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A NOTE ON CONTRIBUTION OF TORSIONAL EXCITATION TO EARTHQUAKE RESPONSE OF SIMPLE SYMMETRIC BUILDINGS

by

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ABSTRACT

It is shown that the torsional excitations may cause significant increase in maximum shear forces and bending moments in the structural elements which are located at the periphery of the buildings. Earlier studies have associated the torsional response with the asymmetry of the structure, i.e. the lack of coincidence between the centers of resistance and mass of the structures. The present study shows how the torsional response of symmetric structures arises from the torsional accelerations of strong ground motion associated with SH and Love-waves.

Introduction

Free-field earthquake ground motion has three translational and three rotational components. Though at present there are practically no recorded strong motion rotational accelerograms, using the theory of elastic wave propagation in layered ground, it can be shown that the rotational components are related to the translational components of ground motion (Trifunac, 1982). Therefore, one can generate the torsional accelerograms from the recorded components of translational acceleration (Lee and Trifunac, 1985, 1987).

It has been recognized by many investigators that in addition to translational excitation, the torsional excitation may contribute significantly to the overall response of structures under strong motion earthquake excitation. Shibata et al. (1969) showed that an unsymmetric building subjected to idealized half-sine ground excitation fails predominantly due to torsion. Hart et al. (1975) studied the torsional response of buildings during ambient as well as earthquake vibrations. They

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concluded that the torsional response of buildings, is influenced significantly by the torsional component of ground motion, in addition to the building asymmetry. Abdel-Ghaffar (1984) has highlighted the importance of studying torsional excitation of long-span suspension bridges. Many other works (e.g. Elms, 1976; Humar and Awad, 1983; Kobori and Shinozaki, 1973; Luco, 1976; Prasad and Jagdish, 1977; Reinhorn et al., 1977; and TSO, 1975) have also described the importance of studying torsional response of structures.

Even for dynamically symmetric structures,most of the building codes recommend the torsional seismic force in the form of accidental eccentricity only, but do not mention the torsional ground motion. Newmark (1969) was the first to propose an approximate analysis for torsion in symmetrical buildings, by using horizontally travelling seismic shear waves of constant shape and velocities. He found that maximum contribution to stresses from torsion comes to the corner elements and to the end shear walls. He also introduced the concept of accidental eccentricity. His recommendations are valid mainly for simple structures which primarily vibrate in their fundamental modes.

For multi-degree-of-freedom systems, the simplest and the most commonly used practice for analyzing the structural response under translational excitation is the use of some response spectrum superposition method. Therefore, it is natural to extend the spectrum approach for analyzing the torsional responses also.

Amini and Trifunac (1981, 1985) and Gupta and Trifunac (1987a, b, c, d) have developed a new statistical approach for response spectrum superposition, which gives excellent agreement with the time history solutions for the case of translational response. This approach can also provide the amplitudes of all the significant peaks of the response which is not possible from other conventional methods. It will be shown here that the statistical method of spectrum superposition can be used for studying the torsional responses also. An example will be presented to show the contribution of torsional excitation to the total maximum response of structures under earthquake excitation. Effects of increasing the dimensions of the floors, and varying the minimum shear wave velocity in the layered ground, on the additional contribution due to free-field torsional excitations, will be investigated. This will provide an idea about the conditions under which torsional response is significant.

Torsional Spectra

To use the statistical method of spectrum superposition, to find the torsional response of a structure, one should have the Fourier and the response spectra of the input torsional ground motion. Several investigators have proposed analytical procedures for finding the torsional ground motions and their spectra, directly from the translational ground motion (e.g. Lee and Trifunac, 1985).

Hart et al. (1975) constructed the time history of the freefield rotational motion by differentiating two orthogonal horizontal components of ground motion. Following the ideas of Newmark (1969), Nathan and Mackenzie (1975) found the time history of rotational motion from the two translational components by using the differences between the translations at opposite edges of the foundation, and thus they accounted for the averaging effects of the foundation size. In these two methods for generating the rotational accelerograms, translational time histories have to be differentiated. Tso and Hsu (1978) presented a scheme for computing the rotational spectra which eliminates differentiation of the acceleration records. Rutenberg and Heidebrecht (1985) used directly the response spectra of translational accelerograms for computing the torsional and rocking response spectra.

The above methods for computing rotational ground motions and their response spectra assume horizontally travelling seismic waves of constant shapes and velocities, and thus, neglect the dispersive nature of seismic waves. In this paper we shall find the torsional spectra by using a procedure based on the studies by Lee and Trifunac (1985). They have shown that for torsion, ψ_{13} (Figure 1) is related to the translational component u_3 by.

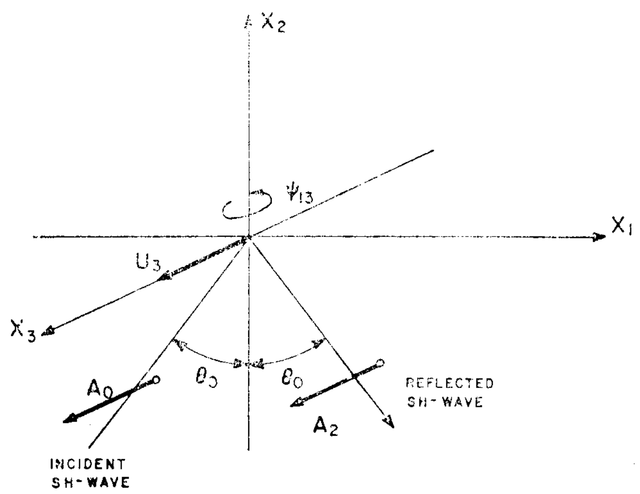

Fig 1. Coordinate system for incident SH-wave(from) Trifunac, 1982).

$$\psi_{13} = -\frac{i\omega}{2c}u_3 \qquad (1)$$

In this expression, ω is the circular frequency and c is the phase velocity of SH or Love waves. $i=\sqrt{-1}$ and u_3 is the surface displacement associated with incident SH or Love waves.

If the m-th mode of the surface wave contributes an amplitude $A_m(\omega_n)$ to the total translational spectrum amplitude $A(\omega_n)$ at frequency ω_n, and if $\psi_m(\omega_n)$ is the corresponding contribution to the total torstional spectrum amplitude $\psi(\omega_n)$, then for torsional spectra one can write (from equation (1))

$$\psi_m(\omega_n) = \frac{-i}{2}\;\frac{\omega_n}{C_m(\omega_n)}\;A_m(\omega_n), \qquad (2)$$

where $C_m(\omega_n)$ is the phase velocity of the m-th mode of Love surface wave at frequency ω_n. Summing over all the modes at frequency ω_n, the total torsional spectrum amplitude is given by (Lee and Trifunac, 1985)

$$|\psi_{13}(\omega_n)| = \frac{\omega_n}{2}\left|\sum_{m=1}^{M} A_m(\omega_n)/C_m(\omega_n)\right| \qquad (3)$$

As $\omega_n \to 0$, the only mode present in a layered medium is the first mode, and its phase velocity approaches β_{max}, the largest (normally for the half space) shear wave velocity in the layered model, Thus

$$\frac{|\psi_{13}(\omega_n)|}{|A_3(\omega_n)|} = \frac{\omega_n}{2\beta_{max}}, \text{ as } \omega_n \to 0 \qquad (4)$$

where $|A_3(\omega_n)| = \left|\sum_{m=1}^{M} A_m(\omega_n)\right|$ is the total spectral amplitude of u_3, at frequency ω_n (Lee and Trifunac, 1985).

At the high frequency end, the phase velocities of all the modes of surface waves approach β_{min}, the minimum (usually top layer) shear wave velocity in the layered model. Thus

$$\frac{|\psi_{13}(\omega_n)|}{|A_3(\omega_n)|} = \frac{\omega_n}{2\beta_{min}}, \text{ as } \omega_n \to \infty \qquad (5)$$

From equations (4) and (5) it is seen that the velocity representing the overall trend of the ratios of rotational to translational spectra is not constant. It changes from β_{min} (for $\omega\to\infty$) to β_{max} (for $\omega\to 0$). A good approximation for the variation of velocity with frequency may be the harmonic mean of the phase velocities of various modes available at a particular frequency. Lee and Trifunac (1985) have shown that approximating the behavior of $\log_{10}[|\psi|/|A|]$ versus $\log_{10}$[Period] by a straight line joining the asymptotic levels, is also a good approximation. Therefore, in the present study, we shall compute the rotational spectra by using their approximation.

Probabilistic Theory for Torsional Response

Once the Fourier and the response spectra for free-field torsional ground motion are known, and if it is assumed that the effects of soil structure interaction can be ignored, it is straightforward to extend the results of Gupta and Trifunac (1987b) to the torsional response of a multi-degree-of-freedom structure. For this purpose, a multi-story structure is modeled by masses concentrated at the floors and connected with massless torsional springs and dashpots as shown in Figure 2. For the i-th story of the structure, J_i is the mass moment of inertia of the floor about vertical axis through its center of mass, k_{iT} is the stiffness of the torsional spring, and c_{iT} is the torsional viscous damping. If $\ddot{\theta}_T$ is the input torsional ground acceleration and ψ_i the relative rotation of i-th floor, then the system of equilibrium equations for different floors can be written as

$$[J]\{\ddot{\psi}\} + [C_T]\{\dot{\psi}\} + [K_T]\{\psi\} = -\ddot{\theta}_T[J]\{I\} \qquad (6)$$

In this equation, $\{\psi\}$ is the relative torsional displacement vector, $\{I\}$ is a unity vector, and matrices $[J]$, $[C_T]$, and $[K_T]$ are defined as follows

$$[J] = \begin{pmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_n \end{pmatrix} \qquad (7)$$

$$[C_T] = \begin{pmatrix} C_{1T} & -C_{1T} & 0 & \\ -C_{1T} & C_{1T}+C_{2T} & -C_{2T} & \\ 0 & -C_{2T} & C_{2T}+C_{3T} & \\ & & & \ddots \end{pmatrix} \quad \text{and} \qquad (8)$$

$$[K_T] = \begin{pmatrix} K_{1T} & -K_{1T} & 0 & \\ -K_{1T} & K_{1T}+K_{2T} & -K_{2T} & \\ 0 & -K_{2T} & K_{2T}+K_{3T} & \\ & & & \ddots \end{pmatrix} \qquad (9)$$

Equations (6) can be uncoupled by using the transformation,

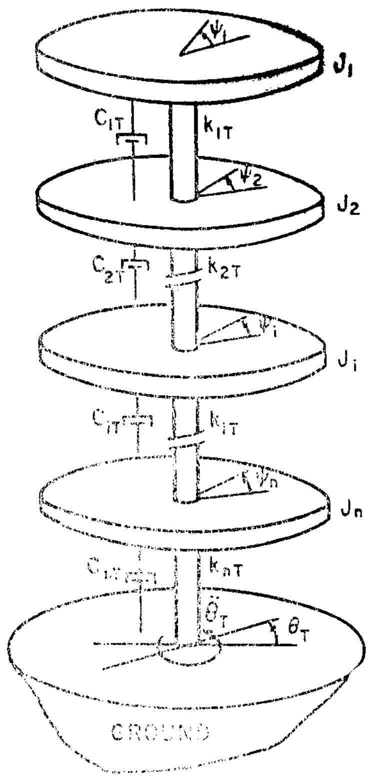

Figure 2. A Multi-Degree-of-Freedom Torsional System.

$$\{\psi\}=[A_T]\{\phi\}, \tag{10}$$

where $[A_T]$ is the matrix with its columns consisting of torsional mode shape vectors. Applying this transformation, equation of motion for the j-th torsional mode becomes

$$\ddot{\phi}_j+2\zeta_{jT}\omega_{jT}\dot{\phi}_j+\omega_{jT}^2\phi_j=-\ddot{\theta}_T\alpha_{jT} \tag{11}$$

where,

$$\alpha_{jT}=\frac{\{A_T^j\}^T[J]\{I\}}{\{A_T^j\}^T[J]\{A_T^j\}}, \tag{12}$$

is the participation factor for this mode of vibration. ω_{jT} is the modal torsional frequency and ζ_{jT} is the fraction of critical damping for the j-th mode.

From equation (11) the transfer function for the response of the i-th floor due to the j-th torsional mode of vibration can be written as

$$H_{ij}(\omega)=\frac{-A_{ijT}\alpha_{jT}}{\omega_{jT}^2-\omega^2+2i\zeta_{jT}\omega_{jT}\omega}. \tag{13}$$

Similar to the case of the translational response (Amini and Trifunac, 1985; Gupta and Trifunac, 1987d), one can write the delta function approximation for the above transfer function as

$$|H_{ij}(\omega)|^2=\frac{\pi A_{ijT}^2\alpha_{jT}^2}{4\zeta_{jT}\omega_{jT}^3}\delta(\omega-\omega_{jT}). \tag{14}$$

Thus the contribution of the j-th mode to the energy density function of the orsional displacement response at the i-th floor of the structure can be written as

$$E_{ij}(\omega)=\frac{1}{T}\,\frac{A_{ijT}^2\alpha_{jT}^2\bar{\Theta}_T^2(\omega_{jT})}{4\zeta_{jT}\omega_{jT}^3}\delta(\omega-\omega_{jT}), \tag{15}$$

where $\bar{\Theta}_T$ is the average value of the Fourier spectrum amplitudes, Θ_T, of the torsional accelerogram $\ddot{\theta}_T$, over the frequency band of width $\pi\zeta_{jT}\omega_{jT}$ centered at ω_{jT}. Then the energy density of the total torsional displacement response at the i-th floor of the structure is given by

$$E\psi_i(\omega)=\sum_{j=1}^{n}E_{ij}(\omega)=\frac{1}{T}\sum_{j=1}^{n}\frac{A_{ijT}^2\alpha_{jT}^2\bar{\Theta}_T^2(\omega_{jT})}{4\zeta_{jT}\omega_{jT}^3}\delta(\omega-\omega_{jT}). \tag{16}$$

The inertial torques and the base torques for different levels of the structure are found from the following expressions

$$\{\tau(t)\}=[K_T]\{\psi(t)\}, \tag{17}$$

and

$$\{T(t)\}=[S]\{\tau(t)\}, \tag{18}$$

where matrix $[S]$ has ones at and below the diagonal and zeros above.

Equation (17) can also be written as

$$\{\tau(t)\} = [J][A_T][\omega_T^2]\{\phi(t)\}, \tag{19}$$

where matrices $[J]$, $[A_T]$ and $\{\phi(t)\}$ have been defined before. Matrix $[\omega_T^2]$ is given by

$$[\omega_T^2] = \begin{pmatrix} \omega_{1T}^2 & & & 0 \\ & \omega_{2T}^2 & & \\ & & \ddots & \\ 0 & & & \omega_{nT}^2 \end{pmatrix} \tag{20}$$

Using expression (19) the base torque $T_i(t)$ at the i-th level becomes,

$$\begin{aligned} T_i(t) = & (J_1A_{11T} + J_2A_{21T} + \cdots\cdots + J_iA_{i1T})\omega_{1T}^2\phi_1(t) \\ & + (J_1A_{12T} + J_2A_{22T} + \cdots\cdots + J_iA_{i2T})\omega_{2T}^2\phi_2(t) \\ & + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & + (J_1A_{1nT} + J_2A_{2nT} + \cdots\cdots + J_iA_{inT})\omega_{nT}^2\phi_n(t). \end{aligned} \tag{21}$$

From this expression, the contribution of the j-th torsional mode to the base torque at the i-th level is given by

$$T_{ij}(t) = (J_1A_{1jT} + J_2A_{2jT} + \cdots\cdots + J_iA_{ijT})\omega_{jT}^2\phi_j(t). \tag{22}$$

The transfer function for this base torque response can be obtained by replacing $\phi_j(t)$ by its transfer function H_{ij}, given by equation (13). Then using the delta function approximation similar to that of equation (14) the energy density of the base torque response at the i-th floor, due to the j-th mode of vibration is given by

$$E_{ij}(\omega) = \frac{1}{T}\left[\frac{(J_1A_{1jT} + J_2A_{2jT} + \cdots\cdots + J_iA_{ijT})^2\alpha_{jT}^2\bar{\Theta}_T^2(\omega_{jT})\omega_{jT}}{4\zeta_{jT}}\right]\delta(\omega - \omega_{jT}). \tag{23}$$

The total energy density function of the base torque response at the i-th level is given by

$$ET_i(\omega) = \frac{1}{T}\sum_{j=1}^{n}\frac{(J_1A_{1jT} + J_2A_{2jT} + \cdots\cdots + J_iA_{ijT})^2\alpha_{jT}^2\bar{\Theta}_T^2(\omega_{jT})\omega_{jT})}{4\zeta_{jT}}\delta(\omega - \omega_{jT}). \tag{24}$$

Knowing the energy density spectra $E\psi_i(\omega)$ and $ET_i(\omega)$ for the torsional displacement and the base torque responses at the i-th level of the structure, one can compute the moments of interest; viz., m_{0i}, m_{2i} and m_{4i} (Amini and Trifunac, 1981; Gupta and Trifunac, 1987a). Then the scaling parameters of the distribution function for the peaks of the response are

$$\bar{a}_i = (2m_{0i})^{1/2}$$

$$\varepsilon_i = \frac{m_{0i}m_{4i} - m_{2i}^2}{m_{0i}m_{4i}},$$

and

$$N_i = \frac{T}{2\pi}\left(\frac{m_{4i}}{m_{2i}}\right)^{1/2}. \tag{25}$$

The definitions of $\bar{a}_i$, ε_i and N_i and their properties are discussed in detail in Amini and Trifunac (1981, 1985) and Gupta and Trifunac (1987a, d). One can also find the modified values $\bar{a}_E$ and $\bar{a}_\mu$ for the parameter $\bar{a}$, using the response spectrum of $\ddot{\theta}_T(t)$, by applying the procedure outlined by Gupta and Trifunac (1987b, d).

From the energy density, $E_{ij}(\omega)$, of the response at the i-th level due to the j-th torsional mode, one can find its moments m_{0ij}, m_{2ij} and m_{4ij}. From these the parameters $\bar{a}_{ij}$, ε_{ij} and N_{ij} of the statistical distribution of the peaks of the response at the i-th level in the j-th mode of vibration can be calculated. Using these parameters, the expected and the most probable values, $E(a_{(1)})_{ij}$ and $\mu(a_{(1)})_{ij}$, of the largest peak (first order peak) normalized by $\bar{a}_{ij}$ can be computed. Then, two new values, $(\bar{a}_E)_{ij}$ and $(\bar{a}_\mu)_{ij}$, of the parameter a_{ij} can be defined for the torsional displacement response as (Amini and Trifunac, 1985; Gupta and Trifunac, 1987d)

$$(\bar{a}_E)_{ij} = \frac{(\psi_{ij})_{max}}{E(a_{(1)})_{ij}}, \text{ and } (\bar{a}_\mu)_{ij} = \frac{(\psi_{ij})_{max}}{\mu(a_{(1)})_{ij}}, \tag{26}$$

In these expressions, $(\psi_{ij})_{max}$ is the maximum torsional displacement at the i-th level of the structure in the j-th mode of vibration. It is found from the torsional response spectral displacement $SD\theta_{jT}$ at frequency ω_{jT},

$$(\psi_{ij})_{max} = A_{ijT}\alpha_{jT}SD\theta_{jT}. \tag{27}$$

From $(\bar{a}_E)_{ij}$ and $(\bar{a}_\mu)_{ij}$, the modified (Amini and Trifunac, 1985; Gupta and Trifunac, 1987d) values $\bar{a}_E$ and $\bar{a}_\mu$ for the total torsional displacement response at the i-th floor are given by

$$(\bar{a}_E)_i = [(\bar{a}_E)^2_{i1} + (\bar{a}_E)^2_{i2} + \cdots\cdots + (\bar{a}_E)^2_{in}]^{1/2} \tag{28}$$

$$(\bar{a}_\mu)_i = [(\bar{a}_\mu)^2_{i1} + (\bar{a}_\mu)^2_{i2} + \cdots\cdots + (\bar{a}_\mu)^2_{in}]^{1/2}. \tag{29}$$

For the base torque response at the i-th level, one has to replace $(\psi_{ij})_{max}$ in equations (26) by $(T_{ij})_{max}$,

$$(T_{ij})_{max} = (J_1A_{1jT} + J_2A_{2jT} + \cdots\cdots + J_iA_{ijT})\alpha_{jT}\omega^2_{jT}SD\theta_{jT}. \tag{30}$$

After evaluating the parameters of the statistical distributions for the peaks of the torsional responses of the structure, one can compute the expected and the most probable values of all the peaks of the response at various levels of the structure. Using order statistics, the expected value and the most probable value can be computed from equations or from the tables presented by Gupta and Trifunac (1987a, d).

As an example, one could study the torsional response of the simple three story structure, which was considered by Gupta and Trifunac (1987b, c) for the translational response. It is assumed here that the floors are rigid and connected by four massless columns as shown in Figure 3. Taking the floor dimensions as 5.38m × 5.38m, floor masses $M_1 = 175.13$ KN-S^2/m, $M_2 = 262.70$ KN-S^2/m and $M_3 = 350.26$ kN-S^2/m and the story heights $L = 3.66$m, the moment of inertia matrix is found to be

$$[J] = 10^5 \times \begin{pmatrix} 8.474 & 0 & 0 \\ 0 & 12.711 & 0 \\ 0 & 0 & 16.948 \end{pmatrix} (\mathrm{kg-m^2}), \tag{31}$$

and

$$[k_T] = 10^9 \times \begin{pmatrix} 1.4259 & -1.4259 & 0 \\ -1.4259 & 4.2777 & -2.8518 \\ 0 & -2.8518 & 7.1295 \end{pmatrix} (\mathrm{N.m/rad}). \tag{32}$$

The torsional stiffnesses at various levels of the structure have been estimated from the translational stiffnesses in x and y directions, as described in the following.

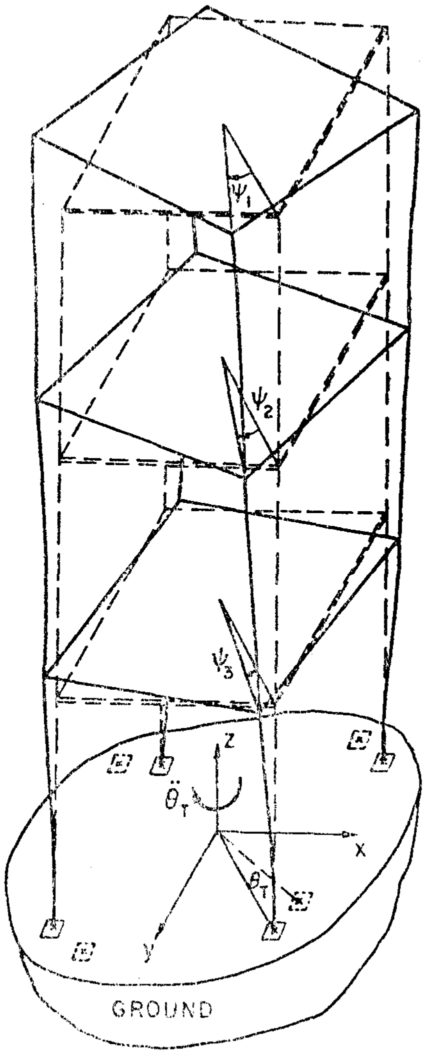

Figure 3. Torsional Deformation of Example Structure.

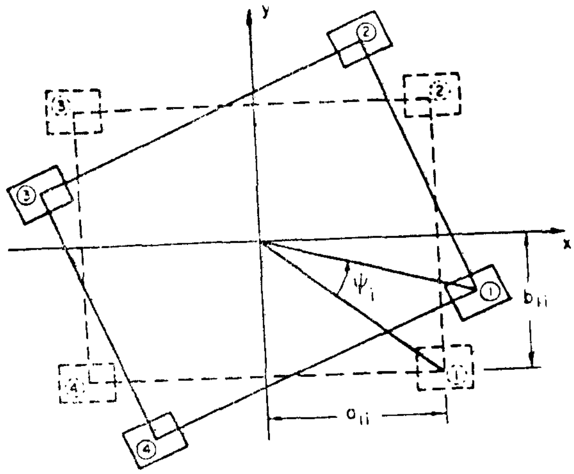

Figure 4. Rotation of a Floor Relative to One Floor Below it.

Consider the relative deflection of the i-th floor as shown in Figure 4. Figure 5 shows the deformations in column 1 due to the rotation. It is seen that the column has lateral deflections u_{1i} and v_{1i} in x and y directions, and also rotates about its axis through an angle ψ_i. Assuming the angle ψ_i is small, the deflections u_{1i} and v_{1i} can be approximated by

$$u_{1i} \cong r\psi_i \cos\phi_i = r\psi_i \frac{b_{1i}}{r} = \psi_i b_{1i}, \quad (33)$$

and

$$v_{1i} \cong r\psi_i \sin\phi_i = r\psi_i \frac{a_{1i}}{r} = \psi_i a_{1i}. \quad (34)$$

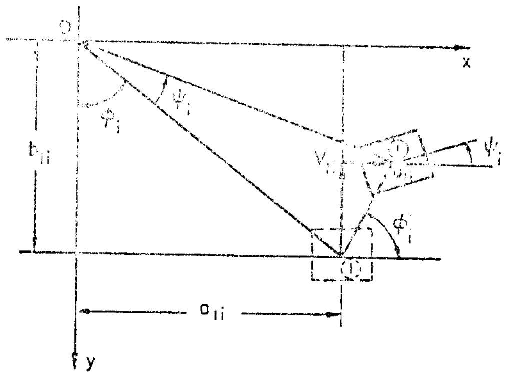

Figure 5. Deformation in a Column Due to Rotation of the Floor.

Similar equations can be written for the other three columns. If K_{iT} is the equivalent torsional stiffness for the i-th level of the structure, then

$$K_{iT}\psi_i = \sum_{j=1}^{4} k_{jx} v_{ji} a_{ji} + \sum_{j=1}^{4} k_{jy} u_{ji} b_{ji} + \frac{4GI_z}{L}\psi_i. \quad (35)$$

In this equation, k_{jx} and k_{jy} are the translational stiffnesses of the j-th column, G is the shear modulus of rigidity, L is the height and I_z is the arca moment of inertia of the cross-section of the column about its longitudinal axis.

Using expressions (33) and (34), equation (35) can be written as

$$K_{iT}\psi_i = \sum_{j=1}^{4} k_{jx}(\psi_i a_{ji}) a_{ji} + \sum_{j=1}^{4} k_{jy}(\psi_i b_{ji}) b_{ji} + \frac{4GI_z}{L}\psi_i, \quad \text{or}$$

$$K_{iT} = \sum_{j=1}^{4} k_{jx} a_{ji}^2 + \sum_{j=1}^{4} k_{jy} b_{ji}^2 + \frac{4GI_z}{L}. \quad (36)$$

This gives the torsional stiffness of the i-th level in terms of the translational stiffnesses. (In this example we have used $E \approx 27\times10^9$ N/m², $G \approx 12\times10^9$ N/m² and $L = 3.66$m). Translational stiffness' in the x and y directions have been taken as $k_{1x} = k_x$, $k_{2x} = 2k_x$, $k_{3x} = 3k_x$ with $k_x = 105\times10^3$ kN/m and $k_{1y} = k_y$, $k_{2y} = 2k_y$ and $k_{3y} = 3k_y$ with $k_y = 70\times10^3$ kN/m.

With matrices [J] and $[K_T]$ given by equations (31) and (32), the modal matrix $[A_T]$, mode participation vector $\{\alpha_T\}$ and the modal frequency vector $\{\omega_T\}$ are found to be

$$[A_T] = \begin{bmatrix} 1.0 & 1.0 & 1.0 \\ .723 & -.738 & -2.487 \\ .325 & -.969 & 2.518 \end{bmatrix} \quad (37)$$

$$\{\alpha_T\} = \begin{Bmatrix} 1.370 \\ -.553 \\ .1004 \end{Bmatrix}, \text{ and } \{\omega_T\} = \begin{Bmatrix} 21.590 \\ 54.083 \\ 76.603 \end{Bmatrix} \text{(rad/s)}. \quad (38)$$

Using the above properties for torsional deformation of the example structure, a synthetic torsional accelerogram developed by Lee and Trifunac(1985) (Figures 6 and 7), and the results on order statistics of peak responses (Gupta and

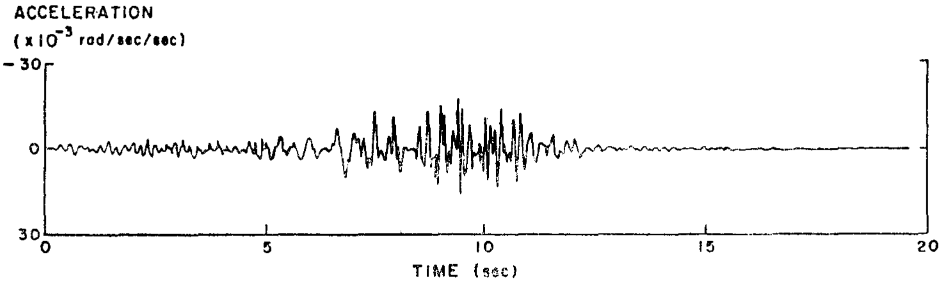

Figure 6. Synthetic Torsional Accelerogram. After Lee and Trifunac (1985).

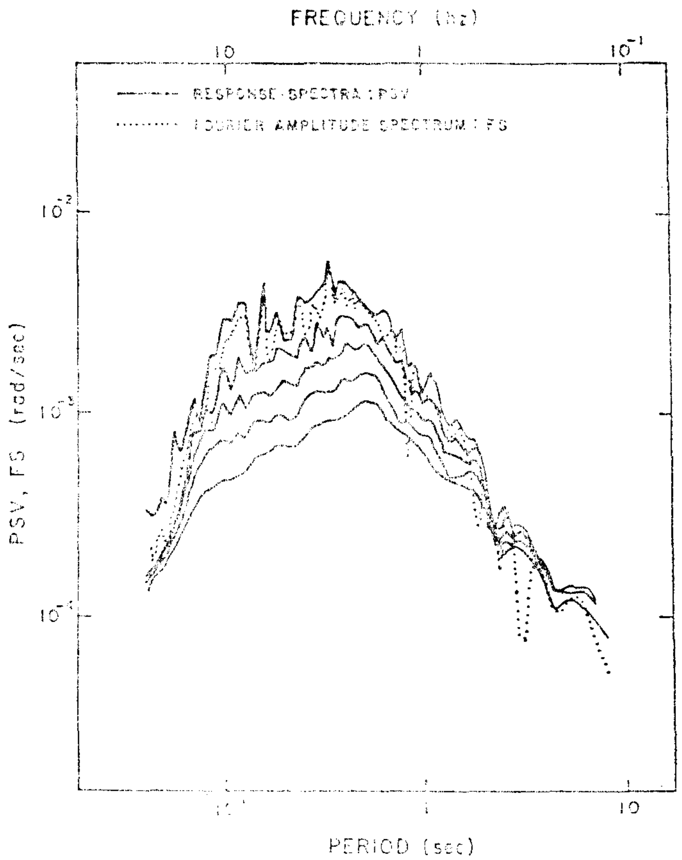

Figure 7. Fourier and Response Spectra for the Synthetic Accelerogram of Figure [illegible]. After Lee and Trifunac (1985).

Trifunac 1987a,b,d,)one can find the expected and the most probable values of the peaks (n = 1 corresponds to the first largest peak, n = 2 to the second largest peak, etc.) of the torsional displacements and the torques at various levels of the structure. These results are shown in Figures 8 and 9, along with the results from the time series solutions. It is observed that the agreement between the results from the statistical theory and those from the time series solution is good. The

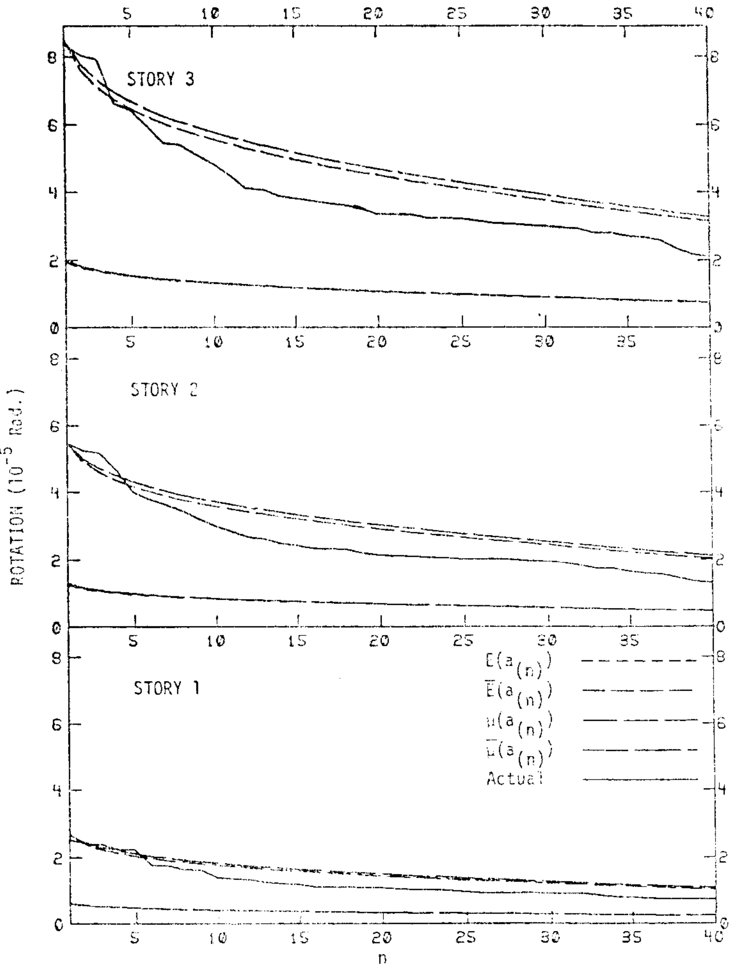

Figure 8. Theoretical and Actual Peaks of Rotations of Floors due to Torsional Excitation (Synthetic Torsional Accelerogram After Lee and Trifunac, 1985).

agreement between the theoretical prediction of peaks and the peaks computed by step by step integration of the response in time,can be further improved by considering the variable $\bar{a}_E^*$ and $\bar{a}_\mu^*$ as in Gupta and Trifunac (1987b). Thus the results of this statistical theory can also be used to analyze the structural response subjected to torsional excitations due to strong motion earthquakes.

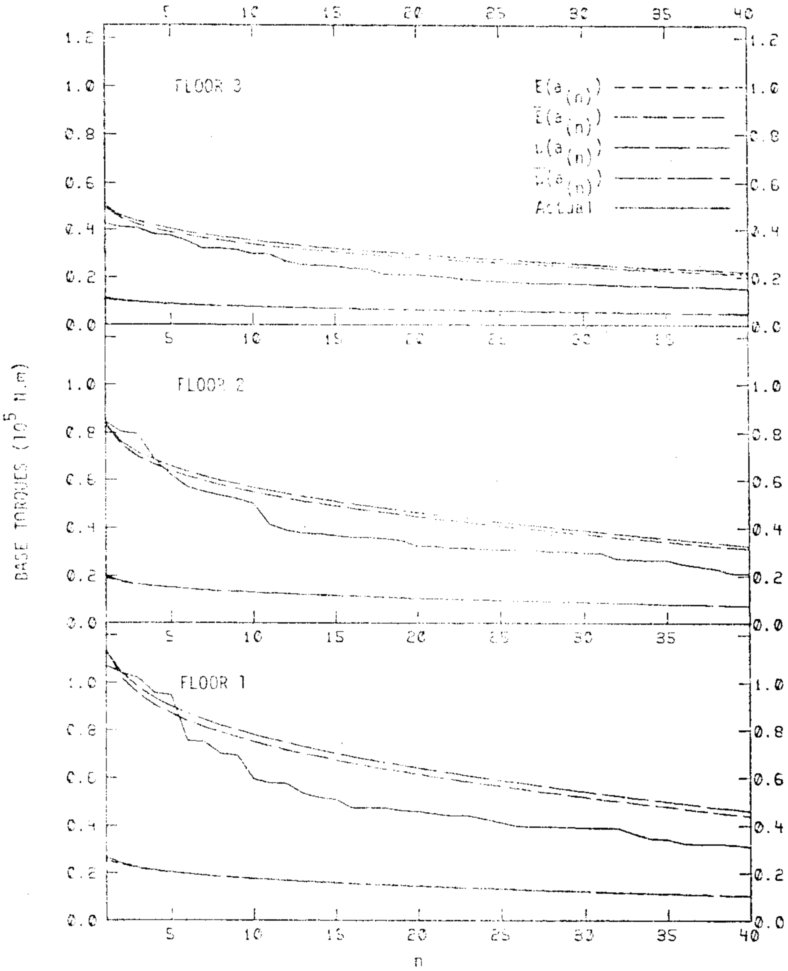

Figure 9. Theoretical and Actual Peaks of Torque at Different Base Floors due to Torsional Excitation (Synthetic Torsional Accelerogram After Lee and Trifunac, 1985).

Increase in Lateral Response Due to Torsion

Equations (33) and (34) show that the lateral displacements u_i and v_i are associated with the torsional displacements ψ_i at different floors of a structure. Corresponding to these displacements, there are base shears and bending moments also. Clearly, these lateral responses due to torsion will be maximum for the end and corner columns of the structure. We now study the lateral response due to torsional excitation of the example structure.

Assuming the values of translational and torsional properties of the structure as those given above, effects of varying the floor dimensions from $b=a=25$m to $b=5a$ and $b=10a$ have been studied for $\beta_{min}=350$m/s, 1750m/s and 3500m/s with $\beta_{max}=3500$m/s. By changing the floor dimensions, the values of the moments of inertia of floors are changed and hence the modal frequencies are also changed. For finding the torsional response of the structure, the torsional spectra have been computed from the spectra of translational component S00E of the El Centro Earthquake. We have also found the maximum translational displacements, shear forces and moments due to this component acting in the x-direction. The results are presented in Tables 1 through 6. It is seen that the increase in the maximum

Table 1 Increase in Maximum Lateral Displacement Response of an End Column Due to Torsional Motion for Different Aspect Ratios of Floors and Different β_{min} and β_{max} in the Layered Ground. (Most Probable Values)

ASPECT RATIO b/a	FLOOR	TRANSLATIONAL DISPL (Cm)	$\beta_{max}=3500$ m/s, $\beta_{min}=350$ m/s		$\beta_{max}=3500$ m/s, $\beta_{min}=1750$ m/s		$\beta_{max}=3500$ m/s, $\beta_{min}=3500$ m/s	
			TOTAL DISPL (Cm)	INCREASE (%)	TOTAL DISPL (Cm)	INCREASE (%)	TOTAL DISPL. (Cm)	INCREASE (%)
1	1	4.577	4.613	0.8	4.584	0.2	4.581	0.1
	2	2.948	2.968	0.7	2.952	0.1	2.950	0.1
	3	1.390	1.400	0.7	1.392	0.1	1.391	0.1
5	1	4.577	5.090	11.2	4.666	1.9	4.620	0.9
	2	2.948	3.273	11.0	3.001	1.8	2.974	0.9
	3	1.390	1.557	12.0	1.414	1.7	1.401	0.8
10	1	4.577	6.227	36.1	4.855	6.1	4.708	2.9
	2	2.948	4.008	36.0	3.120	5.8	3.028	2.7
	3	1.390	1.945	40.0	1.472	5.9	1.426	2.6

Table 2 Increase in Maximum Base Shear Response of an End Column Due to Torsional Motion for Different Aspect Ratios of Floors and Different β_{min} and β_{max} in the Layered Ground.

ASPECT RATIO b/a	FLOOR	TRANSLATIONAL BASE SHEAR (10^6N)	$\beta_{max}=3500$ m/s, $\beta_{min}=350$ m/s		$\beta_{max}=3500$ m/s, $\beta_{min}=1750$ m/s		$\beta_{max}=3500$ m/s, $\beta_{min}=3500$ m/s	
			TOTAL SHEAR (10^6N)	INCREASE (%)	TOTAL SHEAR (10^6N)	INCREASE (%)	TOTAL SHEAR (10^6N)	INCREASE (%)
1	1	1.780	1.797	1.0	1.784	0.2	1.782	0.1
	2	3.321	3.343	0.7	3.326	0.2	3.325	0.1
	3	4.378	4.402	0.5	4.384	0.1	4.381	0.1
5	1	1.780	1.963	10.3	1.810	1.7	1.796	0.9
	2	3.321	3.528	6.2	3.358	1.1	3.341	0.6
	3	4.378	4.640	6.0	4.416	0.9	4.396	0.4
10	1	1.780	2.327	30.7	1.866	4.8	1.821	2.3
	2	3.321	3.979	19.8	3.429	3.3	3.373	1.6
	3	4.378	5.264	20.2	4.504	2.9	4.434	1.3

responses of corner and end elements of a structure may be significant, particularly for large aspect ratios of the floors and for soft ground conditions. A quantitative idea about these increases can be obtained from the results in Tables1 to 6 for the simple three story example structure with just four columns at the corners.

Table 3 Increase in Maximum Dverturning Moment of an End Column Due to Torsional Motion for Different Aspect Ratios of Floors and Different β_{min} and β_{max} in the Layered Ground.

ASPECT RATIO b/a	FLOOR	TRANSLATIONAL MOMENT (10^6N.m)	β_{max}=3500 m/s β_{min}=350 m/s TOTAL MOMENT (10^6N.m)	INCREASE (%)	β_{max}=3500 m/s β_{min}=1750 m/s TOTAL MOMENT (10^6N.m)	INCREASE (%)	β_{max}=3500 m/s β_{min}=3500 m/s TOTAL MOMENT (10^6N.m)	INCREASE (%)
1	1	6.515	6.578	1.0	6.529	0.2	6.523	0.1
	2	18.432	18.606	0.9	18.480	0.3	18.467	0.2
	3	34.107	34.414	0.9	34.183	0.2	34.164	0.2
5	1	6.515	7.183	10.3	6.626	1.7	6.572	0.9
	2	18.432	19.839	7.6	18.726	1.6	18.591	0.9
	3	34.107	36.537	7.1	34.632	1.5	34.385	0.8
10	1	6.515	8.515	30.7	6.829	4.8	6.665	2.3
	2	18.432	22.602	22.6	19.194	4.1	18.823	2.1
	3	34.107	41.293	21.1	35.482	4.0	34.822	2.1

Table 4 Increase in Maximum Lateral Displacement Response of an End Column Due to Torsional Motion for Different Aspect Ratios of Floors and Different β_{min} and β_{max} in the Layered Ground. (Expected Values)

ASPECT RATIO b/a	FLOOR	TRANSLATIONAL DISPL. (Cm)	β_{max}=3500 m/s β_{min}=350 m/s TOTAL DISPL (Cm)	INCREASE (%)	β_{max}=3500 m/s β_{min}=1750 m/s TOTAL DISPL (Cm)	INCREASE (%)	β_{max}=3500 m/s β_{min}=3500 m/s TOTAL DISPL (Cm)	INCREASE (%)
1	1	4.579	4.615	0.8	4.586	0.2	4.583	0.1
	2	2.949	2.929	0.8	2.953	0.1	2.951	0.1
	3	1.391	1.401	0.7	1.393	0.1	1.392	0.1
5	1	4.579	5.092	11.2	4.668	1.9	4.622	0.9
	2	2.949	3.274	11.0	3.003	1.8	2.975	0.9
	3	1.391	1.557	11.9	1.414	1.7	1.401	0.7
10	1	4.579	6.230	36.1	4.858	6.1	4.711	2.9
	2	2.949	4.010	36.0	3.122	5.9	3.029	2.7
	3	1.391	1.946	39.9	1.472	5.8	1.426	2.5

Table 5 Increase in Maximum Base Shear Response of an End Column Due to Torsional Motion for Different Aspect Ratios of Floors and Different β_{min} and β_{max} in the Layered Ground.

ASPECT RATIO b/a	FLOOR	TRANSLATIONAL BASE SHEAR (10^6N)	$\beta_{max}=3500$ m/s $\beta_{min}=350$ m/s		$\beta_{max}=3500$ m/s $\beta_{min}=1750$ m/s		$\beta_{max}=3500$ m/s $\beta_{min}=3500$ m/s	
			TOTAL SHEAR (10^6N)	INCREASE (%)	TOTAL SHEAR (10^6N)	INCREASE (%)	TOTAL SHEAR (10^6N)	INCREASE (%)
1	1	1.781	1.798	1.0	1.785	0.2	1.783	0.1
	2	3.322	3.344	0.7	3.327	0.2	3.326	0.1
	3	4.380	4.404	0.5	4.386	0.1	4.383	0.1
5	1	1.781	1.963	10.2	1.811	1.7	1.797	0.9
	2	3.322	3.530	6.3	3.359	1.1	3.342	0.6
	3	4.380	4.642	6.0	4.418	0.9	4.398	0.4
10	1	1.781	2.327	30.7	1.866	4.8	1.822	2.3
	2	3.322	3.980	19.8	3.431	3.3	3.374	1.6
	3	4.380	5.266	20.2	4.507	2.9	4.436	1.3

Table 6 Increase in Maximum Overturning Moment for an End Column Due to Torsional Motion for Different Aspect Ratios of Floors and Different β_{min} and β_{max} in the Layered Ground.

ASPECT RATIO b/a	FLOOR	TRANSLATIONAL MOMENT (10^6N.m)	$\beta_{max}=3500$ m/s $\beta_{min}=350$ m/s		$\beta_{max}=3500$ m/s $\beta_{min}=1750$ m/s		$\beta_{max}=3500$ m/s $\beta_{min}=3500$ m/s	
			TOTAL MOMENT (10^6N.m)	INCREASE (%)	TOTAL MOMENT (10^6N.m)	INCREASE (%)	TOTAL MOMENT (10^6N.m)	INCREASE (%)
1	1	6.519	6.581	1.0	6.532	0.2	6.527	0.1
	2	18.439	18.614	0.9	18.488	0.3	18.474	0.2
	3	34.119	34.426	0.9	34.195	0.2	34.176	0.2
5	1	6.519	7.185	10.2	6.629	1.7	6.575	0.9
	2	18.439	19.843	7.6	18.735	1.6	18.599	0.9
	3	34.119	36.547	7.1	34.647	1.5	34.398	0.8
10	1	6.519	8.517	30.6	6.828	4.7	6.667	2.3
	2	18.439	22.605	22.6	19.203	4.1	18.832	2.1
	3	34.119	41.295	21.0	35.497	4.0	34.836	2.1

Discussion and Conclusions

From the combined translational and torsional response analysis of the simple example structure in this study, it is found that the torsional excitations may cause significant increase in maximum lateral displacement, shear force and bending moment responses. Contribution from torsion will be maximum for the elements located at the periphery of the building. The effects of torsional excitation are more prominent for low values of the minimum shear wave velocity in the layered medium beneath the structure. Though these conclusions are not new, the present study provides a rational mathematical basis, by using the order statistics theory for response spectrum superposition, for estimating the actual increase in the translational response at any point of the structure. The method used for generating the spectra of rotational ground motion in this study is also based on physical principles of wave propagation in layered ground and it takes into account the dispersive nature of seismic waves, which is typically ignored by most of the methods used for this purpose so far.

Most of the earlier studies have associated the torsional response with the asymmetry of the structure; i.e., the lack of coincidence between the centers of resistance and mass of the structure. However, the present study shows how the torsional displacements for a dynamically symmetric structure arise from the torsional acceleration of strong ground motion associated with SH and Love waves. The effects of asymmetry will augment the response from torsional input ground motion considered here. The effects of soil structure interaction are not considered here.

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APPENDIX:NOTATION

The following symbols have been used in this work:

a_n	n^{th} local peak of f(t)
$a_{(n)}$	n^{th} order peak of f(t) when peaks are arranged in decreasing order of amplitudes
a_{li}	x coordinate of column l w.r.t. floor center
a_x, a_y, a_z	components of ground acceleration
$\bar{a}_i$	parameter $\bar{a}$ for i^{th} story response
$\bar{a}^*$	nonstationary form of parameter $\bar{a}$
$\bar{a}_E$	modified r.m.s. amplitude of the expected values of peaks of f(t)
$(\bar{a}_E)_i$	parameter $\bar{a}_E$ for i^{th} story response
$(\bar{a}_E)_{ij}$	parameter $\bar{a}_E$ for i^{th} story response due to j^{th} mode
$\bar{a}_E^*$	nonstationary form of parameter $\bar{a}_E$
$\bar{a}_\mu$	modified r.m.s. amplitude for most probable values of peaks of f(t)
$(\bar{a}_\mu)_i$	parameter $\bar{a}_\mu$ for i^{th} story response
$(a_\mu)_{ij}$	parameter $\bar{a}_\mu$ for i^{th} story response due to j^{th} mode
$\bar{a}_\mu^*$	nonstationary form of parameter $\bar{a}_\mu$
$A(\omega)$	transform of ground acceleration
$A_m(\omega)$	contribution to $A(\omega)$ from m^{th} mode of surface waves
A_{ijT}	$i\text{-}j^{th}$ element of matrix $[A_T]$
$[A_T]$	modal transformation matrix for torsional response
b_{li}	y coordinate of column l w.r.t. floor center

C	surface phase velocity
C_{iT}	torsional viscous damping coefficient for i^{th} story
$[C_T]$	torsional damping matrix
E	compression modulus
$E(\omega)$	energy density spectrum of $f(t)$
$E[a_{(n)}]$	expected value of $a_{(n)}$
$E[a((_{n})]$	modified expected value of $a_{(n)}$
$E_{ij}(\omega)$	function $E(\omega)$ for i^{th} story response due to j^{th} mode
$E\psi_i(\omega)$	energy spectrum for torsional displacement response of i^{th} story
$ET_i(\omega)$	energy spectrum for base torque response at i^{th} floor
G	shear modulus
$H_{ij}(\omega)$	transfer function for i^{th} story response in j^{th} mode
$[I]$	unit column vector
I_z	moment of inertia of column cross-section
$[J]$	moment of inertia matrix
K_{iT}	torsional stiffness coefficient of i^{th} story
$[K_T]$	torsional stiffness matrix
$[K]_x, [K]_y, [K]_z$	stiffness matrices for the motions in the x, y and z directions
L	column height
N_i	total number of peaks in the i^{th} story response
r	position vector of a column relative to floor center
$SD\theta_T(\omega)$	torsional spectrum displacement
$SD\theta_{jT}$	torsional spectrum displacement at j^{th} modal frequency
T	total duration of the response
$T_i(t)$	base torque response of i^{th} floor
$T_{ij}(t)$	base torque response of i^{th} floor due to j^{th} mode
$(T_{ij})_{max}$	maximum base torque response at i^{th} floor due to j^{th} mode
$\{T(t)\}$	base torque vector
u_1, u_2, u_3	components of ground motion
u_{1i}	lateral column deflection in x direction
v_{1i}	lateral column deflection in y direction
α_{jT}	j^{th} torsional mode participation factor
$\{\alpha_T\}$	torsional mode participation vector
β_{min}, β_{max}	minimum and maximum shear wave velocities in the layered ground
ε	parameter describing the width of energy spectrum
ε_{ij}	parameter ε for i^{th} story response due to j^{th} mode
ζ_{jT}	critical damping ratio for j^{th} torsional mode
$\ddot{\theta}_T(t)$	torsional acceleration
$\Theta_T(\omega)$	transform of $\ddot{\theta}_T(t)$
$\bar{\Theta}_T(\omega)$	mean value of $\Theta_R(\omega)$ over the interval $\pi\zeta_{jT}\omega_{jT}$
$\mu[a_{(n)}]$	most probable value of $a_{(n)}$
$\bar{\mu}[a_{(n)}]$	modified most probable value of $a_{(n)}$
$\{\tau)t)\}$	inertial torque vector
$\{\phi\}$	generalized coordinate vector for torsional response

ψ_i	relative torsional displacement of i^{th} floor
ψ_{13}	rotation about axis 2 from axis 1 to 3 (torsion)
$\psi_m(\omega)$	contribution to $\psi(\omega)$ from m^{th} surface wave mode
$(\psi_{ij})_{max}$	maximum relative torsional displacement of i^{th} floor due to j^{th} mode
$\psi(\omega)$	transform of ground torsion
ω, ω_n	circular frequency
ω_{jT}	j^{th} torsional mode frequency
$\{\omega_T\}$	torsional mode frequency vector

关于扭转扰动对简单对称建筑物地震反应影响的注记

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提 要

研究表明，扭转扰动可能导致建筑物周边的构件中最大剪力和弯矩的显著增加。以前的研究已将扭转反应与结构的非对称性（即结构的抗力中心与质量中心不重合）相联系。本文研究涉及SH波和Love波的强地面运动的扭转加速度是如何引起对称结构的扭转反应的。