

ISOLATION OF SOIL-STRUCTURE INTERACTION EFFECTS BY FULL-SCALE FORCED VIBRATION TESTS

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SUMMARY

Forced vibration tests designed to isolate the effects of soil-structure interaction are described and the results obtained for the nine-storey reinforced concrete Millikan Library Building are analysed. It is shown that it is possible to determine experimentally the fixed-base natural frequencies and modal damping ratios of the superstructure. These values may be significantly different from the resonant frequencies and damping ratios of the complete structure-foundation-soil system. It is also shown that forced vibration tests can be used to obtain estimates of the foundation impedance functions.

In the case of the Millikan Library it is found that during forced vibration tests the rigid-body motion associated with translation and rocking of the base accounts for more than 30 per cent of the total response on the roof and that the deformation of the superstructure at the fundamental frequencies of the system is almost entirely due to the inertial forces generated by translation and rocking of the base.

INTRODUCTION

Full-scale forced vibration tests are commonly used to determine the natural frequencies, modal damping values and mode shapes of structures. The frequent practice in interpreting the results of forced vibration tests is to neglect the effects of the interaction between the structure and the soil. Such a simplifying assumption may lead to serious errors in that resonant frequencies, energy dissipation and other dynamic characteristics of the complete structure-foundation-soil system are ascribed to the superstructure. The typical result is that the fixed-base natural frequencies of the structure are underestimated while the energy dissipation in the structure is overestimated.

The principal objective of this study is to analyse in detail the effects of soil-structure interaction during forced vibration tests. In particular, an attempt is made at extracting structural characteristics, such as fixed-base natural frequencies and energy dissipation mechanism, as well as foundation-soil characteristics, such as foundation impedance functions, from forced vibration test results which involve the complete structure-foundation-soil system.

Although a large number of theoretical studies of the interaction between structures and the supporting soil have been made and a variety of sophisticated analytical models have been proposed, the experimental study of the interaction phenomenon has been very limited. The second objective of this study is associated with the need of illustrating the interaction effects under controlled experimental conditions.

For the purpose of this study the nine-storey reinforced concrete Millikan Library Building was selected as the experimental site. The Millikan Library has been the subject of a large number of forced vibration tests (Kuroiwa,¹² Jennings and Kuroiwa,⁹ Trifunac,¹⁹ Foutch *et al.*,⁵ Luco *et al.*,¹³) and ambient vibration tests (Blandford *et al.*,¹ Trifunac,¹⁹ McLamore,¹⁷ Udawadia and Trifunac²⁰). Accelerograms for the 1968 Borrego Mountain, 1970 Lytle Creek and 1971 San Fernando earthquakes have been recorded in the Library and a

number of studies on its seismic response have been conducted (Crouse,³ Udwadia and Trifunac,²¹ Iemura and Jennings,⁸ Udwadia and Marmarelis,²² Luco¹⁴). In addition, detailed soil mechanics information is available (Converse Foundation Engineers²) and shear wave velocity profiles have been determined in the vicinity of the site (Eguchi *et al.*,⁴ Shannon and Wilson, Inc. and Agabian Assoc.¹⁸). This wealth of information makes the Millikan Library an ideal experimental site. The presentation of tests designed to extract the characteristics of the superstructure and the discussion of the results obtained are preceded by a brief description of the Millikan Library Building and a summary of the results of previous experiments. A detailed study of the contribution of the motion of the base to the total motion at the top of the superstructure is followed by an exploration of the possibility of determining experimentally the foundation impedance functions.

DESCRIPTION OF THE MILLIKAN LIBRARY BUILDING

The Robert A. Millikan Library is a nine-storey reinforced concrete building located on the campus of the California Institute of Technology. The structure has a basement and an enclosed roof area. The typical floor plan covers an area of 21×23 m (69×75 ft) and the building stands 43.9 m (144 ft) above the first floor level and 48.2 m (158 ft) above the basement slab (see Figure 1.). The majority of the lateral loads in the transverse (N-S) direction are resisted by 30 cm (12 in) reinforced concrete shear walls located on the east and west ends of the building. In the longitudinal (E-W) direction the 30 cm (12 in) reinforced concrete walls of the central core provide most of the lateral resistance. The north-side and south-side facades are precast window wall

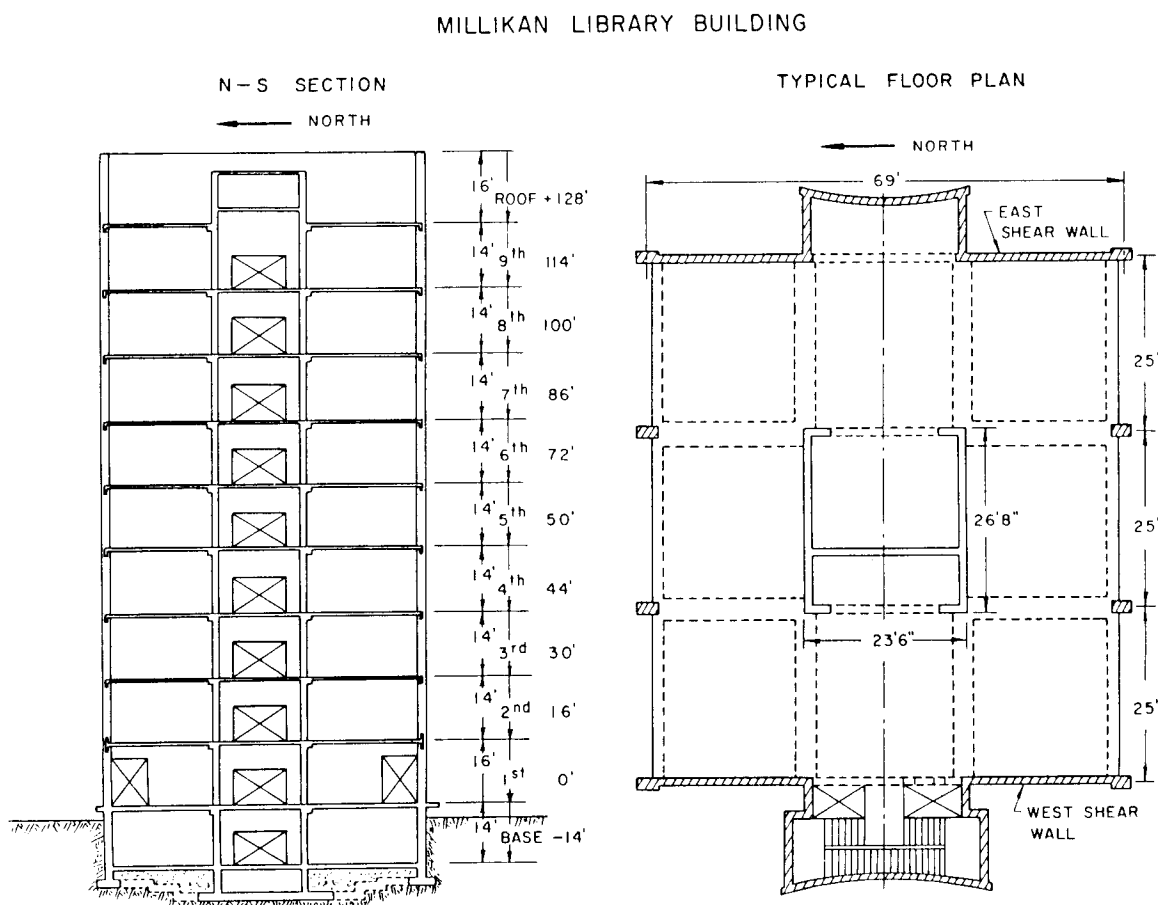


Figure 1. Millikan Library Building: N-S elevation and typical floor plan.

panels connected to the main structure with steel angle clips. The floor system consists of 23 cm (9 in) slabs of lightweight concrete reinforced in two directions and supported by reinforced concrete beams. The total weight of the superstructure is estimated at 1.05×10^8 N (23.5×10^6 lb).

The foundation system of the library consists of a central pad 9.75 m (32 ft) wide and 1.22 m (4 ft) deep which runs in the E-W direction and extends from the east curved shear wall to the west curved shear wall (Figure 2). Also provided are beams 3 m (10 ft) wide by 0.61 m (2 ft) deep which run E-W beneath the rows of columns at the north and south ends of the building. These beams are connected to the central pad by stepped beams. The contact between the central pad and the underlying soil is approximately 7 m (23 ft) below the first-floor level. The plan dimensions of the foundation are approximately 23.3×25.1 m (76.5×82.5 ft) with additional areas of dimensions 9.9×1.7 m (32.5×5.5 ft) and 9.9×3.5 m (32.5×11.5 ft) at the east and west extremes, respectively. The total weight of the foundation is estimated at 0.14×10^8 N (3.2×10^6 lb). The foundation rests on alluvium composed of medium to dense sands mixed with gravel. The alluvium at the site extends about 275 m (900 ft) to bedrock.

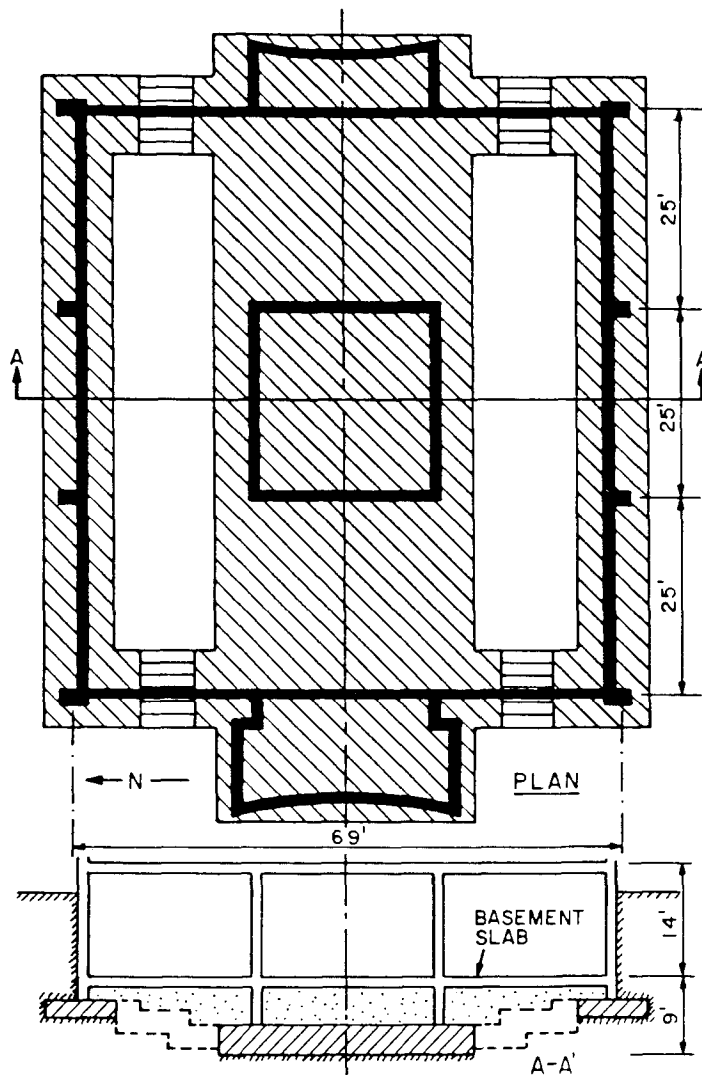


Figure 2. Millikan Library: foundation system.

SUMMARY OF PREVIOUS EXPERIMENTAL STUDIES

Since 1966 the Millikan Library Building has been subjected to a number of ambient, man-excited and shaker-excited vibration tests. A list of the experimentally determined fundamental system frequencies in the N-S and E-W directions is presented in Table I. The results of forced vibration tests conducted in 1975 indicate fundamental system frequencies of 1.8 and 1.2 cps in the N-S and E-W directions, respectively. Inspection of the results presented in Table I reveals that the most recent values of the resonant frequencies are somewhat lower than those determined in the initial forced and ambient vibration tests. In particular, comparison of the results of ambient vibration tests before and after the 1971 San Fernando earthquake indicates reductions of 5 and 14 per cent for the resonant frequencies in the N-S and E-W directions. Detailed moving window analyses of the response during the San Fernando earthquake show marked reductions of the system frequencies within the duration of the strong shaking followed by partial recovery at the end of the excitation (Udwadia and Trifunac²¹). Whether the observed reductions in resonant frequencies correspond mainly to changes in the soil, foundation or superstructure remains an interesting question (Foutch and Jennings,⁷ Luco,¹⁴ Luco *et al.*^{15, 16}).

Vibration tests of the Millikan Library Building have also provided estimates of the system modal damping in the neighbourhood of the first resonant frequency, as listed in Table I. These values range from 0.7 to 1.8 per cent. Studies of the E-W response of the Library during the San Fernando earthquake indicate that the apparent damping during the first 43 sec of the strong motion portion of the excitation may have been as high as 5.5 per cent (Udwadia and Marmarelis²²) or 13.0 per cent (Iemura and Jennings⁸).

In an experiment performed in 1974 the Millikan Library Building was forced into resonance by a vibration generator located at the roof and the three-dimensional motion at 51 locations on each of the 2nd, 4th, 6th and 8th floors, the basement slab and the roof was measured for shaking in both the N-S and E-W directions (Foutch *et al.*⁵). The patterns of deformation of the west shear wall for N-S excitation and of a section through the central core for E-W excitations are shown in Figure 3. The observations indicate that for N-S excitations each floor remains essentially plane, experiencing an almost uniform translation and an almost uniform rotation about an E-W axis. For excitation in the E-W direction, each floor experiences an almost

Table I. Fundamental system frequencies and system modal damping ratios

Test	Frequency (cps)		System modal damping (per cent)	
	N-S	E-W	N-S	E-W
Forced vibration test (1966-67) ^{9, 12}	1.89-1.98	1.46-1.51	1.2-1.8	0.7-1.7
Ambient vibration test (March 1967) ¹	1.91	1.49	1.6	1.5
Ambient vibration test (April 1968) ²⁰	1.89	1.45	—	—
Lytle Creek earthquake transfer function (Sept. 1970) ²¹	1.90-2.00	1.30-1.50	—	—
San Fernando earthquake transfer function (Feb. 1971) ²¹	1.50-1.90	1.00-1.50	—	—
San Fernando earthquake (Feb. 1971) ⁸	—	0.82-1.43	—	1.0-13.0
San Fernando earthquake (Feb. 1971) linear model ²²	—	1.02-1.11	—	3.5-5.5
Ambient vibration test (Feb. 1971) ¹⁷	1.80	1.25	—	—
Ambient vibration test (March 1971) ²⁰	—	1.30	—	—
Man excited vibration test (Dec. 1972) ²⁰	1.77	1.37	—	—
Ambient vibration test (April 1973) ²²	—	1.28	—	1.3
Forced vibration test (1974) ⁵	1.76	1.21	—	—
Forced vibration test (July 1975)	1.79	1.21	1.8	1.8
[Present study]				

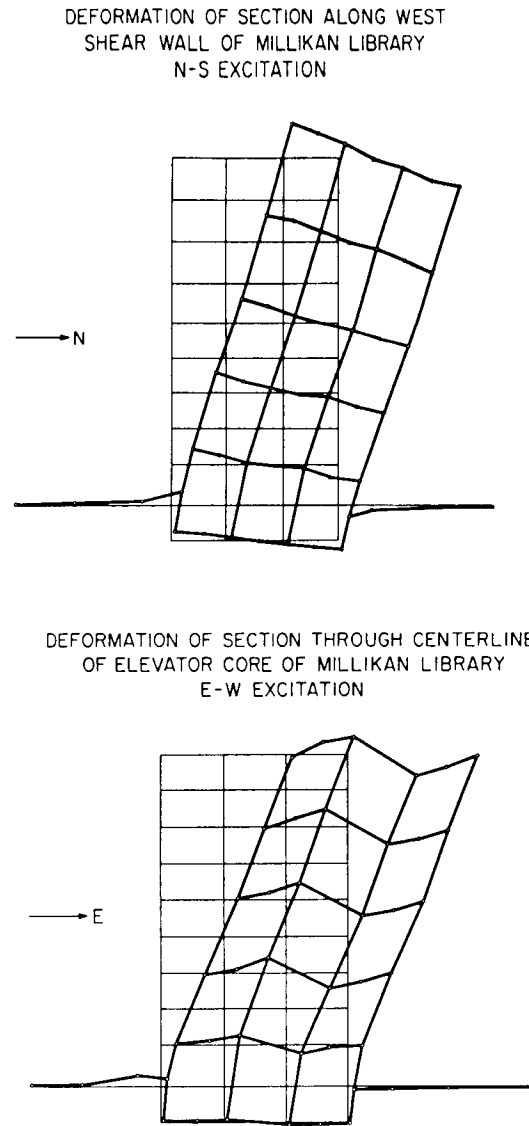


Figure 3. (a) Deformation of section along west shear wall: N-S excitation. (b) Deformation of section through centreline of elevator core: E-W excitation

uniform translation while the average rotation is almost zero. In this case, the central core introduces a marked bending of the floor slabs.

The patterns of deformation of the basement slab for excitation in the N-S and E-W directions are shown in Figures 4 and 5, respectively. For vibrations in the N-S direction, the stiff shear walls on the east and west ends of the building cause an almost rigid translation of the basement slab together with an almost uniform rotation about the E-W axis of symmetry of the base (Figure 4). Some deviations from this average rigid-body motion may be observed at the location of the central core and at the north and south ends of the slab. For vibrations in the E-W direction, the central core induces large localized deformation of the basement slab (Figure 5).

The results of the experiment conducted by Foutch *et al.*⁵ indicate that the interaction of the structure and the soil has a marked effect on the response during forced vibration tests. In particular, for N-S vibrations, it was found that the translation of the basement slab was 4 per cent of the motion at the roof and that the average rotation of the basement slab multiplied by the height of the structure amounted to 25 per cent of the roof motion. Thus, the rigid body motion of the structure due to soil compliance contributed 29 per cent of the roof

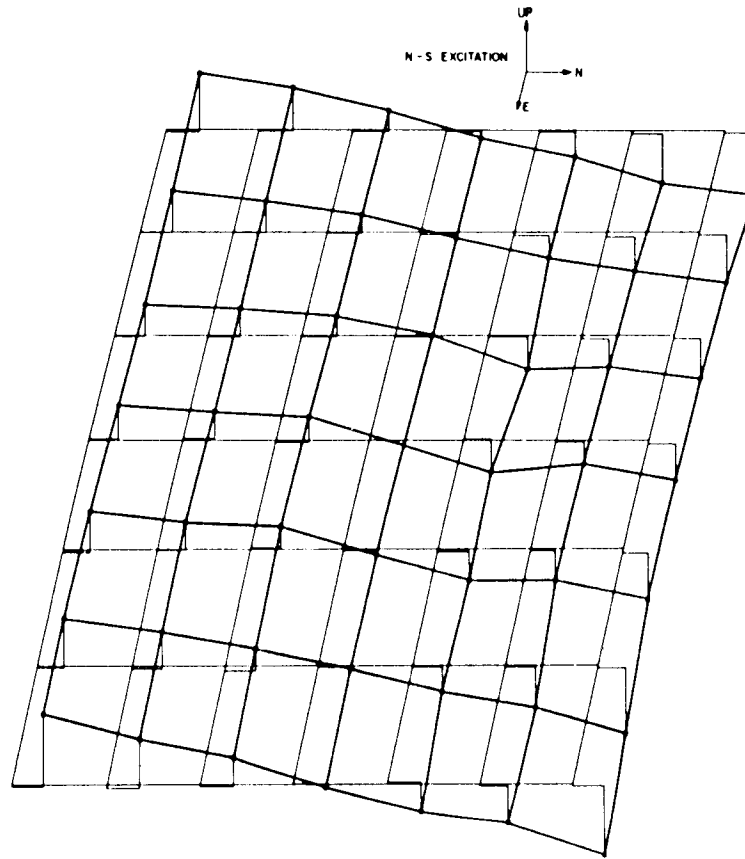


Figure 4. Deformation of the basement slab: N-S vibrations

response. These results are in sharp contrast with those of a previous study (Kuroiwa,¹² Jennings and Kuroiwa⁹) which indicate that the rigid body motion contributes less than 3 per cent to the N-S response at the roof and which led Jennings and Kuroiwa⁹ to the conclusion that the structure behaved essentially as if it were fixed at the foundation level. The present study is motivated in part by the need to explain this discrepancy.

In related experiments, the three-dimensional motion of the soil surface generated by the forces that the foundation exerts on soil during forced vibration tests was recorded in the immediate neighbourhood of the Library and in one quadrant of the Pasadena area extending to a distance of six kilometres (four miles) from the building (Foutch *et al.*⁵ Luco *et al.*¹³). These studies as well as the previous work of Jennings¹⁰ reveal that the motion of the foundation distorts the soil surface in the vicinity of the building and generates Rayleigh and Love waves which can be recorded at considerable distances from the structure.

EXPERIMENTAL DETERMINATION OF THE FREQUENCY RESPONSE

A set of forced vibration tests designed with the purpose of isolating the effects of soil-structure interaction was conducted by the authors in 1975. In these tests, the N-S and E-W steady-state response of the Millikan Library to a harmonic exciting force produced by an eccentric-mass vibration generator mounted on the roof was determined. The response of the building was recorded at four points, three of which were located on the basement slab (stations 1, 2 and 3) while the fourth point was located on the roof (station 4). The locations of the recording stations for vibrations in the N-S and E-W directions are shown in Figure 6. The horizontal motions of the basement slab and of the roof were recorded at station 1 and 4, respectively, while the vertical motion of the basement slab was recorded at stations 2 and 3. For vibrations in the N-S direction, the response

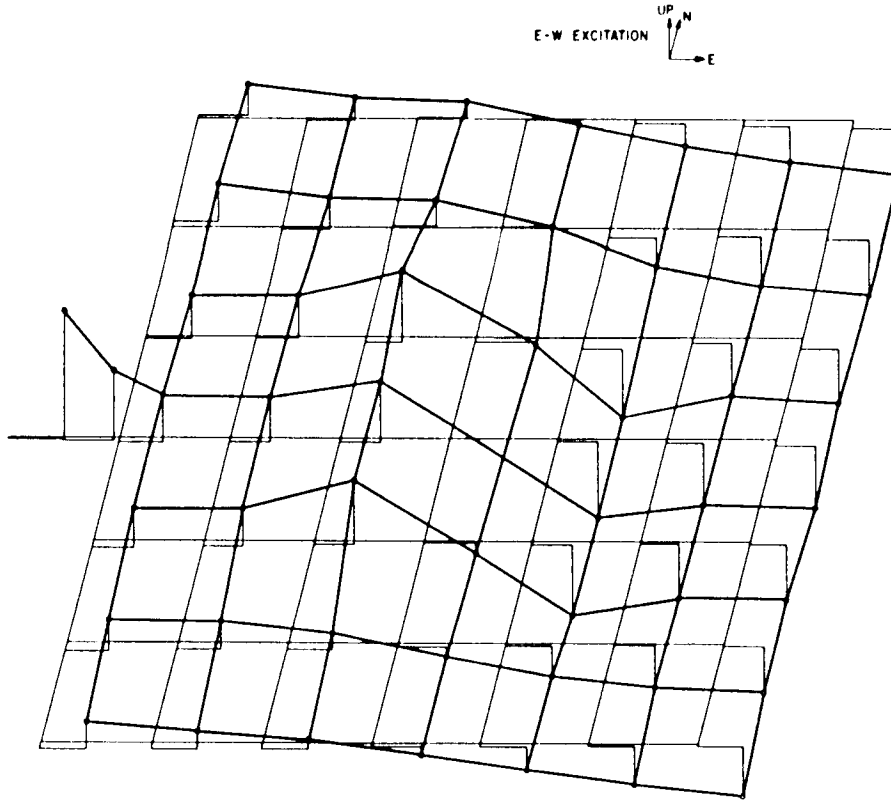


Figure 5. Deformation of the basement slab: E-W vibrations

was determined at 116 distinct frequencies in the range 0.8 to 2.50 Hz. For vibrations in the E-W direction, the response was recorded at 67 frequencies in the range from 0.8 to 1.75 Hz.

The recording system consisted of four Model SS-1 Ranger-type seismometers (moving coil, velocity-type transducers with natural period in the vicinity of 1 sec), an Earth Sciences SC-201A signal conditioner, an Ampex SP-300 tape recorder and two Brush recorders. In addition, an instrument designed to determine the phase of the forcing function with respect to its peak value was also used. For each frequency of excitation, the output of the four seismometers amplified by the signal conditioner was recorded on a common time basis on the tape recorder and plotted on the Brush recorders. The phase of the forcing function was also recorded. The data were corrected to account for the effects introduced by the seismometers and the signal conditioner. Details of the corrections used are presented elsewhere (Luco *et al.*¹⁵).

The resulting corrected data corresponding to the displacements at the four recording stations can be described by the functions

$$y_j(\omega) = Y_j(\omega) \exp \{i[\omega t - \phi_j(\omega)]\}, \quad (j = 1, 4) \quad (1)$$

where ω represents the frequency of the excitation, $Y_j(\omega)$ denotes the amplitude of the displacement at the j th station and $\phi_j(\omega)$ represents the phase of the motion at the j th station with respect to the force that the vibration generator exerts on the roof. The harmonic force that the vibration generator applies on the roof is given by

$$f_T(\omega) = F_T(\omega) \exp(i\omega t) \quad (2)$$

where $F_T(\omega) = 103.6 \omega^2 \text{ N}$ ($23.3 \omega^2 \text{ lb}$) (Keightley *et al.*¹¹).

Assuming that the deformation patterns of the basement and roof slabs determined in previous experiments (Foutch *et al.*⁵) remain unchanged for frequencies in the neighbourhood of the first resonant frequency, it is

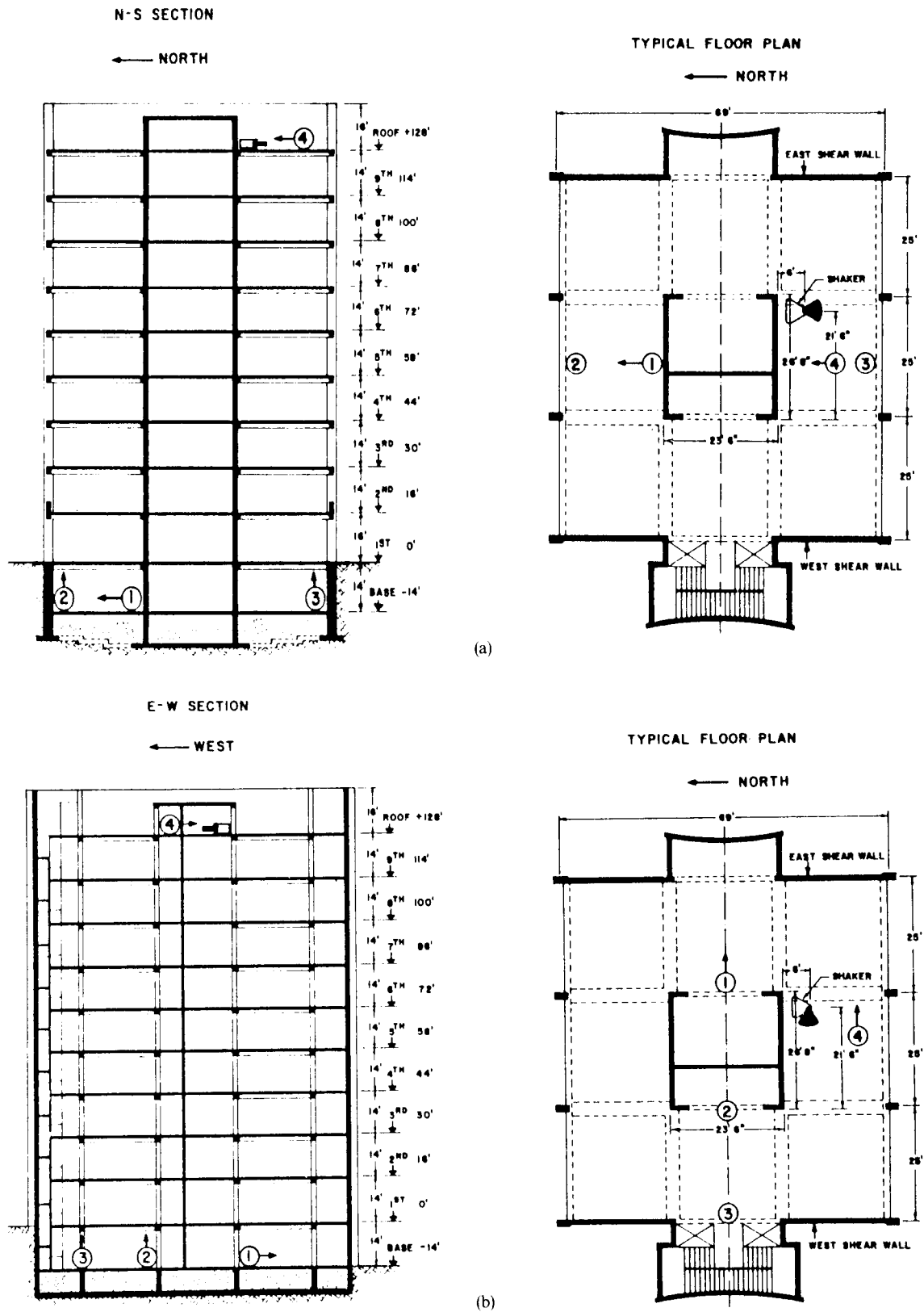


Figure 6. Location of the four Ranger seismometers and of the shaker for (a) N-S and (b) E-W vibrations

possible to use the response at the four recording stations to estimate the translation and rotation of the basement slab and the translation of the roof. In particular, the average translation and rotation of the basement slab, $\bar{U}_b e^{i\omega t}$ and $\bar{\Phi}_b e^{i\omega t}$, and the average total translation of the roof, $\bar{U}_T e^{i\omega t}$, can be expressed in the form

$$\bar{U}_b = 1.10 Y_1 e^{-i\phi_1}, H\bar{\Phi}_b = \frac{1}{2} (-6.14 Y_2 e^{-i\phi_2} + 6.68 Y_3 e^{-i\phi_3}), \bar{U}_T = Y_4 e^{-i\phi_4} \quad (3)$$

for N-S vibrations, and

$$\bar{U}_b = 1.14 Y_1 e^{-i\phi_1}, H\bar{\Phi}_b = \frac{1}{2} (3.40 Y_2 e^{-i\phi_2} + 4.28 Y_3 e^{-i\phi_3}), \bar{U}_T = Y_4 e^{-i\phi_4} \quad (4)$$

for E-W vibrations. In equations (3) and (4), $H = 43.3$ m (142 ft) denotes the height of the roof slab with respect to the basement slab. The coefficients appearing in equations (3) and (4) were obtained by calculating the average translation and rotation of the foundation on the basis of the deformation patterns shown in Figures 4 and 5 and by dividing by the amplitude of the response at the location of the recording stations.

For vibrations in the N-S direction, the response of the superstructure is controlled by the motion of the shear walls along the east and west ends of the building. The translation at the base of the shear walls, $U_b e^{i\omega t}$, can be approximated by the average translation of the foundation, $\bar{U}_b e^{i\omega t}$, i.e. $U_b = \bar{U}_b$. The rotation of the base of the shear walls about an E-W axis, $\Phi_b e^{i\omega t}$, can be expressed in terms of the average rotation of the foundation, $\bar{\Phi}_b e^{i\omega t}$, by

$$\Phi_b = \alpha \bar{\Phi}_b \quad (5)$$

where $\alpha = 1.3$.

For E-W vibrations, the response of the superstructure is highly affected by the motion of the central core. The translation at the base of the central core, $U_b e^{i\omega t}$, can be approximated by the average E-W translation of the foundation ($U_b = \bar{U}_b$). The rotation of the base of the central core about a N-S axis, $\Phi_b e^{i\omega t}$, can be expressed in terms of the average rotation of the complete foundation, $\bar{\Phi}_b e^{i\omega t}$, as in equation (5), where, in this case, $\alpha = 3.33$. The values of the coefficient α appearing in equation (5) and connecting the rotation of the shear walls and of the core walls to the average rotation of the foundation have also resulted from the data in Figures 4 and 5.

The amplitudes and phases of \bar{U}_b , $H\bar{\Phi}_b$, $H\Phi_b$ and \bar{U}_T for vibrations in the N-S and N-W directions are plotted versus frequency in Figures 7 and 8. In Figure 7(a) the amplitudes of the motion at the basement slab level and at the roof for N-S excitation are plotted on a logarithmic scale versus the frequency of excitation. The average rotation of the base, $\bar{\Phi}_b$, and the rotation of the base of the shear walls, Φ_b , have been multiplied by $H = 43.3$ m (142 ft), the height of the roof with respect to the basement slab. The results shown indicate that the response at both the roof and the basement slab reach peak amplitudes at the same frequency of 1.79 Hz, which corresponds to the fundamental resonant frequency of the complete soil-structure system for vibrations in the N-S direction. By considering the width of the frequency response curves, it is found that the complete system damping ratio has a value of 1.8 per cent. At the resonant frequency the amplitude of the basement translation is 4.0 per cent of the amplitude of the total roof translation. The contribution of the rigid-body rotation, $H\Phi_b$, associated with rotation of the base of the shear walls amounts to 33.1 per cent of the total roof motion. If the average rotation of the basement slab is used as reference, the contribution of the rigid-body rotation, $H\bar{\Phi}_b$, amounts to 25.6 per cent of the total roof motion. These results confirm the experimental results of Foutch *et al.*⁵ and reveal significant soil-structure interaction effects during forced vibration tests. For N-S vibrations, the rigid-body motion of the superstructure associated with compliance of the soil corresponds to 37 per cent of the roof response, if it is assumed that the superstructure is driven by the shear walls, and up to 30 per cent, if it is assumed that the structure reacts to the average basement rotation.

The phases of the N-S response at the roof and basement levels with respect to the forcing function (e.g. $\bar{U}_T = |\bar{U}_T| \exp[-i\phi_4]$) plotted versus the frequency of excitation are shown in Figure 7(b). Inspection of Figure 7(b) indicates that the different response components are essentially in phase with each other (except for the translation of the base which lags the other components). For low frequencies, the response of the system is

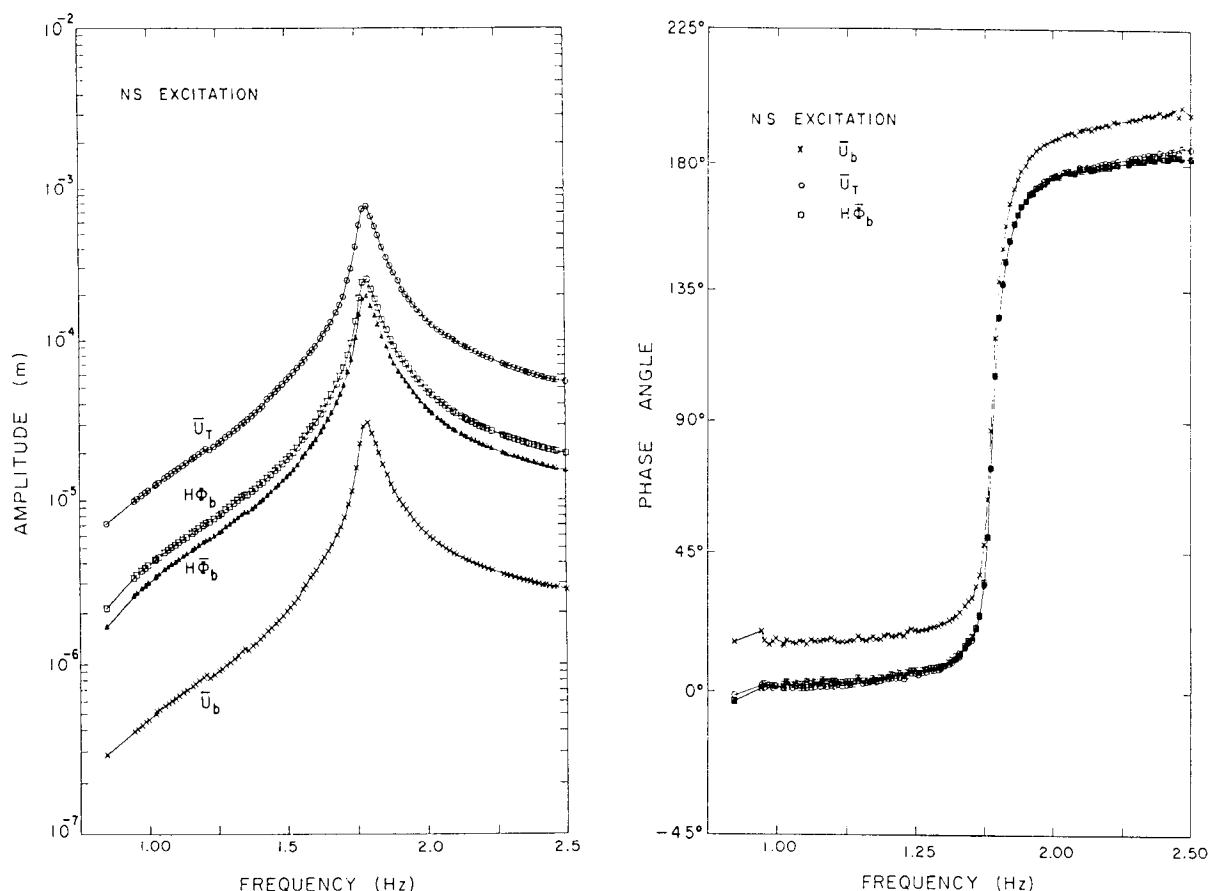


Figure 7. (a) Amplitude and (b) phase of the response for N-S excitation

in phase with the forcing function. At the resonant system frequency the response is 90° out of phase with the force at the roof while for higher frequencies the phase difference becomes 180° .

The results shown in Figures 8(a) and 8(b) indicate that for E-W vibrations the fundamental system frequency corresponds to 1.21 Hz and that the system damping is also 1.8 per cent. For E-W vibration, the translation of the basement slab amounts to 1.2 per cent of the roof response while the rigid-body rotation, $H\Phi_b$, associated with rotation of the central core amounts to 20.7 per cent of the total roof response. If the average rotation of the basement slab is taken as reference, the rigid-body rotation $H\bar{\Phi}_b$ contributes 6.2 per cent of the total roof motion. In summary, for vibrations in the E-W directions, the total rigid-body motion of the structure associated with foundation compliance amounts to 22 per cent of the roof response, if it is assumed that the response of the superstructure is controlled by the motion of the base of the central core, and to 7 per cent, if it is assumed that the superstructure is driven by the average motion of the basement slab. The lower interaction effects for E-W vibrations can be attributed to the higher flexibility of the superstructure in that direction.

The response of the system at the fundamental resonant frequencies is summarized in Table II, which includes values for the relative displacement

$$U_T = \bar{U}_T - U_b - H\Phi_b \quad \text{or} \quad U'_T = \bar{U}_T - U_b - H\bar{\Phi}_b \quad (6)$$

measured with respect to a frame of reference attached to the foundation.

A MODEL FOR THE RESPONSE OF THE SUPERSTRUCTURE

To study the effects that the base translations and rotations may have on the structural response it is convenient to consider the model of the superstructure shown in Figure 9. The superstructure is represented by a lumped

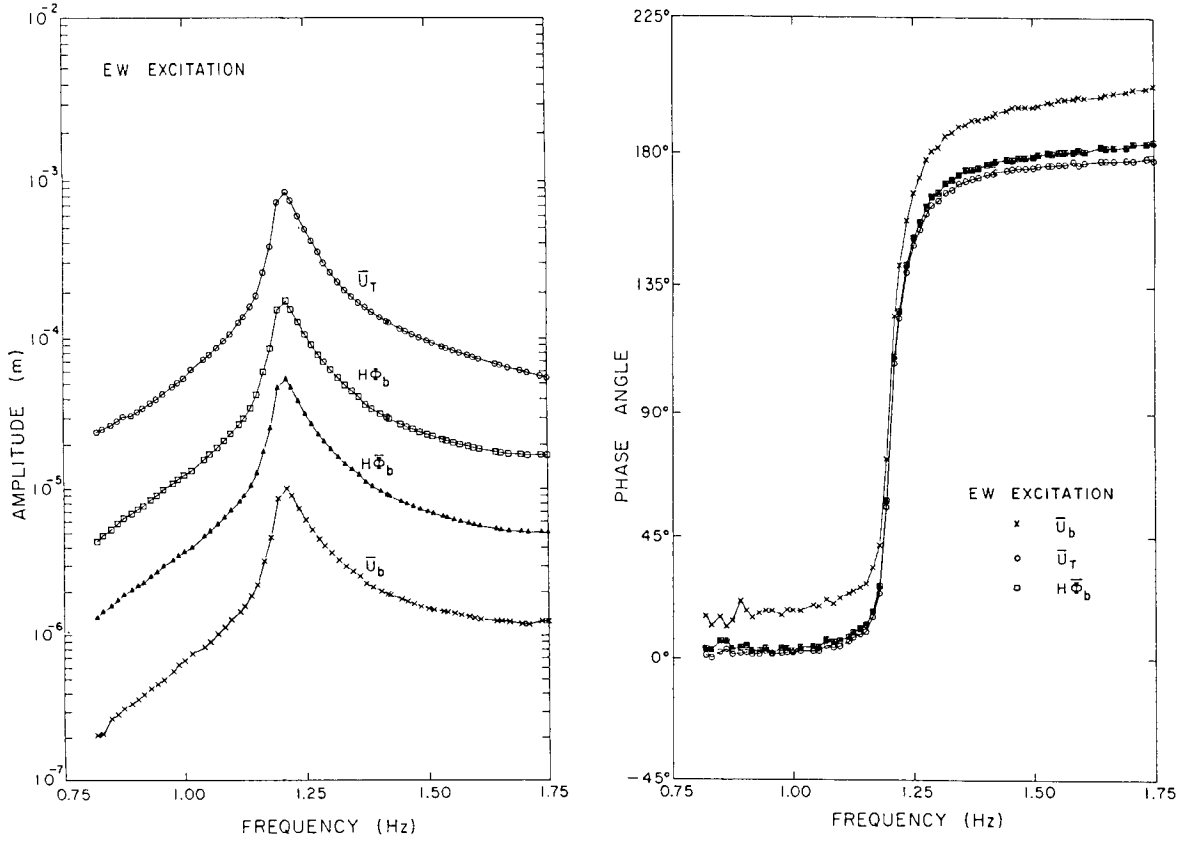


Figure 8. (a) Amplitude and (b) phase of the response for E-W excitation

mass model excited at the top level by the force, $F_T e^{i\omega t}$, that the harmonic vibration generator applies on the roof. The harmonic translation of the base is represented by $U_b e^{i\omega t}$ and the harmonic rotation of the base about a horizontal axis is represented by $\Phi_b e^{i\omega t}$.

The total harmonic displacement at the j th level, $\bar{U}_j e^{i\omega t}$ ($j = 1, N$), may be written in the form

$$\bar{U}_j e^{i\omega t} = (U_b + h_j \Phi_b + U_j) e^{i\omega t} \quad (7)$$

in which $U_j e^{i\omega t}$ represents the displacement at the j th level associated with deformation of the superstructure and h_j denotes the height of the j th level with respect to the basement slab.

The equation of motion for harmonic vibrations of the superstructure, considering the translation U_b and the rotation Φ_b of the base, can be written in terms of the fixed-base properties of the superstructure. The resulting equation is

$$-\omega^2 [M] \{\bar{U}\} + i\omega [C] \{U\} + [K] \{U\} = \{F\} \quad (8)$$

where $\{\bar{U}\}$ and $\{U\}$ represent the total and relative displacement vectors, respectively, and

$$\{F\} = (0, 0, 0, \dots, F_T)^T \quad (9)$$

denotes the vector of external forces applied to the superstructure. The $N \times N$ matrices $[M]$, $[C]$ and $[K]$ correspond to the mass, damping and stiffness matrices for the superstructure on a rigid base. Substitution from equation (7) into equation (8) leads to

$$(-\omega^2 [M] + i\omega [C] + [K]) \{U\} = \{F\} + \omega^2 [M] (\{1\} U_b + \{h\} \Phi_b) \quad (10)$$

in which

$$\{1\} = (1, 1, \dots, 1)^T \quad (11)$$

and

$$\{h\} = (h_1, h_2, \dots, h_N)^T$$

Table II. Response at the fundamental system frequency

		N-S vibrations	E-W vibrations
Fundamental system frequency $\tilde{\omega}_1 = \tilde{\omega}_1/2\pi$ (Hz)		1.79	1.21
System damping $\tilde{\xi}_1$		0.018	0.018
$ \bar{U}_T $	(m)	7.59×10^{-4}	8.49×10^{-4}
$ U_T = \bar{U}_T - H\Phi_b - U_b $	(m)	4.79×10^{-4}	6.63×10^{-4}
$ H\Phi_b $	(m)	2.51×10^{-4}	1.76×10^{-4}
$ U'_T = \bar{U}_T - H\bar{\Phi}_b - U_b $	(m)	5.36×10^{-4}	7.86×10^{-4}
$ H\bar{\Phi}_b $	(m)	1.94×10^{-4}	0.53×10^{-4}
$ U_b $	(m)	0.306×10^{-4}	0.100×10^{-4}
$F_T/\omega^2 M_b^*$	(m)	0.0973×10^{-4}	0.0973×10^{-4}
$ U_T/\bar{U}_T $		0.631	0.781
$ H\Phi_b/\bar{U}_T $		0.331	0.207
$ U_b/\bar{U}_T $		0.040	0.012
$ U'_T/\bar{U}_T $		0.706	0.926
$ H\bar{\Phi}_b/\bar{U}_T $		0.256	0.062
$ U_b/\bar{U}_T $		0.040	0.012
$ F_T/\omega^2 M_b \bar{U}_T $		0.013	0.011

* M_b denotes the total mass of the superstructure [$M_b g = 1.05 \times 10^8 \text{ N}$ ($23.5 \times 10^6 \text{ lb}$)].

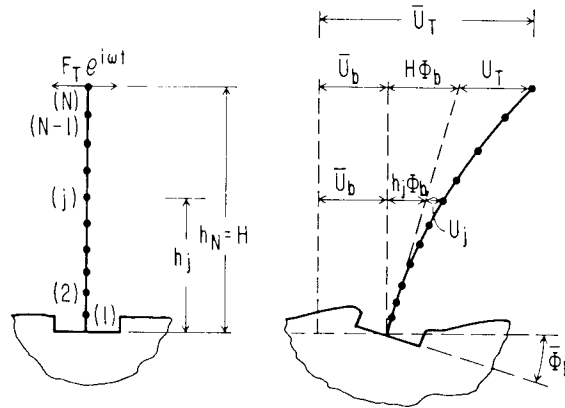


Figure 9. Model of the superstructure

Assuming that the superstructure, when fixed at the base, has classical normal modes it is possible to introduce the change of variables

$$\{U\} = \sum_{r=1}^N \{\phi^{(r)}\} \eta_r \quad (12)$$

where $\{\phi^{(r)}\}$ is the r th fixed-base mode. After substitution from equation (12) into equation (10), it is found that

$$\eta_r = \frac{(\omega/\omega_r)^2}{1 - (\omega/\omega_r)^2 + 2i\tilde{\xi}_r(\omega/\omega_r)} \left(\frac{F_T}{\omega^2 M_r} + \beta_r U_b + \gamma_r H\Phi_b \right) \quad (13)$$

where

$$M_r = \{\phi^{(r)}\}^T [M] \{\phi^{(r)}\}, \omega_r^2 = \frac{1}{M_r} \{\phi^{(r)}\}^T [K] \{\phi^{(r)}\}, \xi_r = \frac{1}{2\omega_r M_r} \{\phi^{(r)}\}^T [C] \{\phi^{(r)}\} \quad (14)$$

and

$$\beta_r = \frac{1}{M_r} \{\phi^{(r)}\}^T [M] \{1\}, \gamma_r = \frac{1}{H M_r} \{\phi^{(r)}\}^T [M] \{h\} \quad (15)$$

In the equations above, M_r denotes modal mass while ω_r and ξ_r correspond to the r th fixed-base natural frequency and modal damping ratio, respectively. For convenience, it is assumed that the modes are normalized to unity at the top of the structure ($\phi_N^{(r)} = 1$)

By use of equations (7), (12) and (13), it can be shown that the total displacement at the j th level of the superstructure is given by

$$\bar{U}_j = U_b + h_j \Phi_b + \sum_{r=1}^N \frac{(\omega/\omega_r)^2}{1 - (\omega/\omega_r)^2 + 2i\xi_r(\omega/\omega_r)} \left(\frac{F_T}{\omega^2 M_r} + \beta_r U_b + \gamma_r H \Phi_b \right) \phi_j^{(r)} \quad (16)$$

where $\phi_j^{(r)}$ denotes the amplitude of the r th fixed-base mode at the j th level. It is important to note that this relation between the response of the superstructure, the force acting on it and the translation and rotation of the base does not involve explicitly the characteristic of the foundation or the soil.

FIXED-BASE NATURAL FREQUENCIES AND MODAL DAMPING VALUES

One of the objectives of forced-vibration tests is the experimental isolation of the fixed-base natural frequencies and modal damping values of the superstructure. The model just described, and, in particular equation (16), provide the basis for a procedure to accomplish this objective. Equation (16) involves the total motion at a given level of the structure \bar{U}_j , and the translation U_b and rocking angle Φ_b of the base. These quantities can be determined experimentally over a frequency range. In addition to the known amplitude F_T of the force that the vibration generator exerts on the roof, equation (16) also involves the modal quantities M_r , β_r and γ_r , and the modal amplitudes $\phi_j^{(r)}$. The modes of vibration can be obtained experimentally or they can be estimated from a fixed-base model of the structure. The quantities M_r , β_r and γ_r depend on the geometry and mass distribution of the superstructure and on the fixed-base modes of vibration, as indicated in equations (14) and (15). Fortunately, at least for the fundamental mode, these quantities are not very sensitive to the details of the structure and can be easily estimated. The values of M_r/M_b (where M_b is the total mass of the superstructure), β_r and γ_r for the first two modes of the Millikan Library are listed in Table III. Since the values in the E-W and N-S directions are very similar, average values of $M_1/M_b = 0.35$, $\beta_1 = 1.42$ and $\gamma_1 = 1.07$ will be used in the subsequent analysis for both directions. The corresponding modal quantities for a uniform shear wall, a structure with a uniform mass distribution and a linear first mode, and a uniform beam are also listed in Table III for the purpose of comparison.

The above discussion indicates that all the quantities appearing in equation (16), except for the fixed-base frequencies ω_r and the damping ratios ξ_r , can be determined experimentally or estimated from the structural properties. Equation (16) can then be used to determine the modal constants ω_r and ξ_r . Consider the case in which the frequency of the excitation is in the neighbourhood of the fundamental fixed-base natural frequency of the superstructure. In this case, the contribution of the higher modes to the total response at the top of the structure ($j = N$) can be neglected and equation (16) reduces to

$$\bar{U}_T = U_b + H \Phi_b + \frac{(F_T/\omega^2 M_1) + \beta_1 U_b + \gamma_1 H \Phi_b}{(\omega_1/\omega)^2 - 1 + 2i\xi_1(\omega_1/\omega)} \quad (17)$$

where $\bar{U}_T = \bar{U}_N$ corresponds to the total motion at the top of the structure. Defining the function

$$A(\omega) = \frac{(F_T/\omega^2 M_1) + \beta_1 U_b + \gamma_1 H \Phi_b}{\bar{U}_T - U_b - H \Phi_b} \quad (18)$$

Table III. Modal characteristics of the Millikan Library

		First mode			Second		
		M_1/M_b^*	β_1	γ_1	M_2/M_b^*	β_2	γ_2
Millikan Library							
N-S	(1)	0.38	1.42	1.02	—	—	—
	(2)	0.35	1.42	1.12	—	—	—
E-W	(1)	0.33	1.44	1.08	0.40	-0.63	-0.08
	(2)	0.36	1.38	1.04	—	—	—
Average values used for N-S and E-W vibrations		0.35	1.42	1.07	0.40	-0.63	-0.08
Uniform shear wall		0.50	1.27	0.81	0.50	-0.42	0.09
Uniform mass distribution and linear first mode		0.33	1.50	1.00	—	—	0.00
Uniform beam		0.25	1.57	1.14	0.25	-0.87	-0.18

* M_b total mass of the superstructure.

(1) Based on experimental mode shapes reported by Jennings and Kuroiwa.⁹

(2) Based on experimental mode shapes reported by Foutch *et al.*⁵

which depends only on known quantities, it is possible to write equation (17) in the form

$$(\omega_1/\omega)^2 - 1 + 2i\xi_1(\omega_1/\omega) = A(\omega) \quad (19)$$

Taking real and imaginary parts in equation (19) and solving for ω_1 and ξ_1 leads to the following estimates for ω_1 and ξ_1 :

$$\omega_1 = \omega[1 + \operatorname{Re} A(\omega)]^{1/2} \quad (20)$$

$$\xi_1 = \frac{\operatorname{Im} A(\omega)}{2[1 + \operatorname{Re} A(\omega)]^{1/2}} \quad (21)$$

The frequency response data presented in the previous section have been used to compute the estimates of the fixed-base natural frequency ω_1 and of the structural modal damping ratio ξ_1 as defined by equations (20) and (21). The results obtained are shown in Figures 10(a) to 10(d) plotted versus the frequency of excitation. It must be pointed out that independent estimates of ω_1 and ξ_1 are obtained for each value of the excitation frequency.

Two sets of estimates have been obtained: the first set is based on the assumption that the superstructure responds to the average rotation of the foundation [i.e., $\alpha = 1$ in equation (5)]; the second estimate is based on the assumption that the superstructure is driven by the rotation of the base of the shear walls for N-S vibrations ($\alpha = 1.3$) and by the rotation of the base of the central core for E-W vibrations ($\alpha = 3.33$). In Figures 10(a) to 10(c), as well as in the rest of this paper, the first estimate is labelled 'rigid foundation' while the second estimate is labelled 'flexible foundation'.

The results presented in Figure 10(a) for vibrations in the N-S direction indicate that the estimates of the fixed-base natural frequency under the 'rigid foundation' assumption range from 2.16 to 2.29 Hz while the corresponding estimates under the 'flexible foundation' assumption range from 2.30 to 2.44 Hz. At the fundamental system frequency of 1.79 Hz, the estimates for the two conditions are 2.16 and 2.30 Hz, respectively. These estimates of the fixed-base natural frequencies are 21 and 28 per cent higher than the current (1975) fundamental system frequency and indicate that the interaction between the structure and the soil is significant for vibrations in the N-S direction. The results presented in Figure 10(b) indicate that estimates of

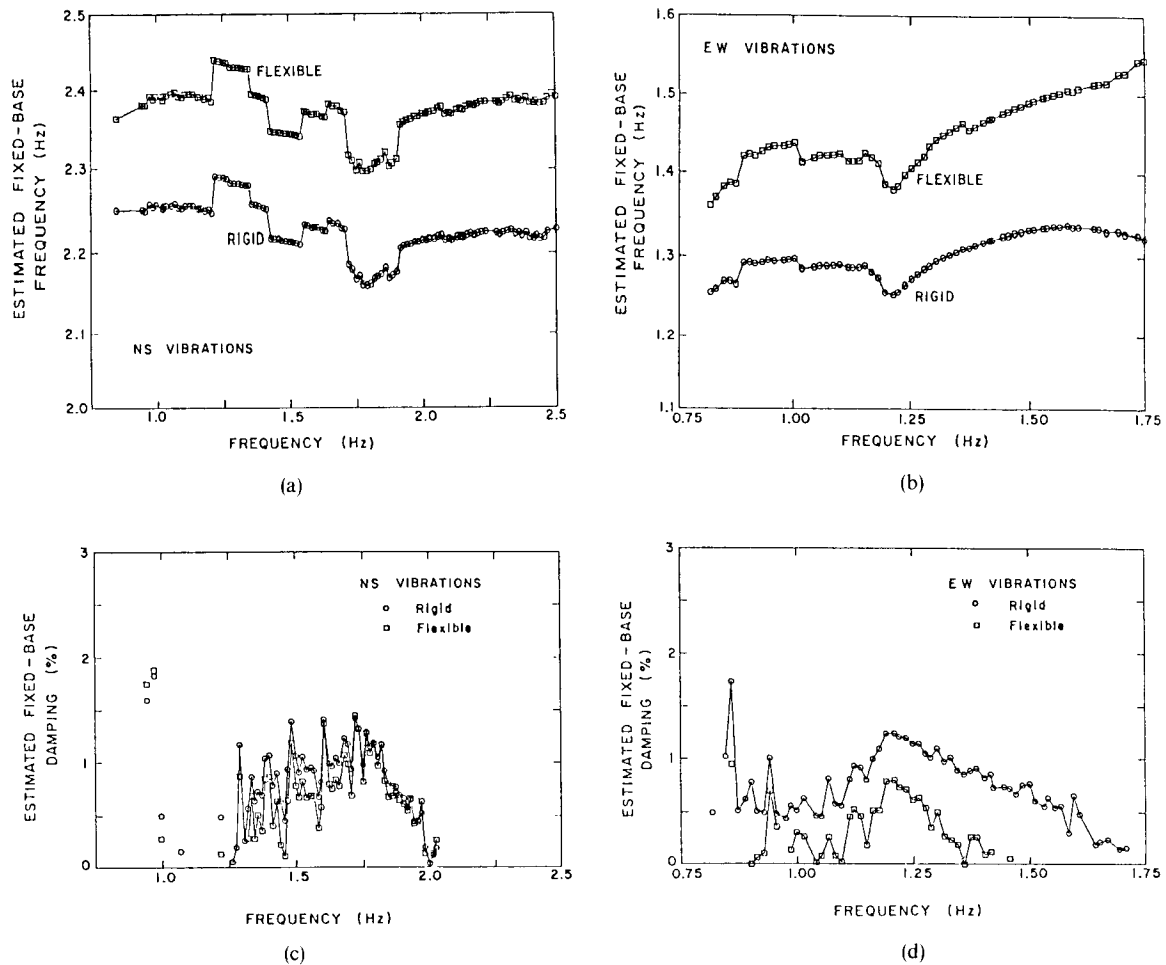


Figure 10. Estimates of the N-S and E-W fixed-base natural frequency and modal damping ratio

the E-W fixed-base natural frequency at the fundamental system frequency are 1.26 and 1.38 Hz, corresponding to the rigid and flexible foundation assumption, respectively. These values are 4 and 14 per cent higher than the current E-W fundamental system frequency of 1.21 Hz. Since earlier vibration tests indicate fundamental system frequencies in the E-W direction in excess of 1.45 Hz it seems that the assumption of a flexible foundation is more representative. The estimates of the fixed-base natural frequencies presented in Figures 10(a) and 10(b) exhibit minima in the vicinity of the fundamental system frequencies where the response of the structure is maximum. These minima may be associated with non-linear response of the superstructure. Given the limitations imposed by the accuracy of the experimental procedure it is not possible to arrive at a firmer conclusion.

The estimates of the fixed-base structural modal damping ratios for vibrations in the N-S and E-W directions are shown in Figures 10(c) and 10(d), respectively. The estimates obtained range from 0 to 1.5 per cent of critical damping. For N-S vibrations, a representative value in the vicinity of the fundamental system frequency is 1.2 per cent. For vibrations in the E-W direction the estimates of the structural damping ratio depend on whether the foundation is assumed rigid or flexible. If the foundation is assumed rigid a representative value for ξ_1 in the vicinity of the fundamental system frequency is 1.2 per cent. The corresponding value under the assumption of a flexible foundation is 0.8 per cent. These estimates are significantly lower than the damping ratio of the complete soil-structure system (1.8 per cent). The results obtained indicate that damping values obtained from forced vibration tests without correction for the effects of soil-structure interaction may overestimate the energy dissipation in the superstructure.

The estimates of ξ_1 for frequencies close to the system frequencies (1.79 Hz and 1.21 Hz for N-S and E-W vibrations, respectively) are very sensitive to the phase differences between the response at the top and base of the structure. For frequencies away from the system frequencies, the estimates of ξ_1 are highly sensitive to the phase difference between the response at the top and the forcing function. Under these conditions small instrumental errors or errors in the parameters used to correct for instrument response may have a significant effect on the estimated damping values. Owing to the accuracy limitations of the phase measurements it is not possible to draw conclusions as to the nature of the energy dissipation mechanism in the structure. In particular, it is not possible to say whether attenuation should be modelled as viscous or hysteretic.

CONTRIBUTION OF THE BASE MOTION TO THE TOTAL RESPONSE

For frequencies in the neighbourhood of the fundamental system frequency, the total displacement at the top of the structure \bar{U}_T , given by equation (16), can be written in the form

$$\bar{U}_T = \bar{U}_b + H\Phi_b + [(F_T/\omega^2 M_1) + \beta_1 U_b + \gamma_1 H\Phi_b] Z_1(\omega) \quad (22)$$

where

$$Z_1(\omega) = [(\omega_1/\omega)^2 - 1 + 2i\xi_1(\omega_1/\omega)]^{-1} \quad (23)$$

The term $(U_b + H\Phi_b)$ represents the contribution of the rigid-body motion due to soil compliance to the total motion at the top. The last term in equation (22), i.e. $[(F_T/\omega^2 M_1) + \beta_1 U_b + \gamma_1 H\Phi_b] Z_1(\omega)$, reflects the contribution of the deformation of the superstructure to the total motion at the top. Equation (22) reveals that the deformation of the superstructure results from the effects of the force applied at the top $[(F_T/\omega^2 M_1) Z_1(\omega)]$ and from the effects of the inertial forces associated with translation and rocking of the base $[\beta_1 U_b + \gamma_1 H\Phi_b] Z_1(\omega)$.

It must be pointed out that the interpretation of forced vibration tests is often made on the basis of the assumption that the base motion is negligible. In this case, the total motion at the top of the superstructure in the neighbourhood of the fundamental frequency is approximately given by

$$\bar{U}_T = (F_T/\omega^2 M_1) Z_1(\omega) \quad (\text{fixed base}) \quad (24)$$

Comparison of equations (22) and (24) indicates that the effects of the base motion can be neglected only if (i) $|U_b + H\Phi_b| \ll |\bar{U}_T|$ and (ii) $|\beta_1 U_b + \gamma_1 H\Phi_b| \ll |F_T/\omega^2 M_1|$. The first condition is satisfied in many cases, i.e. the rigid-body motion associated with ground compliance may be a small percentage of the total response. The second condition, however, is generally not satisfied. The term $F_T/\omega^2 M_1$ is proportional to the ratio of the eccentric mass of the vibration generator to the total mass of the superstructure. In most cases this ratio is extremely small and, consequently, a small amount of translation or rocking of the foundation is sufficient to invalidate the second condition.

In the case of the Millikan Library, and for vibrations in the N-S direction, the rigid-body motion amounts to 37 per cent of the total motion at the top, already indicating significant interaction effects. The most important result, however, corresponds to the fact that at the fundamental system frequency (1.79 Hz) the quantity $|\beta_1 U_b + \gamma_1 H\Phi_b|$ is 11.2 times larger than the term $|F_T/\omega^2 M_1|$, indicating that the deformation of the superstructure, at that frequency, is almost entirely caused by the inertial forces associated with motion of the foundation ($F_T/\omega^2 M_1 = 0.278 \times 10^{-4}$ m). In the E-W direction, the rigid-body motion associated with translation and rotation of the base of the central core contributes 22 per cent of the total response at the roof. The deformation of the superstructure at the E-W system frequency (1.21 Hz) is again essentially controlled by the inertial forces due to the motion of the base. The term $|\beta_1 U_b + \gamma_1 H\Phi_b|$ is, in this case, 7.3 times larger than $|F_T/\omega^2 M_1|$.

A vector representation of the complex quantities appearing in equation (22) is shown in Figures 11(a) and 11(b). Figure 11(a) represents the roof response at the fundamental system frequency ($\omega = \tilde{\omega}_1$), while Figure 11(b) represents the response at the fundamental fixed-base natural frequency of the superstructure ($\omega = \omega_1$), both for vibrations in the N-S direction. According to equation (22), the total motion at the top \bar{U}_T

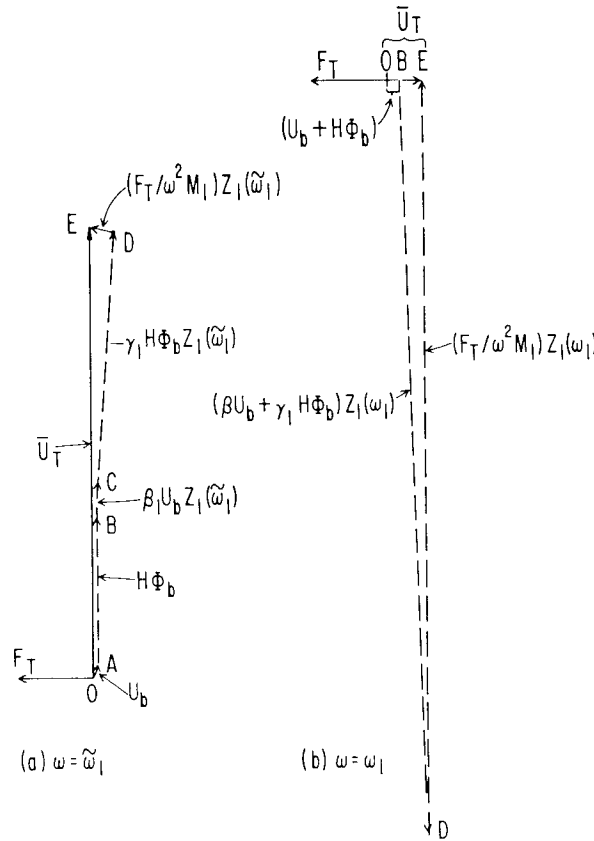


Figure 11. Vector representation of the contributions of the base translation and rotation to the total response at the top for N-S vibrations: (a) for $\omega = \tilde{\omega}_1$ and (b) for $\omega = \omega_1$

corresponds to the vector sum of the terms U_b , $H\Phi_b$, $\beta_1 U_b \dot{Z}_1(\omega)$, $\gamma_1 H\Phi_b Z_1(\omega)$ and $(F_T/\omega^2 M_1) Z_1(\omega)$. These terms are represented in Figures 11(a) and 11(b) by the vectors OA, AB, BC, CD and DE, respectively. The total motion at the top, \bar{U}_T , is represented then by the vector OE, while the relative motion at the top, U_T , is represented by the vector BE. If the structure were supported on a rigid base, the total response at the top of the structure would correspond to the vector DE. Figure 11(a) illustrates that the motion of the base is responsible for almost the entire response at the fundamental system frequency (1.79 Hz). In particular, the total contribution of the motion of the base is represented by the vector OD which has an amplitude only slightly lower than the amplitude of the total response OE.

At the N-S fixed-base natural frequency of the superstructure (2.30 Hz) the deformation caused by the inertial forces due to motion of the base (vector BD) essentially cancels the deformation due to the force applied at the top (vector DE) and yields the total deformation at the top represented by the vector BE [Figure 11(b)]. It may be appreciated that the motion of the base has significantly modified the response of the structure at this frequency. Comparison of the vector DE in Figure 11(b) with the total response represented by vector OE in Figure 11(a) indicates that the soil-structure interaction effects have not only introduced a frequency shift, but have also modified the amplitude of the peak response.

EXPERIMENTAL EVALUATION OF IMPEDANCE FUNCTIONS

The foundation impedance functions play an important role in soil-structure interaction studies. It is of interest to explore the possibility of determining these functions experimentally. To achieve this purpose it is convenient to refer to previous studies of the response of the foundation of the Millikan Library (Wong, *et*

al.²⁴) which indicate that, although the foundation experiences some deformation, the relation between the base forces and moments and the average translation and rotation of the base can be obtained by use of a rigid foundation model. Thus, if $H_s e^{i\omega t}$ and $M_s e^{i\omega t}$ represent the force and moment that the foundation exerts on the soil and $\bar{U}_b e^{i\omega t}$ and $\bar{\Phi}_b e^{i\omega t}$ represent the average translation and rotation of the base, it is possible to write

$$H_s = GL(K_{HH}\bar{U}_b + K_{HM}L\bar{\Phi}_b) \quad (25)$$

$$M_s = GL^2(K_{MH}\bar{U}_b + K_{MM}L\bar{\Phi}_b) \quad (26)$$

where K_{HH} , $K_{HM} = K_{MH}$, K_{MM} represent the complex frequency-dependent horizontal, coupling and rocking impedance functions for the foundation assumed rigid. The impedance functions are normalized by a reference shear modulus G and a characteristic length L . The horizontal force H_s and the moment M_s that the foundation exerts on the soil can also be obtained by considering the variation of the linear and angular momenta of the superstructure and foundation. For the lumped mass model of the superstructure shown in Figure 9, it is found that

$$H_s = F_T + \omega^2 M_o \bar{U}_b + \omega^2 \{1\}^T [M] \{\bar{U}\} \quad (27)$$

$$M_s = HF_T + \omega^2 I_o \bar{\Phi}_b + \omega^2 I_{ob} \Phi_b + \omega^2 \{h\}^T [M] \{\bar{U}\} \quad (28)$$

where M_o corresponds to the mass of the foundation, I_o to the mass moment of inertia of the foundation with respect to a horizontal axis through the centre of the basement slab, and I_{ob} to the sum of the moments of inertia of all floors with respect to horizontal axes through the centres of each floor. In equation (28) the contribution of the rotary inertia associated with deformation of the superstructure has been neglected.

When the frequency of the excitation is in the neighbourhood of the fundamental fixed-base natural frequency, it is possible to approximate the total displacement vector $\{\bar{U}\}$ appearing in equations (27) and (28) by

$$\{\bar{U}\} = \{1\} \bar{U}_b + \frac{1}{H} \{h\} \alpha H \bar{\Phi}_b + \{\phi^{(1)}\} (\bar{U}_T - \bar{U}_b - \alpha H \bar{\Phi}_b) \quad (29)$$

Substitution from equation (29) into equations (27) and (28) leads to the approximations (for $\omega \ll \omega_2$)

$$\frac{H_s}{\omega^2 M_b} = \frac{F_T}{\omega^2 M_b} + \left(1 + \frac{M_o}{M_b} - \beta_1 \frac{M_1}{M_b}\right) \bar{U}_b + \alpha \left(\frac{S_b}{HM_b} - \beta_1 \frac{M_1}{M_b}\right) H \bar{\Phi}_b + \beta_1 \frac{M_1}{M_b} \bar{U}_T \quad (30)$$

$$\frac{M_s}{\omega^2 M_b H} = \frac{F_T}{\omega^2 M_b} + \left(\frac{S_b}{HM_b} - \gamma_1 \frac{M_1}{M_b}\right) \bar{U}_b + \alpha \left(\frac{\alpha^{-1} I_o + I_{ob} + I_b}{H^2 M_b} - \gamma_1 \frac{M_1}{M_b}\right) H \bar{\Phi}_b + \gamma_1 \frac{M_1}{M_b} \bar{U}_T \quad (31)$$

in which $M_b = 10.7 \times 10^6$ kg (730×10^3 lb) is the total mass of the superstructure, $H = 43.3$ m (142 ft) and

$$S_b = \{h\}^T [M] \{1\}, \quad I_b = \{h\}^T [M] \{h\} \quad (32)$$

The expressions for H_s and M_s given by equations (30) and (31) involve quantities that can be estimated (M_o , I_o , M_b , S_b , I_b , I_{ob} , M_1/M_b , β_1 , γ_1 , α) or experimentally determined (\bar{U}_b , $\bar{\Phi}_b$, \bar{U}_T). The values of S_b/HM_b , $I_b/H^2 M_b$ and M_o/M_b are 0.554, 0.395 and 0.136, respectively. The values of $I_{ob}/H^2 M_b$ and $I_o/H^2 M_b$ are, respectively, 0.027 and 0.044 for N-S vibrations and 0.030 and 0.005 for E-W vibrations. For the particular vibration generator used in the experiment $F_T/\omega^2 M_b = 0.973 \times 10^{-5}$ m.

Once H_s and M_s have been calculated, the problem of determining the impedance functions reduces to solving the two linear equations (25) and (26) for the three unknowns K_{HH} , K_{MM} and $K_{HM} = K_{MH}$. It is apparent that it is not possible to obtain a unique solution. It is possible, however, to define the following approximations to the horizontal and rocking impedances functions:

$$K'_{HH} = \frac{H_s}{GL\bar{U}_b} \quad (33)$$

$$K'_{MM} = \frac{M_s}{GL^3 \bar{\Phi}_b} \quad (34)$$

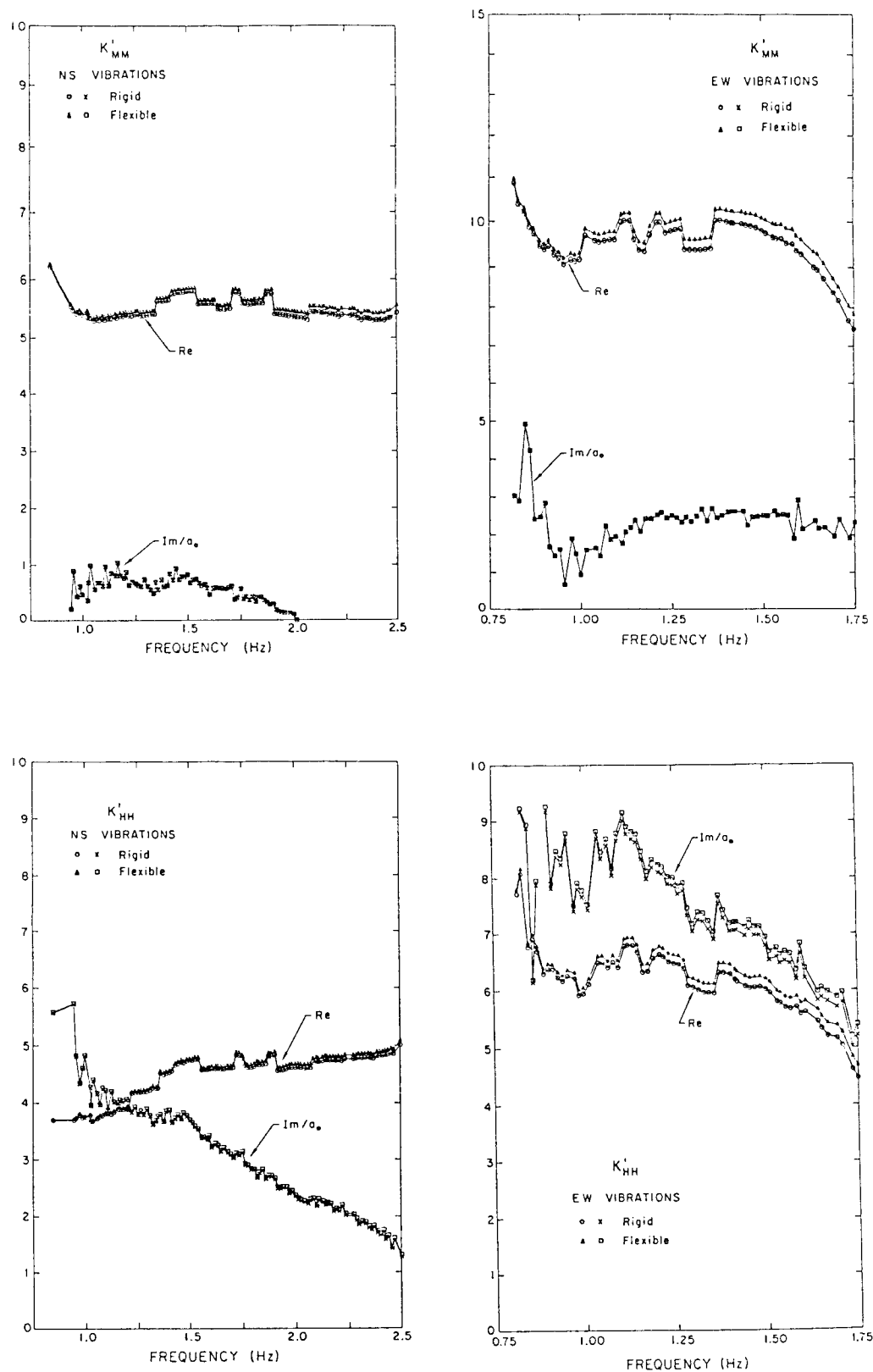


Figure 12. Approximate rocking and horizontal impedance functions for N-S and E-W vibrations

These approximations are based on the assumption that the coupling terms $K_{HM} = K_{MH}$ are considerably smaller than K_{HH} and K_{MM} .

Estimates of the approximate impedance functions K'_{MM} and K'_{HH} , based on the experimental data and the use of equations (30), (31), (33), and (34), are shown versus frequency in Figure 12. In each figure, the real part and the imaginary part divided by the dimensionless frequency $a_0 = \omega L/\beta$, where β is a shear wave velocity of reference, are shown for the 'rigid' and 'flexible' foundation models. The impedance functions are normalized by use of the shear modulus of reference $G = 2.68 \times 10^8 \text{ N/m}^2$ ($38.9 \times 10^3 \text{ psi}$), shear wave velocity of reference $\beta = 382 \text{ m/sec}$ (1253 ft/sec) and length of reference $L = 13.7 \text{ m}$ (45 ft). The length of reference L corresponds approximately to the radius of an equivalent circular foundation of the same area as the actual foundation. The shear modulus G and shear wave velocity of reference β correspond to the average of these soil properties at three sites close to the Library and at the depth of the bottom of the foundation (Eguchi *et al.*⁴ Shannon and Wilson, Inc.¹⁸).

The experimental estimates of the impedance functions shown in Figure 12 conform with the expectations based on theoretical studies of the response of foundations. It appears that the proposed experimental approach can lead to reliable estimates of the foundation impedance functions.

CONCLUSIONS

It has been shown that estimates of the fixed-base fundamental frequencies of the superstructure, the fixed-base modal damping ratios of the superstructure and the foundation impedance functions can be determined experimentally by use of forced vibration tests. The procedure is based on recording the amplitude and phase of the translational response at the top of the superstructure as well as the amplitudes and phases of the translational and rotational response of the base. Analyses of the results of forced vibration tests of the nine-storey reinforced concrete Millikan Library Building reveal that interpretations of forced vibration tests which do not include soil-structure interaction effects underestimate the fixed-base natural frequencies of the superstructure and overestimate the energy dissipation within the structure.

The experimental results obtained indicate that the interaction between structures and the soil may have significant effects on the response during forced vibration tests. In addition to the effects already mentioned, the translation and rotation of the foundation associated with ground compliance lead to a rigid-body motion of the superstructure that may account for a significant portion of the total response. In the case of the Millikan Library Building, the rigid-body motion by itself accounts for more than 30 per cent of the total N-S roof response. Perhaps even more important is the finding that the inertial forces associated with the rigid-body motion account for a major portion of the deformation of the superstructure at the resonant system frequency.

From the experimental point of view, it was found that reliable estimates of the damping ratio of the superstructure and of the foundation damping coefficients (associated with the imaginary part of the impedance functions) can be obtained only if the phase of the different response components with respect to the forcing function is recorded with high accuracy.

ACKNOWLEDGEMENT

The experimental work described herein was planned in 1974 and conducted in 1975. The analysis of the data has been performed on an intermittent basis between 1975 and 1986. Partial support during this period from contracts with the U.S. Geological Survey, Earthquake Research Affiliates Program at the California Institute of Technology, and more recently from the Argonne National Laboratory is gratefully acknowledged.

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