A comparison of soil-structure interaction calculations with results of full-scale forced vibration tests

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The capability of a simple soil-structure interaction model to predict the response of structures during forced vibration tests is evaluated by comparison of the theoretical and experimental response of a nine-storey reinforced concrete building. A very good agreement between theoretical and experimental results was obtained for vibrations in which the foundation acts as a rigid body. It was also found that simple models may not be applicable when the foundation is highly deformable.

INTRODUCTION

Recently, the authors[1] have reported a complete set of experimental results obtained during forced-vibration tests of the nine-storey reinforced concrete Millikan Library Building (Fig. 1). The results presented include amplitude and phase (with respect to the forcing function) of the translational response of the roof, translational response of the base and rocking response of the base over a range of frequencies for excitation in two horizontal directions. On the basis of this information, the authors were able to isolate experimentally structural properties such as fixed-base natural frequencies and fixed-base modal damping ratios. Experimental estimates of the horizontal and rocking foundation impedance functions were also obtained. The objective of this study is to assess the capability of simple mathematical soil-structure interaction models to reproduce the experimental response during forced-vibration tests. Of particular interest is to determine whether simple foundation and soil models can result in reasonable estimates of the foundation impedance functions.

The objectives of the study are accomplished by detailed comparison of theoretical and experimental results. These comparisons are preceeded by a presentation of the mathematical soil-structure interaction model used in the calculation of the response and by a description of the properties of the Millikan Library system.

BASIC INTERACTION EQUATIONS

To study the effects of soil-structure interaction on structural response during forced-vibration tests it is convenient to consider the model of the superstructure shown in Fig. 2. The superstructure is represented by a lumped mass model excited at the top level by the force $F_T e^{i\omega t}$ that the harmonic vibration generator applies on the roof. The harmonic translation of the base is represented by $U_b e^{i\omega t}$ and the harmonic rotation about a horizontal axis of the active elements of the base is represented by $\Phi_b e^{i\omega t}$ in which ω is the frequency.

The total harmonic displacement of the jth level, $\bar{U}_j e^{i\omega t}$ $(j=1, N)$, can be written in the form

$$\bar{U}_j e^{i\omega t} = (U_b + h_j\Phi_b + U_j) e^{i\omega t} \qquad (1)$$

in which h_j denotes the height of the jth level with respect to the basement slab and $U_j e^{i\omega t}$ represents the displacement of the jth level relative to a frame of reference attached to the active elements of the foundation.

The equation of motion for small harmonic vibrations of the superstructure is

$$-\omega^2[M]\{\bar{U}\} + i\omega[C]\{U\} + [K]\{U\} = \{F\} \qquad (2)$$

where $\{\bar{U}\}$ and $\{U\}$ represent the total and relative displacement vectors, respectively, and

$$\{F\} = (0, 0, 0, \ldots, F_T)^T \qquad (3)$$

denotes the vector of external forces applied to the superstructure. The $N \times N$ matrices $[M]$, $[C]$, and $[K]$ correspond to the mass, damping and stiffness matrices for the superstructure on a fixed-base.

Expansion of the relative displacement vector in terms of the fixed-base modes of vibration of the superstructure

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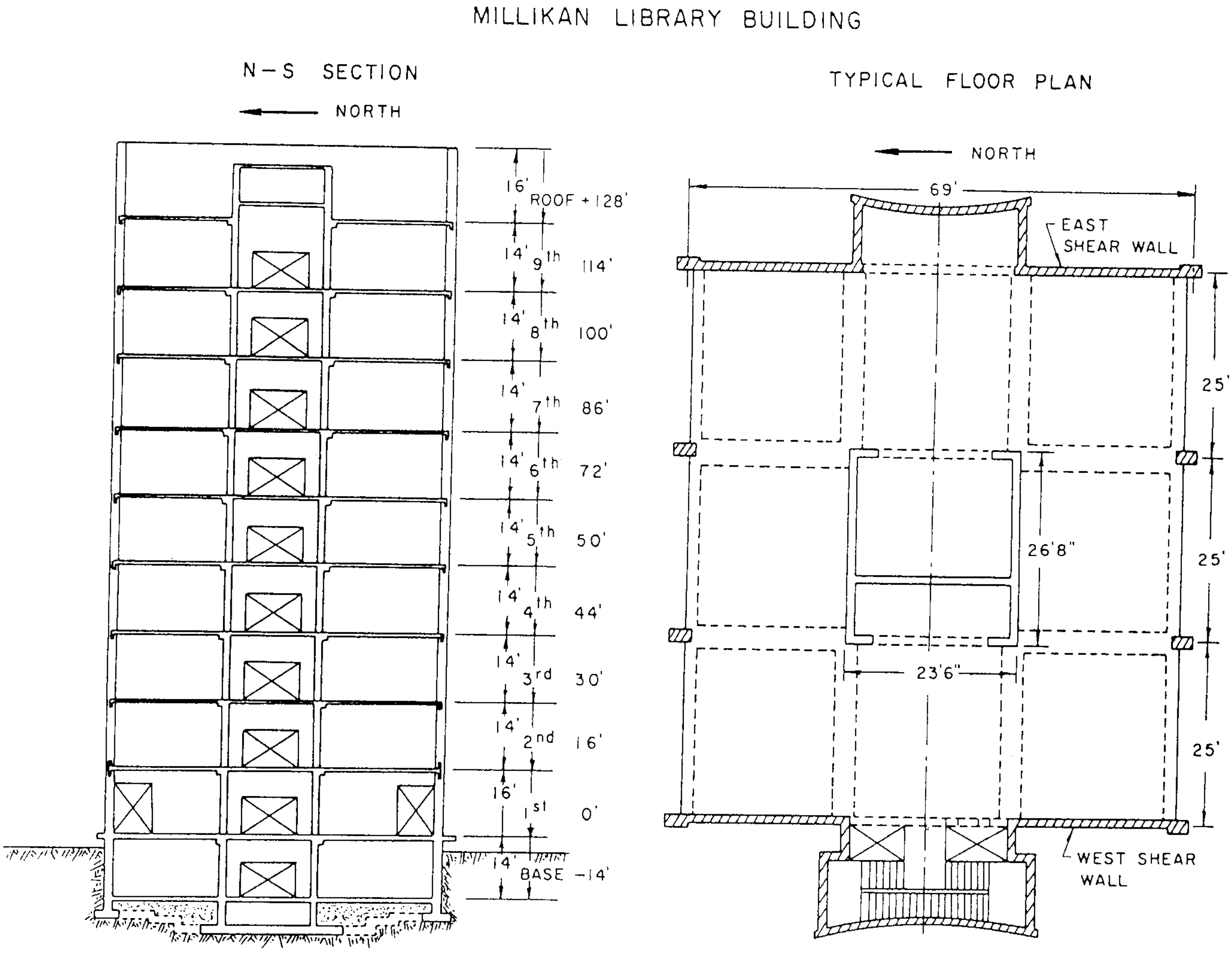

Fig. 1. *Millikan Library Building: NS elevation and typical floor plan*

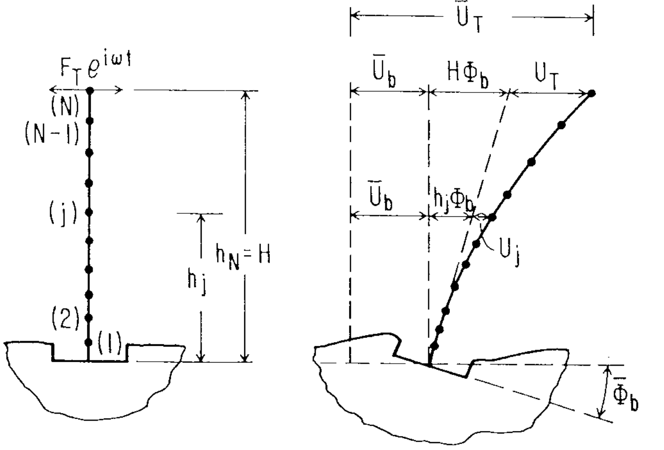

Fig. 2. *Model of the superstructure*

and use of equations (1), (2) and (3) leads to

$$\bar{U}_j = U_b + h_j\Phi_b + \sum_{r=1}^{N} Z_r(\omega/\omega_r, \xi_r) \times \left(\frac{F_T}{\omega^2 M_r} + \beta_r U_b + \gamma_r H\Phi_b\right)\phi_j^{(r)} \tag{4}$$

where $\phi_j^{(r)}$ denotes the amplitude of the rth fixed-based mode at the jth level and $Z_r(\omega/\omega_r, \xi_r)$ is the dynamic amplification factor

$$Z_r(\omega/\omega_r, \xi_r) = \frac{(\omega/\omega_r)^2}{1-(\omega/\omega_r)^2 + 2i\xi_r(\omega/\omega_r)} \tag{5}$$

The rth modal mass M_r and rth fixed-base natural frequency and modal damping ratio are defined by

$$M_r = \{\phi^{(r)}\}^T[M]\{\phi^{(r)}\}, \quad \omega_r^2 = \frac{1}{M_r}\{\phi^{(r)}\}^T[K]\{\phi^{(r)}\},$$

$$\xi_r = \frac{1}{2\omega_r M_r}\{\phi^{(r)}\}^T[C]\{\phi^{(r)}\} \tag{6}$$

The rth participation factor β_r and the modal height $\gamma_r H$ are given by

$$\beta_r = \frac{1}{M_r}\{\phi^{(r)}\}^T[M]\{1\}, \quad \gamma_r = \frac{1}{HM_r}\{\phi^{(r)}\}^T[M]\{h\} \tag{7}$$

in which H is the total height of the superstructure. It is assumed that the modes are normalized to unity at the top of the structure ($\phi_N^{(r)} = 1$), and the vectors $\{1\}$ and $\{h\}$ are given by

$$\{1\} = (1, 1, \ldots, 1)^T, \qquad \{h\} = \{h_1, h_2, \ldots, h_N\}^T \tag{8}$$

Equation (4) shows that the total displacement at the jth level may be thought as being formed by two parts: the first part $(U_b+h_j\Phi_b)$ corresponds to a rigid-body motion while the second part, given by the term with the summation sign, corresponds to the deformation of the superstructure. The form of this last term indicates that the deformation of the superstructure arises from the force applied at the top (terms proportional to F_T) and from the inertial forces associated with translation and rocking of the base (terms proportional to U_b and Φ_b).

The horizontal force $H_S e^{i\omega t}$ and the moment $M_S e^{i\omega t}$ that the foundation exerts on the soil can be obtained by considering the linear and angular momenta of the superstructure and foundation. For the lumped mass model of the superstructure shown in Fig. 2, it is found that

$$H_S=F_T+\omega^2 M_0\bar{U}_b+\omega^2\{1\}^T[M]\{\bar{U}\} \tag{9}$$

$$M_S=HF_T+\omega^2 I_0\bar{\Phi}_b+\omega^2 I_{0b}\Phi_b+\omega^2\{h\}[M]\{\bar{U}\} \tag{10}$$

where $\bar{U}_b e^{i\omega t}$ and $\bar{\Phi}_b e^{i\omega t}$ are, respectively, the average translation and rotation of the base, M_0 corresponds to the mass of the foundation, I_0 to the mass moment of inertia of the foundation with respect to a horizontal axis through the centre of the basement slab, and I_{0b} to the sum of the moments of inertia of all floors with respect to horizontal axes through the centres of each floor. In equation (10) the contribution of the rotary inertia associated with deformation of the superstructure has been neglected.

To complete the formulation of the interaction problem it is necessary to invoke the relations between H_S, M_S, $\bar{U}_b$, $\bar{\Phi}_b$, U_b and Φ_b resulting from the flexibility of the soil. Previous studies of the response of the foundation of the Millikan Library[2] indicate that although the foundation experiences some deformation, the relation between the base forces and moments and the average translation and rotation of the base can be obtained by use of a rigid foundation model. Thus, it is possible to write

$$H_S=GL(K_{HH}\bar{U}_b+K_{HM}L\bar{\Phi}_b) \tag{11}$$

$$M_S=GL^2(K_{MH}\bar{U}_B+K_{MM}L\bar{\Phi}_b) \tag{12}$$

where K_{HH}, $K_{HM}=K_{MH}$, K_{MM} represent the normalized complex frequency-dependent impedance functions for the foundation assumed rigid. The impedance functions are normalized by a reference shear modulus G and a characteristic length L and depend on the shape of the foundation and the characteristics of the soil. In the case of the Millikan Library Building, the motion of U_b, Φ_b of the active elements of the foundation, i.e., those elements directly connected with motion of the superstructure, and the average motion $\bar{U}_b$, $\bar{\Phi}_b$ of the foundation are related in the form

$$U_b=\bar{U}_b,\qquad \Phi_b=\alpha\bar{\Phi}_b \tag{13}$$

where α is a flexibility parameter determined experimentally.

Equations (4) and (9)–(13) determine completely the soil-structure interaction problem and permit the response of the foundation and superstructure to be obtained in terms of the applied force F_T. In particular, it is found that the motion of the base (U_b, Φ_b) and top $(\bar{U}_T=\bar{U}_N)$ of the superstructure is given by

$$\begin{pmatrix} U_b \\ H\Phi_b \end{pmatrix}=\left(\frac{\omega}{\omega_1}\right)^2\kappa\left([\bar{K}]-\left(\frac{\omega}{\omega_1}\right)^2\kappa[\bar{M}]\right)^{-1}\{\bar{F}\}\left(\frac{F_T}{\omega^2 M_b}\right) \tag{14}$$

and

$$\bar{U}_T/(F_T/\omega^2 M_b)=\left(\sum_{r=1}^{N}\frac{M_b}{M_r}Z_r\right)+\left(\frac{\omega}{\omega_1}\right)^2\kappa\{\bar{F}\}^T \times\left([\bar{K}]-\left(\frac{\omega}{\omega_1}\right)^2\kappa[\bar{M}]\right)^{-1}\{\bar{F}\} \tag{15}$$

where M_b is the total mass of the superstructure and κ is a dimensionless parameter defined by

$$\kappa=\frac{\omega_1^2 M_b}{GL} \tag{16}$$

characterizing the relative stiffness of the structure as compared with that of the soil. The vector $\{\bar{F}\}$ and the matrices $[\bar{M}]$ and $[\bar{K}]$ are defined by

$$\{\bar{F}\}=\begin{Bmatrix}1\\1\end{Bmatrix}+\sum_{r=1}^{N}\begin{Bmatrix}\beta_r\\ \gamma_r\end{Bmatrix}Z_r \tag{17}$$

$$[\bar{M}]=\begin{bmatrix}\left(1+\dfrac{M_0}{M_b}\right) & \left(\dfrac{S_b}{HM_b}\right)\\ \dfrac{S_b}{HM_b} & \dfrac{\alpha^{-1}I_0+I_{0b}+I_b}{H^2M_b}\end{bmatrix}+\sum_{r=1}^{N}\begin{bmatrix}\beta_r^2 & \beta_r\gamma_r\\ \beta_r\gamma_r & \gamma_r^2\end{bmatrix}\times\frac{M_r}{M_b}Z_r \tag{18}$$

$$[\bar{K}]=\begin{bmatrix}K_{HH} & \alpha^{-1}\gamma K_{HM}\\ \gamma K_{MH} & \alpha^{-1}\gamma^2 K_{MM}\end{bmatrix} \tag{19}$$

in which $\gamma=L/H$ and

$$S_b=\{h\}^T[M]\{1\},\qquad I_b=\{h\}^T[M]\{h\} \tag{20}$$

CHARACTERIZATION OF THE MILLIKAN LIBRARY BUILDING

To calculate the response of the Millikan Library Building during forced vibration tests it is necessary to assign values to the various structural, foundation and soil parameters appearing in the mathematical model described above. Fortunately, a wealth of information is readily available.

Structural properties

The Robert A. Millikan Library is a nine-storey reinforced concrete building located on the campus of the California Institute of Technology. The structure has a basement and an enclosed roof area. The typical floor plan covers an area of 21 × 23 m (69 × 75 ft) and the building stands 43.9 m (144 ft) above the first floor level and 48.2 m (158 ft) above the basement slab (see Fig. 1). The majority of the lateral loads in the transverse (N–S) direction are resisted by 30 cm (12 in) reinforced concrete shear walls located on the east and west ends of the building. The 30 cm (12 in) reinforced concrete walls of the central core provide most of the lateral resistance in the longitudinal (E–W) direction. The north-side and south-side facades are precast window wall panels connected to the main structure with steel angle clips. The floor system consists of 23 cm (9 in) slabs of lightweight concrete reinforced in two directions and supported by reinforced concrete beams.

The mathematical model of the superstructure involves the parameters M_b, H, S_b, I_b, I_{0b}, and the modal quantities $\omega_r, \xi_r, M_r, \beta_r, \gamma_r$. The first set of parameters was evaluated on the basis of the geometry of the structure and from the mass distribution reported by Jennings and Kuroiwa[3]. The resulting values are: $M_b=10.7\times10^6$ kg $(0.73\times10^6$ lb sec^2/ft), $H=43.3$ m (142 ft), $S_b/HM_b=0.544$, $I_b/H^2M_b=0.395$ and $I_{0b}/H^2M_b=0.027$ (for NS vibrations) and 0.030 (for EW vibrations). The quantities M_r/M_b, β_r and γ_r for the first two fixed-base modes were obtained from knowledge of the geometry and mass distribution of the superstructure and from the experimental mode shapes reported by Jennings and Kuroiwa[3] and Foutch *et al.*[4]. The average values of these parameters do not differ much from those for the N–S and E–W directions[5] and will be used for both directions. These values are: $M_1/M_b=0.35$, $\beta_1=1.42$, $\gamma_1=1.07$, $M_2/M_b=0.40$, $\beta_2=-0.63$ and $\gamma_2=-0.08$.

The values for the fundamental fixed-base frequencies (ω_1) and fixed-base modal damping ratios (ξ_1) were based on estimates obtained by the authors[1] by a process which isolates these fixed-base structural properties from the results of forced vibration tests. Two sets of estimates were obtained. The first set is based on the assumption that the response of the superstructure is driven by the average translation and rotation of the foundation. This set of estimates is designated as 'rigid foundation' estimates and corresponds to a value of $\alpha=1$ in equation (13). The second set of estimates is based on the assumption that the response of the superstructure is controlled by the motion of the base of the shear walls for N–S vibrations and by the motion of the base of the central core for E–W vibrations. These estimates, labelled as 'flexible foundation' estimates, correspond to experimentally determined values of $\alpha=1.30$ for N–S vibrations and $\alpha=3.33$ for E–W vibrations. The resulting values of ω_1 and ξ_1 are listed in Table 1. The estimates for ω_2 were based on the results of forced vibrations tests and the value of ξ_2 was set equal to ξ_1 (Table 1). For reasons to be discussed later on, the values of ξ_1 and ξ_2 for E–W vibrations appearing in Table 2 were replaced by a value of 0.015 for the actual calculations.

No attempt was made to calculate the fixed-base natural frequencies and mode shapes directly from the elastic properties of the superstructure.

Table 1. Estimates of the fixed-base frequencies and modal damping ratios

	N–S		E–W	
	Rigid	Flexible	Rigid	Flexible
α	1.00	1.33	1.00	3.33
$\omega_1/2\pi$ (Hz)	2.16	2.30	1.26	1.38
ξ_1	0.012	0.012	0.012	0.008
$\omega_2/2\pi$ (Hz)	10.0	10.0	6.20	6.20
ξ_2	0.012	0.012	0.012	0.008

Foundation model

The foundation system of the Millikan Library consists of a central pad 9.75 m (32 ft) wide and 1.22 m (4 ft) deep which runs in the E–W direction and extends from the east curved shear wall to the west curved shear wall (Fig. 3). Also provided are beams 3 m (10 ft) wide by 0.61 m (2 ft) deep which run E–W beneath the rows of columns at the north and south ends of the building. These beams are connected to the central pad by stepped beams. The contact between the central pad and the underlying soil is approximately 7 m (23 ft) below the first-floor level. The plan dimensions of the foundation are approximately 23.3×25.1 m $(76\times82.5$ ft) with additional areas of dimensions 9.9×1.7 m $(32.5\times5.5$ ft) and 9.9×3.5 m $(32.5\times11.5$ ft) at the east and west extremes, respectively. The foundation rests on alluvium composed of medium to dense sands mixed with gravel extending about 275 m (900 ft) to bedrock.

The radius of a circular foundation of equal area is 14.2 m (46.7 ft), while the radii for circular foundations with equal moments of inertia about E–W and N–S axes are 13.6 m (44.7 ft) and 15.2 m (49.7 ft), respectively. The embedment depth varies from about 4.0 m (13 ft) along the north and south ends to about 5.5 m (18 ft) on the central E–W foundation pad. For the purpose of

Table 2. Shear wave velocities

Depth range m (ft)	Shear wave velocities, m/sec (ft/sec)			
	Millikan Library[a]	Athenaeum Building[a]	Arms Laboratory[b]	Model
0–1.83 (0–6)	298.7 (980)	262.1 (860)	192.0 (630)	298.7 (980)
1.83–2.74 (6–9)	298.7 (980)	262.1 (860)	338.3 (1110)	298.7 (980)
2.74–5.49 (9–18)	298.7 (980)	365.8 (1200)	338.3 (1110)	298.7 (980)
5.49–7.01 (18–23)	387.1 (1270)	420.6 (1380)	338.3 (1110)	387.1 (1270)
7.01–9.75 (23–32)	387.1 (1270)	420.6 (1380)	487.7 (1600)	387.1 (1270)
9.75–13.41 (32–44)		420.6 (1380)	487.7 (1600)	454.2 (1490)
13.41–20.12 (44–66)			487.77 (1600)	487.7 (1600)
20.12–102.41 (66–336)			609.6 (2000)	609.6 (2000)
102.41–118.57 (336–389)			762.0 (2500)	762.0 (2500)
118.57– (389–)			944.8 (3100)	944.8 (3100)

[a] Eguchi *et al.* (1976)
[b] Shannon and Wilson, Inc. and Agbabian Associates (1976)

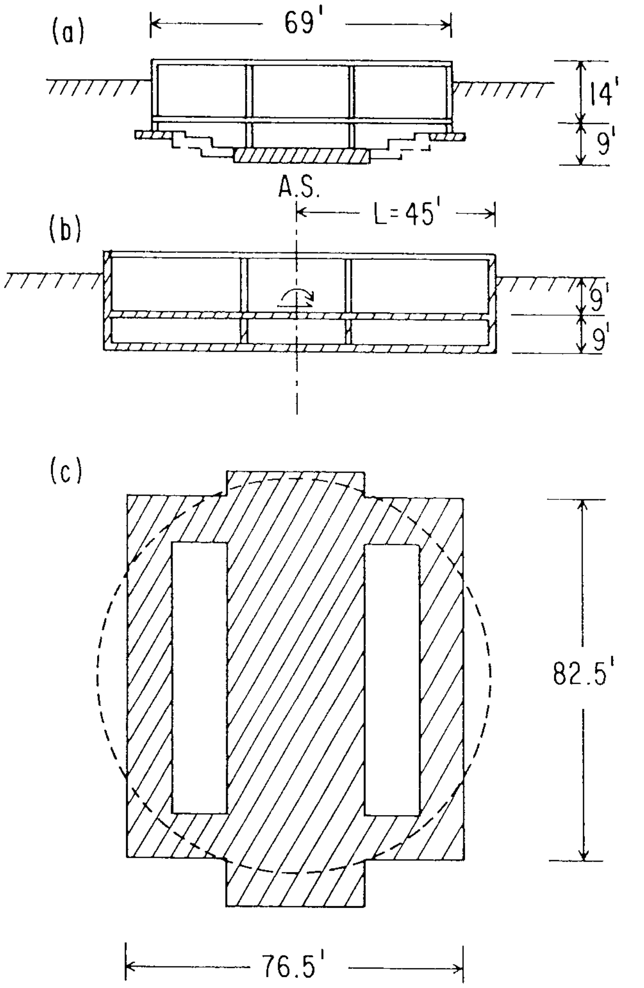

Fig. 3. Foundation system of the Millikan Library Building. (a) NS section of actual foundation, (b) section of model for EW vibrations, and (c) plan view of foundation and model

calculating the impedance functions in the N–S direction, the foundation was modelled as a rigid cylinder with a basal radius of 13.7 m (45 ft) embedded in the soil to a depth of 4.0 m (13 ft). For vibrations in the E–W direction, the foundation was modelled as a rigid cylinder with a radius of 13.7 m (45 ft) embedded to a depth of 5.5 m (18 ft). The embedment depths of the equivalent cylinders were selected to correspond to the embedment depths of the active elements of the actual foundationn for rocking vibrations in the N–S and E–W directions.

The foundation model is characterized by the parameters $M_0/M_b = 0.136$ and $I_0/H^2M_b = 0.004$ for N–S vibrations and $I_0/H^2M_b = 0.005$ for E–W vibrations. The flexibility of the foundation is characterized by the parameters α defined by equation (13) ($\alpha = 1$ for the 'rigid' foundation model). The values of α for the 'flexible' foundation model were determined on the basis of the observed deformation of the basement slab[4] and correspond to $\alpha = 1.30$ for N–S vibrations and $\alpha = 3.33$ for E–W vibrations.

Soil characteristics

The soil properties at the site were investigated by Converse Foundation Engineers[6] six years before the construction of the library. A summary of test results has been presented by Jennings and Kuroiwa[3]. The soil consists of medium to dense sands with gravel. Firm soil was found at a depth of about 5.5 m (18 ft) below the ground level. The shear wave velocities in the upper layers at the Millikan Library and at the Athenaeum Building (located 370 m from the Library) have been determined by Eguchi *et al.*[7]. The shear wave velocities at a boring next to the Arms Laboratory at a distance of 76 m from the library have been measured by Shannon and Wilson and Agbabian Associates[8]. The shear wave velocities at these three sites are listed in Table 2. For the purpose of the model studied here, the shear waves velocities below the depth of 9.75 m (32 ft) were averaged to obtain the velocity profile listed in the last column of Table 2. The velocity profile thus obtained is consistent with the soil mechanics information available. The soil model considered consists of nine viscoelastic layers resting on a uniform viscoelastic half-space. The unit weight of soil in all layers is taken to be 18.1×10^3 N/m^3 (115 lb/ft^3) and the P wave velocities were taken to be twice the corresponding shear wave velocities. It was assumed that the hysteretic damping ratios for P and S waves are equal, i.e., $\xi_P = \xi_S$. Calculations were performed for a value of $\xi_S = 0.02$ for all layers.

Impedance functions

To complete the characterization of the Millikan Library Building it is necessary to calculate the horizontal, rocking and coupling foundation impedance functions for the foundation and soil models described above. The impedance functions for the rigid foundation were obtained by use of an indirect boundary integral equation which involves the Green's functions for the layered viscoelastic soil model[9]. The calculated impedance functions were referred to the centre of the basement slab located 4.3 m (14 ft) below the first floor slab as illustrated in Fig. 3. The horizontal impedance functions were multiplied by the factor (14.23/13.72) to account for the difference between the equivalent radius for a circular foundation of equal area ($a_{eq} = 14.23$ m) and the radius of the model ($a = 13.72$ m). Similarly, the rocking impedance functions for vibrations in the N–S and E–W directions were multiplied by the factors $(13.62/13.72)^3$ and $(15.15/13.72)^3$, respectively. The coupling impedance functions were left unmodified.

The real and imaginary parts of the resulting corrected impedance functions are shown versus frequency in Figs 4 and 5. The imaginary parts are divided by the dimensionless frequency $a_0 = \omega L/\beta$, where $L = 13.7$ m (45 ft) and $\beta = 382$ m/sec (1253 ft/sec) are a length and shear wave velocity of reference. The impedance functions are normalized by the shear modulus of reference $G = 2.68 \times 10^8$ N/M^2 (38.9×10^3 psi) and the length of reference L. The shear modulus G and the shear wave velocity of reference β used to render the impedance functions dimensionless correspond to average soil properties at the depth of the bottom of the foundation.

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

The structural, foundation and soil models previously described have been used to calculate the response at the base $U_b(\omega)$, $H\Phi_b(\omega)$ and top $\bar{U}_T(\omega)$ of the Millikan Library Building during forced vibration tests. The calculations are based on use of equations (14) and (15) for a forcing function $f_T = F_T(\omega)\,e^{i\omega t}$ where $F_T(\omega) = 103.6$

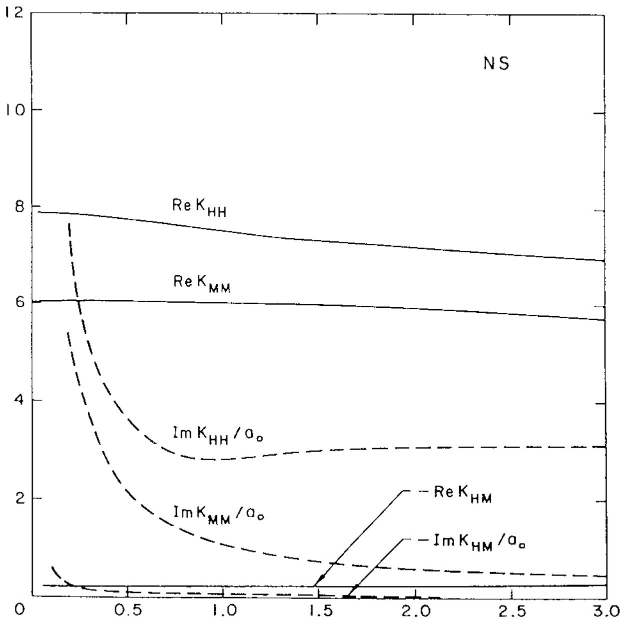

Fig. 4. Theoretical impedance functions for NS vibrations

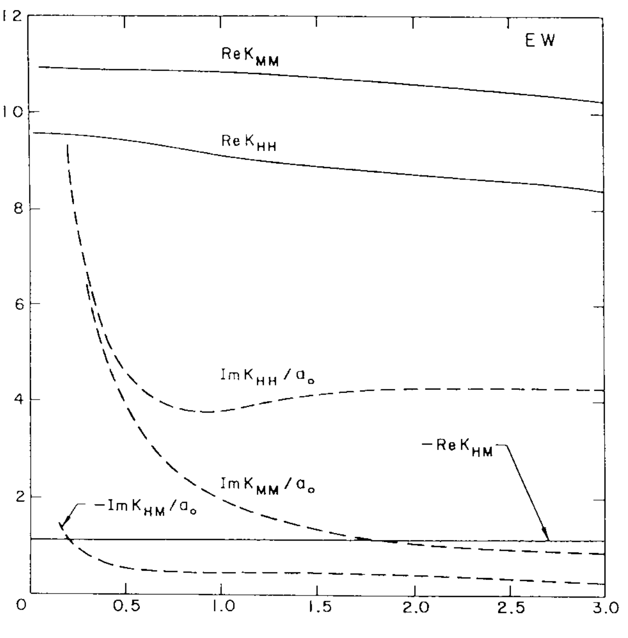

Fig. 5. Theoretical impedance functions for EW vibrations

ω^2 Newtons (23.3 ω^2 lb)[10]. In this case, $F_T/\omega^2 M_b = 0.973 \times 10^{-5}$ m.

Before proceeding with the comparisons of the response at the base and top of the Library it is instructive to consider comparisons of theoretical and experimental estimates of the foundation impedance functions.

Comparison of experimental and theoretical estimates of the impedance functions

Forced vibration tests in which the response at the top of the structure and the translational and rocking response at the base are recorded can be used to calculate the force and moment that the foundation exerts on the soil. Although this information is not sufficient to determine K_{HH}, K_{MM} and $K_{HM} = K_{MH}$ experimentally, approximations to the horizontal and rocking impedance functions defined by

$$K'_{HH} = \frac{H_S}{GL\bar{U}_b} \tag{21}$$

$$K'_{MM} = \frac{M_S}{GL^3\bar{\Phi}_b} \tag{22}$$

can be calculated from knowledge of H_S, M_S, $\bar{U}_b$ and $\bar{\Phi}_b$. The meaning of these approximations can be obtained by use of equations (11), (12). (21) and (22) which lead to

$$K'_{HH} = K_{HH} + K_{HM}(L\bar{\Phi}_b/\bar{U}_b) \tag{23}$$

$$K'_{MM} = K_{MM} + K_{MH}(\bar{U}_b/L\bar{\phi}_b) \tag{24}$$

Equations (23) and (24) indicate that the error of the estimates depends on the ratio of the rocking of the foundation to the corresponding translation and on the ratios (K_{HM}/K_{HH}) and (K_{MH}/K_{MM}). The theoretical results shown in Figs 4 and 5 indicate that the ratios of coupling impedances to horizontal or rocking impedances are small. Also, it can be shown that for a superstructure of height H, the ratio $L\bar{\Phi}_b/\bar{U}_b$ is approximately proportional to the slenderness ratio H/L. For tall structures, K'_{MM} will provide an accurate approximation to K_{MM} while K'_{HH} may not approximate K_{HH} as closely. In the case of the Millikan Library, the amplitude of the ratios $(K_{MH}\bar{U}_b/K_{MM}L\bar{\Phi}_b)$ and $(K_{HM}L\bar{\Phi}_b/K_{HH}\bar{U}_b)$ are, respectively, 0.02 and 0.06 for N–S vibrations and 0.06 and 0.19 for E–W vibrations.

In a previous study[5], the authors have obtained experimental estimates of K'_{MM} and K'_{HH} based on equations (21) and (22). These estimates are shown versus frequency in Figs 6 through 9. In each figure, the real parts (stiffness coefficients) and the imaginary parts divided by the dimensionless frequency $a_0 = \omega L/\beta$ (damping coefficients) are shown for the 'rigid' and 'flexible' foundation models. No significant differences were found between the experimental results for these two foundation models. Also shown in Figs 6 through 9 are the theoretical estimates of K'_{MM} and K'_{HH} obtained by use of equations (23) and (24) and based on the theoretical impedance functions shown in Figs 4 and 5 and on the theoretical value of $(L\bar{\Phi}_b/\bar{U}_b)$ for the 'flexible' foundation case.

A first feature of the comparison between experimental and theoretical impedance functions is the excellent agreement obtained for the rocking stiffness in both the N–S and E–W directions (Figs 6 and 7). A reasonable agreement is also found for the imaginary parts of the rocking impedance functions in both directions. The comparisons in Fig. 8 indicate that the theoretical horizontal stiffness in the N–S direction overestimates the corresponding experimental results by about 50%. At the N–S resonant frequency (1.79 Hz) the theoretical and experimental imaginary parts of the N–S horizontal impedance functions are in excellent agreement but this agreement deteriorates at other frequencies. Finally, the comparisons shown in Fig. 9 indicate that the theoretical horizontal stiffness in the E–W direction is only about 10 percent higher than the corresponding experimental

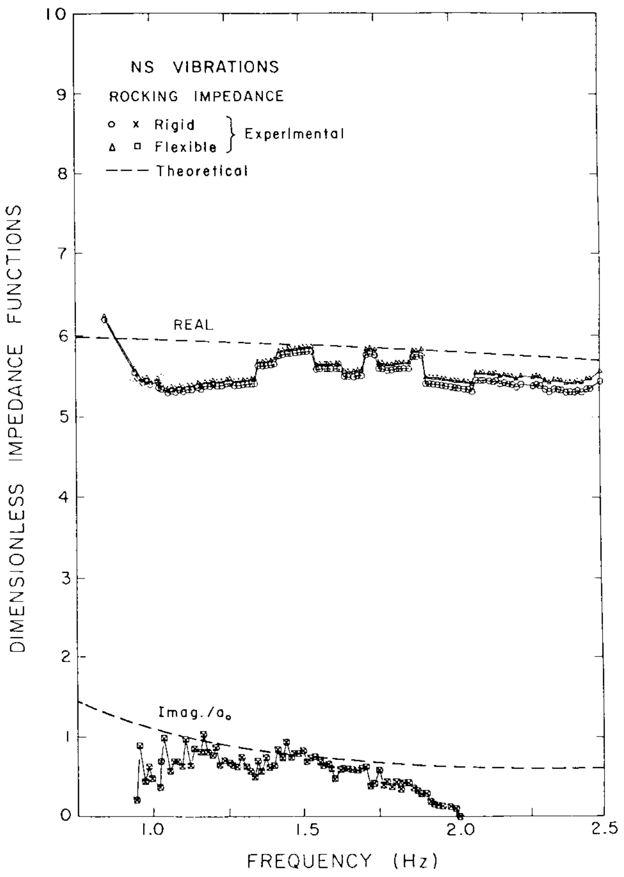

Fig. 6. Comparison of theoretical and experimental estimates of the NS rocking impedance functions

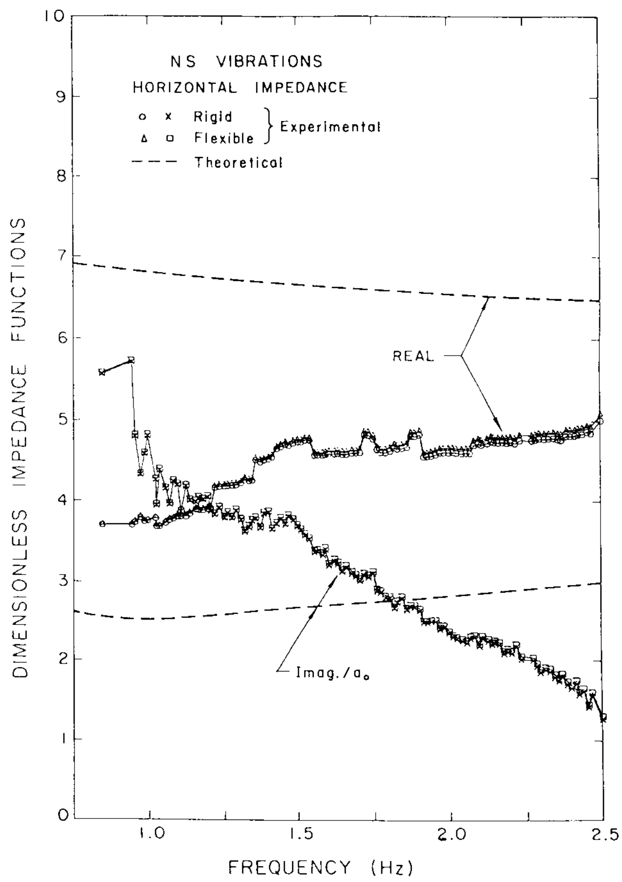

Fig. 8. Comparison of theoretical and experimental estimates of the NS horizontal impedance functions

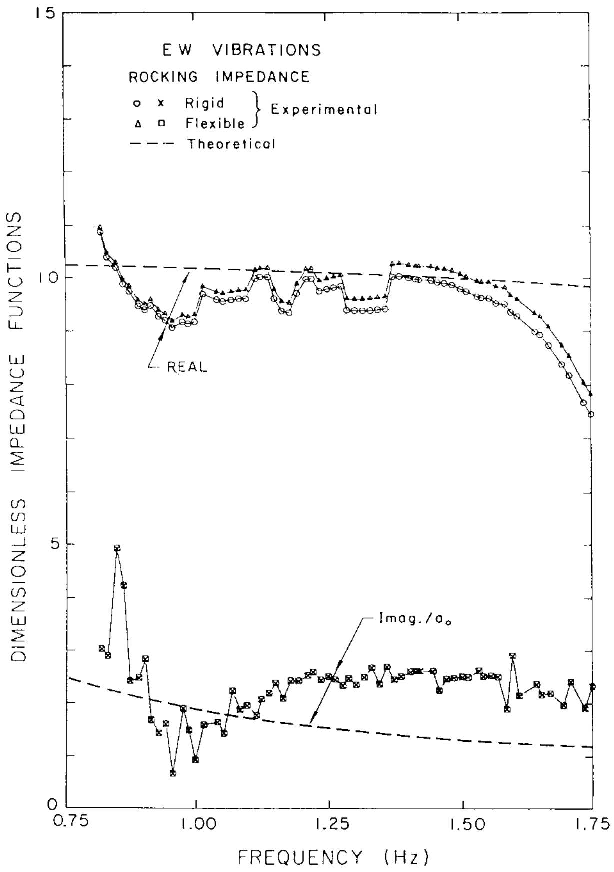

Fig. 7. Comparison of theoretical and experimental estimates of the EW rocking impedance functions

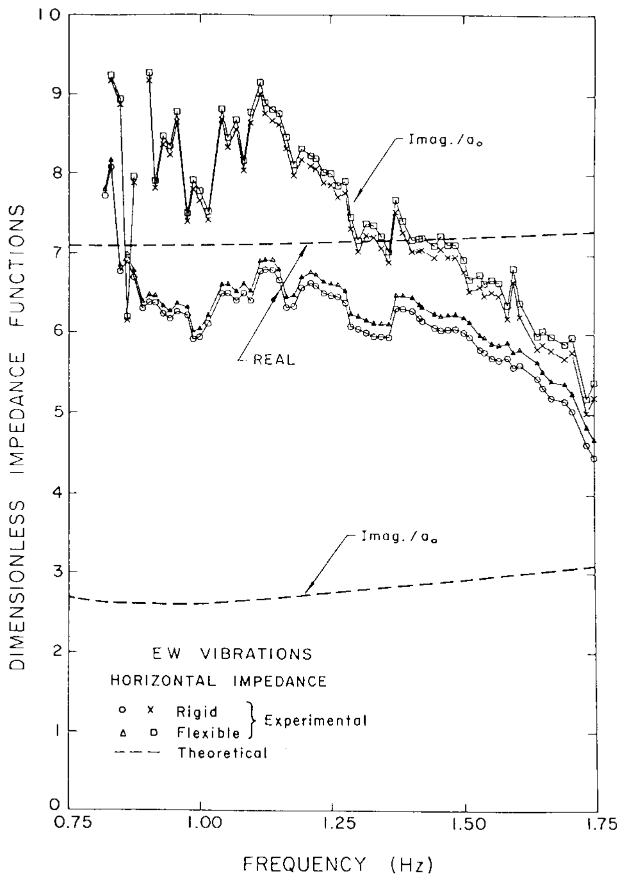

Fig. 9. Comparison of the theoretical and experimental estimates of the EW horizontal impedance functions

Table 3. Comparison of theoretical and experimental response at the system frequency

	N–S			E–W		
	Calculated			Calculated		
	Rigid	Flexible	Observed	Rigid	Flexible	Observed
f_L	1.810	1.810	1.79	1.205	1.205	1.21
$\lvert U_T\rvert$ (10^{-4} m)	7.99	7.57	7.59	8.76	8.58	8.49
$\lvert H\Phi_b\rvert$ (10^{-4} m)	–	2.53	2.51	–	1.81	1.76
$\lvert H\bar{\Phi}_b\rvert$ (10^{-4} m)	1.99	1.90	1.94	0.54	0.54	0.53
$\lvert U_b\rvert$ (10^{-4} m)	0.218	0.225	0.306	0.103	0.102	0.100

value at the system frequency (1.21 Hz). On the other hand, the theoretical values for the imaginary parts of the E–W horizontal impedance function are less than half of the experimental values. The underestimation of the imaginary part of the horizontal impedance function in the E–W direction may have resulted from the presence of the foundations for a one-storey structure and a pond located to the east of the Library and to other details of the foundation not included in the theoretical model.

In conclusion, it can be stated that theoretical estimates of the impedance functions based on simplified models of the foundation match very closely the experimental rocking impedance functions but deviate from the corresponding horizontal impedance functions.

Comparison of experimental and theoretical structural response

Comparisons of theoretical and experimental results for the amplitudes of $U_T(\omega)$, $H\Phi_b(\omega)$ or $H\bar{\Phi}_b(\omega)$ and $\bar{U}_b(\omega)$ are presented in Table 3 and in Figs 10 through 13. The comparisons include vibrations in the N–S and E–W directions and 'flexible' and 'rigid' foundation models. Considering first the comparisons for N–S vibrations, it is observed that the theoretical calculations for both the 'flexible' and 'rigid' foundation models lead to a system frequency of 1.81 Hz which is only one percent larger than the observed system frequency of 1.79 Hz. The amplitudes of the total response at the top $\bar{U}_T$, normalized base rotation $H\Phi_b$ and normalized average rotation of the foundation $H\bar{\Phi}_b$ agree closely with the observed amplitudes at the system frequency (Table 3). The calculated translation of the base $\bar{U}_b$ at the system frequency, on the other hand, is 26 to 29 percent lower than the observed response. The comparisons of the frequency response curves for 'flexible' and 'rigid' foundation models shown in Figs 10 and 11, respectively, also illustrate excellent agreement between theoretical and experimental values for H_T, $H\Phi_b$ and $H\bar{\Phi}_b$ and some discrepancy on the calculated value of the base translation $\bar{U}_b$. The discrepancy between calculated and observed values for $\bar{U}_b$ may be associated with error in the measurement of $\bar{U}_b$ or failure of the foundation model to predict the horizontal stiffness of the foundation in the N–S direction as shown in Fig. 8.

For E–W vibrations it is found that the calculated system frequency is 1.205 Hz which is only half-of-one-percent lower than the observed value of 1.21 Hz. At the system frequency, the calculated amplitudes of $\bar{U}_T$, $H\Phi_b$, $H\bar{\Phi}_b$ and $\bar{U}_b$ match the corresponding observed amplitudes with an error of less than 3 percent (Table 3). The comparisons of the frequency response curves

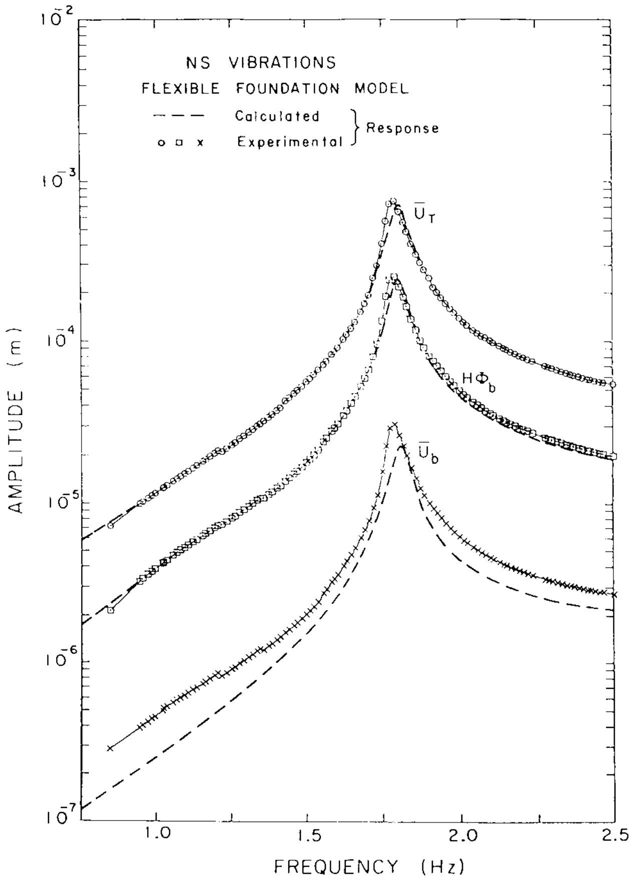

Fig. 10. Comparison of the theoretical and experimental response in the NS direction (flexible foundation model)

presented in Figs 12 and 13 show excellent agreement between calculated and observed responses for frequencies below 1.21 Hz. For higher frequencies, the theoretical results are lower than the observed values. The discrepancy may be associated with the failure of the theoretical model to predict the large observed radiation damping for E–W horizontal vibrations. It has been indicated that the fixed-base modal damping value for E–W vibrations ξ_1 was taken to be 1.5 percent instead of the values of 0.8 percent (flexible foundation) and 1.2 percent (rigid foundation) which were obtained[5] from analysis of the experimental data. The larger value of $\xi_1 = 0.015$ for the damping in the structure leads to the excellent agreement in response at the system frequency shown in Figs 12 and 13 but cannot compensate for the lack of radiation damping at frequencies higher than 1.21 Hz.

In general, given the lack of experimental accuracy in

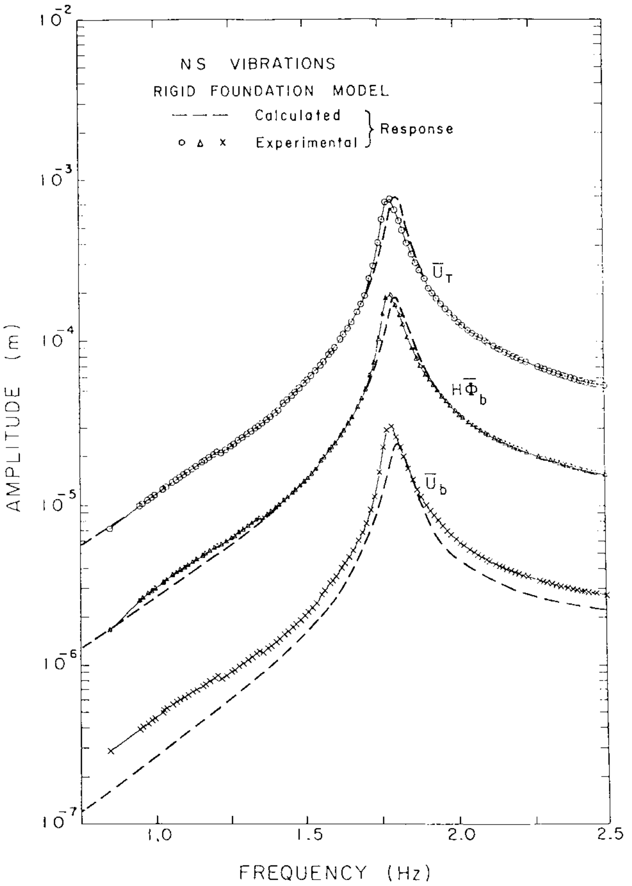

Fig. 11. Comparison of the theoretical and experimental response in the NS direction (rigid foundation model)

determining damping values (as a result of the experimental difficulty of accurately determining the phase of the response), the agreement between theoretical and observed results can be considered encouraging.

Finally, it is necessary to discuss the effects of the value assumed for the material damping in the soil ξ_S and its interdependence with the fixed-base damping value ξ_1 assumed for the superstructure. In Fig. 14, the calculated amplitude $|\bar{U}_T(\tilde{\omega}_1)|$ of the total response on the roof at the system frequency is shown versus the value of ξ_1 for soil material damping ratios ξ_S of 2.0 and 2.5 percent. Considering first the results for N–S vibrations, it is found that if $\xi_S = 0.02$, then, for the calculations to match the observed response it is necessary that $\xi_1 = 0.012$. If, on the other hand, $\xi_S = 0.025$, *then the required value for* ξ_1 *is* 0.0077. *A similar situation exists for E–W vibrations. In this case, if* $\xi_S = 0.02$ then ξ_1 must be 0.0153 while if $\xi_S =$ 0.025 then ξ_1 must be 0.0133. These examples show that different combinations of material damping in the soil and in the superstructure may lead to the same value of the response amplitude at resonance. Clearly, it can be concluded that, the process of matching response amplitudes by itself is not sufficient to completely determine the damping in the superstructure.

CONCLUSIONS

Comparisons of the theoretical and experimental response of a nine-storey reinforced concrete building during forced vibration tests have been presented. The theoretical calculations include the effects of soil-

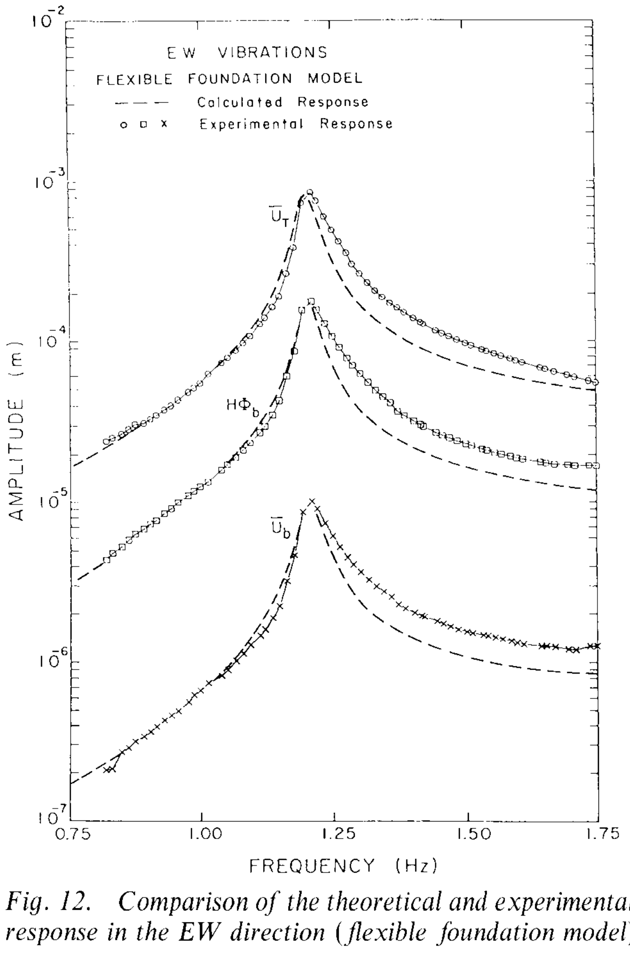

Fig. 12. Comparison of the theoretical and experimental response in the EW direction (flexible foundation model)

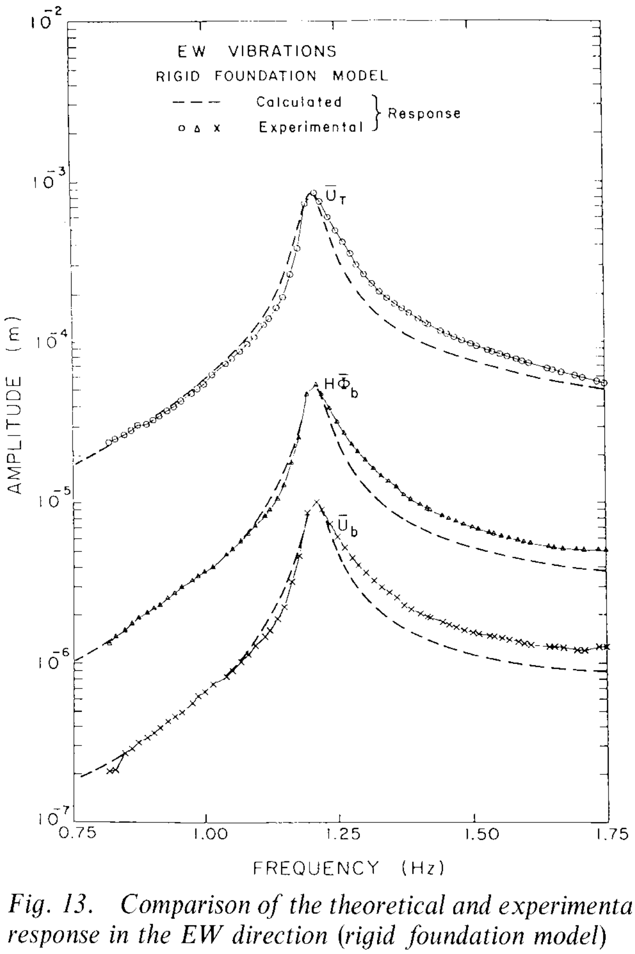

Fig. 13. Comparison of the theoretical and experimental response in the EW direction (rigid foundation model)

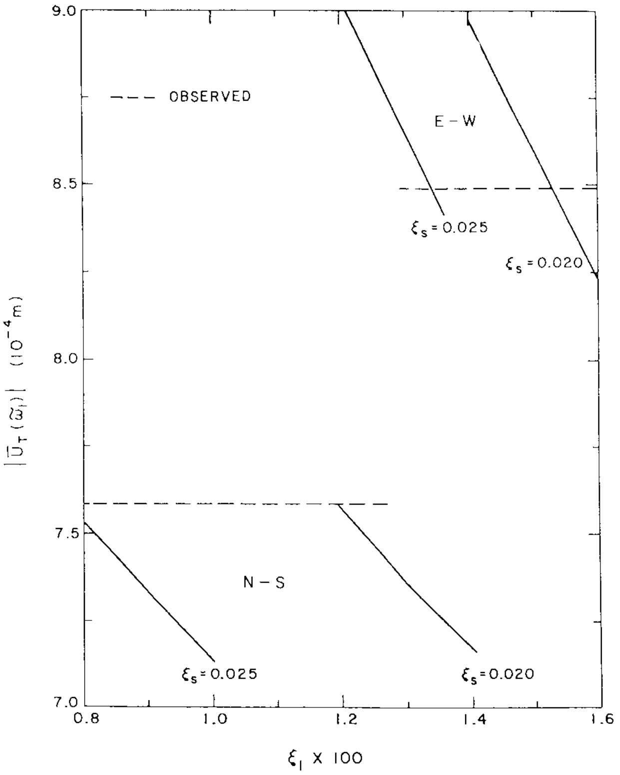

Fig. 14. Effect of ξ_1 and ξ_S on the resonant response at the top of the building

structure interaction and are based on relatively simple models of the superstructure, foundation and soil. For vibrations in the N–S direction, in which the foundation acts almost as a rigid body, excellent agreement was found between the theoretical and experimental values for the translation of the roof, rotation of the base and rocking foundation impedance function. Some related discrepancies were found in the translation of the base and in the horizontal foundation impedance functions. For vibrations in the E–W direction, the central elevator core produces a major out-of-plane deformation of the foundation. In this case, the results of the comparisons were more difficult to interpret. For E–W vibrations, excellent agreement was found between theoretical and experimental values for the translation of the roof and the translation and rotation of the base for frequencies equal to or lower than the resonant system frequency of 1.21 Hz. At higher frequencies, the theoretical response underestimates the experimental values. The agreement at lower frequencies was based on the use of a damping in the superstructure of 1.5 percent which differs from the value of 0.8 to 1.2 percent isolated from the experimental data. These discrepancies in the E–W direction are associated with the failure of the simple foundation model to account for the flexibility of the actual foundation and for the large radiation damping in horizontal vibrations obtained experimentally. It seems then, that if the foundation acts as a rigid body it is possible to predict quite accurately the effects of soil-structure interaction during forced vibration tests by use of simple models. Analytical models more complex than those used in this study may be required for highly flexible foundations.

It has also been found that matching of response amplitudes does not fully constrain the value of the fixed-base structural damping. In the presence of soil-structure interaction effects, the value of the structural damping obtained by matching response amplitudes depends on the value assumed for the material damping in the soil.

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