buildings with soft first floor has finite phase velocities in the horizontal direction, then it may happen that: (1) The wave energy does not propagate from the ground into the building; (2) it propagates only into the "soft" first floor; and (3) it propagates into the whole building. In the case where it propagates only into the soft first floor, the first floor may experience very large displacements while the displacements of the upper part of the building are small and exponentially decaying towards the top of the building. The first floor may then act as an "isolator" for the upper part of the building. Of course, this will occur at the expense of large deformations of the structural members in the first floor. These large displacements cannot be and are not considered in conventional earthquake response analyses that are equivalent to the case for nearly vertical wave incidence, when the wave energy always propagates into the whole building. The action of the "soft" first floor as an "isolator" in the aforementioned sense depends on the ratios between the x-phase velocities of the ground motion and the equivalent shear wave velocities of the first floor and of the upper part of the building.

In conventional earthquake response analyses, which ignore the propagating wave effects ($\eta = 0$), selecting the soft first floor may reduce the overall design loads on the building if this lengthens the fundamental period of the response beyond the range of periods with largest response spectrum amplitudes. However, under those conditions, our analyses show the nature of the additional deformations of the soft floor that should be included in the selection of the optimum strength and ductility of the first floor structural members.

One-dimensional models of buildings, which are analyzed either by some response spectrum superposition technique or by integration of the differential equation of motion, consider only the contribution of the ground motion close to the natural frequencies of the system. While this may be sufficient for evaluation of the envelopes of maximum displacements, shear forces, and bending moments at upper floors (Gupta and Trifunac 1987, 1988), estimation of and design for P-delta effects in the soft first floor should not ignore the additional wave passage effects, as discussed in this paper.

This analysis also shows that another important consequence of the propagating wave excitation of long buildings is the contribution of antisymmetric modes to the overall response. The design approach, which considers synchronous excitation of the base, ignores this important part of the response and thus may result in underestimation of shear and moment envelopes for design.

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