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EARLY HISTORY OF THE RESPONSE SPECTRUM METHOD

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ABSTRACT

This monograph, written on the occasion of the 75th anniversary of the formulation of the concept of the response spectrum method (RSM), presents three articles. The first is a short history of the conditions that contributed to the creation of the RSM. It describes the key players and the vibrational nature of the method that Maurice Biot chose in 1932 to describe the earthquake response of structures. It also briefly discusses the contemporary role of the RSM. The second article presents a rare biographical sketch of M. Biot, written by his wife Nady Biot, which offers a personal and human view of the life of this talented and unique man. The third article, an unpublished discussion that Biot probably wrote in early 1960s after the study by Merchant and Hudson appeared in 1962, was found by Nady Biot among her husband's papers after his death in 1985. Because this discussion describes his views on the contemporary developments in the RSM, it was felt that it would be of interest to publish it here, especially in the context of this monograph, which is devoted to the same subject.

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by

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> ***Once Einstein was asked by the New York State Education Department what schools should emphasize. "In teaching history," he replied, "there should be extensive discussion of personalities who benefited mankind through independence of character and judgment." (Isaacson 2007).***

Introduction

Writing about the early history and formative ideas of a given subject can be interesting and educational in many different ways. It is also difficult, because the record is always incomplete, because we cannot interview all those involved, and because the risk of presenting yet another version of a "Rashomon" story is always present. Yet, it is useful to document what is known and to search for the key factors that led to the formulation of a new concept, to learn about the key players, their teachers, and the times in which they lived—and perhaps to discover the anatomy of the process that led to the new ideas. In this time, when we study such a broad spectrum of natural and societal phenomena, it should seem only natural and logical to try to research the origins and the conditions surrounding the development of a significant new concept. Knowing about the scientists as men can help us to understand the wellsprings of their science, and such a historical review may also be educational for our students by allowing them to see the training and formal education that contributed to the creation of a new result.

The year 2007 marks the 75th anniversary of the formulation of the concept of the response spectrum method (RSM) in 1932. Since then, the RSM has evolved into the essential tool and the central theoretical framework–in short, a conditio sine qua non—for Earthquake Engineering. The mathematical formulation of the RSM first appeared in the doctoral dissertation of M.A. Biot (1905–1985) in 1932 and in two of his subsequent papers (Biot 1933; 1934).

The RSM remained in the academic sphere of research for many years and did not gain widespread engineering acceptance until the early 1970s. There were two main reasons for this. First, the computation of the response of structures to earthquake ground motion led to "certain rather formidable difficulties" (Housner 1947), and, second, there were only a few well-recorded accelerograms that could be used for that purpose. This started to change in 1960s with the arrival of digital computers and the commercial availability of strong-motion accelerographs. Before the digital computer age, the computation of structural response was time consuming, and the results were unreliable (Trifunac 2003). By the late 1960s and early 1970s, however, the digitization of analog accelerograph records and the digital computation of ground motion and of the response spectra were developed completely and tested for accuracy. Then, in 1971, with the occurrence of the San Fernando, California, earthquake, the modern era of RSM was launched. This earthquake was recorded by 241 accelerographs, and by combining these data with all previous strong-motion records it became possible to perform the first comprehensive empirical scaling analyses of response spectral amplitudes (Lee 2002; 2007).

In the following, we focus only on the early history of the RSM. The description and analyses of the modern developments in the RSM are beyond the scope of this work, but the reader can find papers that describe many contemporary uses of the RSM in the special issue of the *Indian Society of Earthquake Technology (ISET) Journal*, Vol. 44. No. 1, published in 2007.

Engineering Mechanics in the 19th and early 20th centuries

Solution of the type of differential equations that describe the dynamic response of structures can be viewed in terms of (1) waves (D'Alembert (1717–1783) first described this method of solution in a memoir of the Berlin Academy in 1750), or (2) using a vibrational approach in terms of the characteristic functions (mode shapes) (Bernoulli (1700–1782), first wrote about this method in a memoir of the Berlin Academy in 1755). The related problem of the vibrating string was solved analytically by Lagrange (1736–1830) in a memoir of the Turin Academy in 1759, which established the existence of a number of independent frequencies equal to the number of interconnected particles. Mathematical principles and the methods associated with the latter approach have been researched and described extensively by Rayleigh (the first edition of the *Theory of Sound* was published in England in 1877). It is interesting that it took 68 years for the first American edition of the *Theory of Sound* to appear, as a Dover publication, in New York, in 1945.

Lord Rayleigh was born John William Strutt, the eldest son of the Baron Rayleigh of Terling Place. In 1861, at the age of nearly 20, he went to Cambridge and entered Trinity College, where he profited greatly from the lectures of Sir George G. Stokes. In 1866, he made a trip to the United States, during the period of reconstruction after the Civil War, and the first pages of the *Theory of Sound* were written on a houseboat during a trip up the river Nile, late in 1872. Rayleigh was a modest man. When he received the Order of Merit, he remarked that the only merit of which he was personally conscious was that of "having pleased himself by his studies, and any results that may have been due to his researches were owing to the fact that it had been a pleasure to him to become a physicist."

The response spectrum method is based on the vibrational representation of the solution, in which each mode shape and its natural frequency are associated with one equivalent single-degree-of-freedom (SDOF) system. Then, for linear systems the response is represented as a superposition of the responses of those equivalent SDOF systems. Therefore, the analysis of the linear response of an n-degree-of-freedom system can be reduced to a study of individual SDOF systems, one at a time.

A comprehensive review of the conditions that prepared and enabled the key players to formulate the concept of the response spectrum method is beyond the scope of this work. In the following, I will mention only a few examples. The first is that the teaching of physics, and particularly of engineering mechanics and applied mathematics, started to expand in Europe around the end of 19th and the beginning of 20th century (Timoshenko 1968; von Kármán and Edson 1967; Cornwell 2003). The second was the arrival of leading scientists and engineers in earthquake-prone areas (e.g., Milne in Japan, Compte de Montessus de Ballore in Chile, and Millikan, Gutenberg, and von Kármán in Southern California). Their organizational abilities, interest, and curiosity to examine yet another challenging group of physical phenomena created new critical mass, which in turn attracted the next generation of talented students.

At most American universities in the early 1900s, engineering curricula did not include advanced mathematics and mechanics, both essential for teaching analysis of the dynamic response of structures. This deficiency in theoretical preparation is reflected in the view of C. Derleth (1874–1956), civil engineering professor and Dean of the College of Engineering at U.C. Berkeley, who commented after the 1906 earthquake (Derleth 1907):

> Many engineers with whom the writer has talked appear to have the idea that earthquake stresses in framed structures can be calculated, so that rational design to resist earthquake destruction can be made, just as one may allow for dead and live loads, or wind and impact stresses. Such calculations lead to no practical conclusions of value.

A comment by A. Ruge (1940), the first professor of engineering seismology at the Massachusetts Institute of Technology, that “the natural tendency of the average design engineer is to throw up his hands at the thought of making any dynamical analysis at all,” shows that the progress was slow (Reitherman 2006).

Such views, however, had started to change gradually toward the end of 1920s. In 1929, at the University of Michigan in Ann Arbor, the first lectures were organized at the Summer School of Mechanics by S. Timoshenko (1878–1972), with the participation of A. Nádai, R.V. Southwell and H.M. Westergaard. “After the first session of the summer school in 1929, the number of doctoral students in mechanics…started rapidly to increase” (Timoshenko 1968). In the summer of 1932, M.A. Biot was among the young post-doctoral students who took part in Timoshenko’s summer school (Mindlin 1989, Bolley 2005).

In southern California, studies of earthquakes and research in theoretical mechanics were expanded and energized by the arrival of R. Millikan (1868–1953), who became the first president (chair of the executive council) of the California Institute of Technology in 1921. Millikan had completed his Ph.D. studies in physics at Columbia University in 1895, and following the recommendation of his advisor M. Pupin (1854–1935) spent a year in Germany, in Berlin and Göttingen. This visit to Europe appears to have influenced many of Millikan’s later decisions while recruiting the leading Caltech faculty members two decades later. In 1921, H.O. Wood (1879–1958) invited Millikan to serve on the Advisory Committee in Seismology (Geschwind 1996). The work on that committee and Millikan's interest in earthquakes were also significant for several subsequent events. In 1926, J. Buwalda (1886–1954) was asked to set up the division of geological sciences at Caltech, and C. Richter (1900–1985) and B. Gutenberg (1889–1960) joined the seismological laboratory in 1926 and 1930, respectively. In the area of applied mechanics, Millikan invited Theodore von Kármán (1881–1963) to join the Caltech faculty, and in 1930 von Kármán became the first director of the Guggenheim Aeronautical Laboratory. An engineer, applied scientist, teacher, and visionary, von Kármán had a remarkable talent for getting people together across professional, national, and language barriers, and he became one of the foremost leaders in the world of aviation and space technology (see, for example, von Kármán and Edson 1967). But it was Millikan's vision and his ability to anticipate future developments that brought so many leading minds to a common place of work, creating an environment that made the first theoretical formulation of the concept of the response spectrum method possible.

First steps

Several earthquake disasters in densely populated areas in the early 20th century made it clear that defensive mechanisms needed to be developed to prevent future loss of life and property from destructive earthquakes. The first practical steps, which initiated the engineering work on the design of earthquake-resistant structures, accompanied the introduction of the *seismic coefficient* (known as *shindo* in Japan and *rapporto sismico* in Italy) and started to appear following the destructive earthquakes in San Francisco, California in 1906, Messina-Reggio, Italy in 1908 (Sorrentino 2007), and Tokyo, Japan in 1923. The first seismic design code was introduced in Japan in 1924. In California, work on earthquake code development started in 1920s, but it was not until after the Long Beach earthquake in 1933 that the Field Act was finally adopted in 1934 (Reitherman 2006).

Benioff (1934) comments on the seismic coefficient method in the introduction to his paper on seismic destructiveness as follows:

> …engineers have been forced to proceed on an empirical basis. From past experience…it has been found that buildings, which are designed to withstand a constant horizontal acceleration of 0.1 gravity are, on the whole, fairly resistant to seismic damage…. We know that seismic motions do not exhibit constant accelerations; that instead they are made up of exceedingly variable oscillatory movements. A formula based upon constant acceleration may thus lead to large errors, especially when applied to new types of structures, which have not been tested in actual earthquakes.

Suyehiro (1932) also discussed the "static load of the intensity given by the mass of the building multiplied by the horizontal acceleration of the seismic vibration.", and we can speculate that Frank Lloyd Wright may have used it in his design of the Imperial Hotel in Tokyo, especially in the analysis of its "floating" foundation.

In Italy, work on developing building design codes began in 1908, following the Messina disaster, in which more than 100,000 persons were killed; in Japan following the 1923 Tokyo disaster, in which more than 150,000 perished; and in California after the Santa Barbara earthquake of 1925 (Freeman 1932, Suyehiro 1932). In 1927, the "Palo Alto Code," developed with the advice of Professors Willis and Marx of Stanford University, was adopted in the cities of Palo Alto, San Bernardino, Sacramento, Santa Barbara, Klamath, and Alhambra, all in California. It specified the use of a horizontal force equivalent to 0.1 g, 0.15 g, and 0.2 g acceleration on hard, intermediate, and soft ground, respectively.

"Provisions Against Earthquake Stresses," contained in the *Proposed U.S. Pacific Coast Uniform Building Code,* was prepared by the Pacific Coast Building Officials Conference and adopted at its 6th Annual Meeting, in October 1927, but these provisions were not generally incorporated into municipal building laws (Freeman, 1932). The code recommended the use of horizontal force equivalent to 0.075, 0.075, and 0.10 g acceleration on hard, intermediate, and soft ground, respectively. Following the 1933 Long Beach earthquake, the Field Act was implemented, and Los Angeles and many other cities adopted an 8 percent g base shear coefficient for buildings and a 10 percent g for school buildings. In 1943, the Los Angeles Code was changed to indirectly take into account the natural period of vibration. The reader can find a brief review of modern code development in the paper by Freeman (2007).

In the fall of 1931, Professor Kyoji Suyehiro visited the United States and presented a series of three lectures on engineering seismology (Suyehiro 1932), and his third lecture (III) entitled "Vibration of Buildings in an Earthquake" is of particular interest for earthquake engineering. It seems that the term "engineering seismology"—*Jishin Kogaku*—was first used at this time (Kanai 1983). Suyehiro was a member of the Japanese Imperial Academy, a Professor of Applied Mechanics with Tokyo Imperial University, and Director of the Earthquake Research Institute. He died on April 9, 1932, but his lectures made strong and lasting impressions on many American seismologists and engineers who later contributed to the development of earthquake engineering.

Suyehiro's work on his multi-pendulum recorder is sometimes cited as reminiscent of and a predecessor to the concept of the response spectrum, and he (1932) refers to his pendula as a "seismic vibration analyzer" (p. 69, 7th line from top). A few lines later, he states that "each damper is so adjusted as to make the magnification of the amplitude of the resonance vibration of each stylus practically equal." Further down, on the same page, he describes the record at Hongo and how that site has a natural period of about 0.3 s. Suyehiro's discussion is directed toward analyzing periodic content in the recorded motion, as is done in general with vibration analyzers, but he presents no explicit or implicit meaning that could be related to the response spectrum concept. In discussing Suyehiro's work, Housner states (p. 24 of EERI 1997) that the Suyehiro pendula "would give six points on the displacement spectral curve," but because the instrument did not have control of damping "the results could not be applied directly to buildings." It is interesting to note here that the torsional pendula then used by the U.S. Coast and

Geodetic Survey (USC&GS), by Biot at Columbia, and by Housner at Caltech, all had the same problem—damping could not be controlled. The first spectra with controlled damping were computed with an electrical analog computer. Biot was aware of and followed Suyehiro's work, and on page 215 of Biot (1934) he states, "According to recent observations (Suyehiro, Tokyo), there seem to exist characteristic frequencies of the ground at given locations. These frequencies would be given by the peaks in the spectral curves."

Figure 1 outlines selected events in the early work on earthquake engineering, the years of "major" earthquakes (from the engineering point of view), and the year of the First World Conference on Earthquake Engineering, for the period 1920 to 1955.

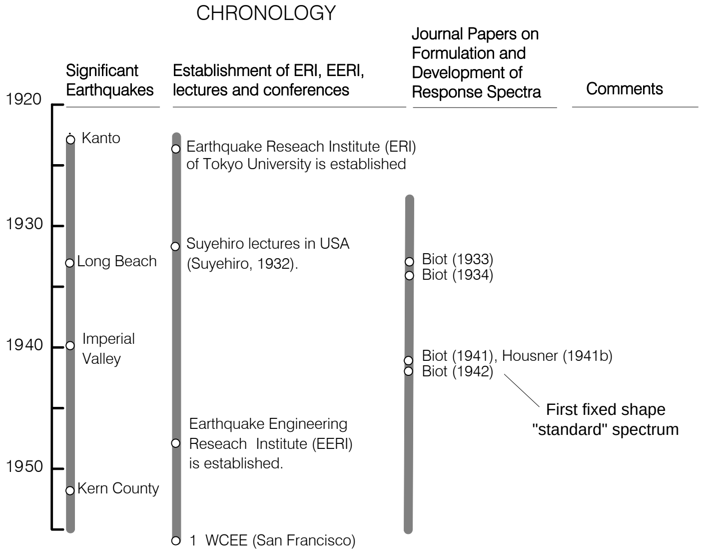


Fig. 1 Relative chronology of significant earthquakes for earthquake engineering, establishment of ERI and EERI, the Suyehiro lectures, the First World Conference on Earthquake Engineering, and journal papers dealing with formulation of the response spectrum method, between 1920 and 1955.

Theodore von Kármán and Maurice Biot

The mathematical formulation of the RSM first appeared in the doctoral dissertation of M.A. Biot in 1932 and in two of his papers (Biot 1933; 1934). Biot defended his Ph.D. thesis at Caltech in June 1932 and presented a lecture on the method to the Seismological Society of America meeting, held at Caltech, the same month. Theodore von Kármán, Biot's advisor, played the key role in guiding his student and in promoting his accomplishments. After the method of solution was formulated, Biot and von Kármán searched for an optimal design strategy. A debate at the time was whether a building should be designed with a soft first floor or be stiff throughout its height, to better resist earthquake forces. An excerpt from the *New York Herald Tribune* in June 1932 illustrates this:

Shock Proof Buildings Sought by Scientists.
Rigid or Flexible Materials, Their Difference in Theory

A building proof against earthquakes is the goal of Dr. Theodor von Kármán and Dr. M. Biot, of the California Institute of Technology. Dr. von Kármán described to the American Society of Mechanical Engineers, whose convention was held recently at Yale, studies of the amount of shock, which various types of buildings have undergone in Japan, South America, and California. Their researches are being conducted at the Institute's Guggenheim Aeronautical Laboratory.

One of the principal problems is to decide whether a rigid or flexible structure is better. Some scientists contend the first is preferable; others would make the ground floor of tall buildings flexible.

Pointing out that the reinforced concrete is superior to steel in absorbing the shocks, Dr. von Kármán's personal belief is that the building should be constructed to shake "with the rhythm of the earth's movements."

Another newspaper article, describing the same meeting, stated:

Quake Strains Discussed
Von Kármán Tells New Haven Meeting Engineers Are Divided Between Rigid and Flexible Buildings

The most interesting piece of research now being conducted at the California Institute of Technology by Dr. M. Biot on the calculation of stresses occurring in buildings during an earthquake was described informally this morning by Dr. T. von Kármán, Director of the Guggenheim Aeronautical Laboratory at the school, under whose direction Dr. Biot is doing the work.

Seek "Quake-Proof" Building.

By a study of past earthquakes in California and Japan and along the Pacific Coast of Central America, engineers interested in building problems have accumulated a record on which they believe they can calculate the rhythm or characteristic of the earth movement in these particular regions. They have sought to evolve an "earthquake-proof" building on the basis of this data.

As a result of this research, said Dr. von Kármán, there have arisen two schools of thought. One asserts that only the most rigid structures should be built in the earthquake regions and the second, which Dr. von Karman supports, contends that a flexible type of building, which can swing with the earthquake, is better.

Biot's interest in the maxima of the transient response in solids and in fluids preceded, and extended beyond, earthquake engineering. After he formulated the concept of the RSM, he extended it to other vibrational problems such as the analysis of aircraft landing gear. He briefly returned to the subject of earthquake engineering almost ten years later, presenting response spectral amplitudes of several earthquakes, which he calculated using the torsional pendulum at Columbia University (Biot 1941). In 1942 he presented a review of the response spectrum method, discussed the effects of flexible soil on the rocking period of a rigid block (Biot 2006), and described the spectrum superposition method based on the sum of absolute modal maxima (Biot 1942). After 1942, Biot moved on to other subjects, making fundamental contributions to many other fields. He did not write papers on earthquake engineering (Trifunac 2005), but followed closely and with interest the work of others.

The principal areas of Biot's opus, his exceptional talent, and his technical views have been described by Mindlin (1989) and by Tolstoy (2006), who wrote: “While Biot’s contributions to science owed much to his command of the sophisticated mathematical tools of theoretical mechanics, they were always rooted in concrete problems of engineering and geophysics. His solutions were firmly based on physical insight. He understood the pitfalls of formalism, but at the same time he appreciated the creative role of mathematical elegance upon which he laid much stress. He was one of the twentieth century's true masters of Lagrangian techniques.” A complete list of Biot’s publications can be found at the end of this monograph and in Trifunac (2006), and a list of his patents and awards is contained in the introduction to Vol. 14 of the *Journal of Mathematical and Physical Sciences*, published in Madras, India, in 1980, on the occasion of his seventieth birthday anniversary.

Biot’s contribution

Biot’s Ph.D. Thesis “Transient Oscillations in Elastic Systems” (Thesis No. 259, Aeronautics Dept., Caltech, 1932) dealt with the general theory of transient response. In Chapter II of his thesis, entitled “Vibration of Buildings during Earthquake,” he introduced the formulation of what would later become known as the Response Spectrum Method (RSM). He fully developed the concept in Biot (1933, 1934).

In Biot (1934), on page 213, he states that “any vibration of an elastic undamped system may always be considered as a superposition of harmonics.” Few lines further down, he continues: “…[a] building, like any elastic system, has a certain number of so called *normal modes* of vibration, and to each of them corresponds a certain frequency…we will show that any motion can be calculated when we know these modes of vibration.” On the next page (p. 214), Biot defines F(v), which he calls the *frequency distribution* or the *spectral distribution* of ground acceleration—which in our modern terms is the Fourier amplitude spectrum of ground acceleration. Near the bottom of the same page, Biot states:

> …we are not interested in the motion itself of the building, bur merely in its maximum amplitude. This maximum is the sum of the amplitudes of each separate free oscillation. It will not always be reached because it supposes that an instant exists for which all of the free oscillations have their maximum deflection simultaneously. However, this maximum will many times be nearly reached in a short time, and in any case it is the highest possible value.

(Today, when we discus methods for superposition of modal responses, we refer to this formulation as *Biot’s sum of absolute maxima*—e.g., Amini and Trifunac 1985). Finally, on page 215, Biot discusses the properties of the *spectral distribution*, and mentions Suyehiro’s observations in Japan. He then concludes: “If we possessed a great number of seismogram spectra we could use their envelope as a standard spectral curve for the evaluation of the probable maximum effect on buildings.”

The above summarizes Biot's formulation of the response spectrum method and of response spectrum superposition concept (written succinctly on three pages!). Physically and mathematically it can be argued that it is superior to our modern use of the relative displacement or velocity spectra because it is directly related to the description of ground motion, via the Fourier amplitude spectrum, F(v), of strong motion acceleration. However, for all practical purposes, the modern approximate procedures are equivalent to Biot's formulation because of (1) close agreement between the amplitudes of F(v) and of the true relative velocity spectra, and (2) because the amplitudes of true velocity spectra and pseudo velocity spectra are also practically the same.

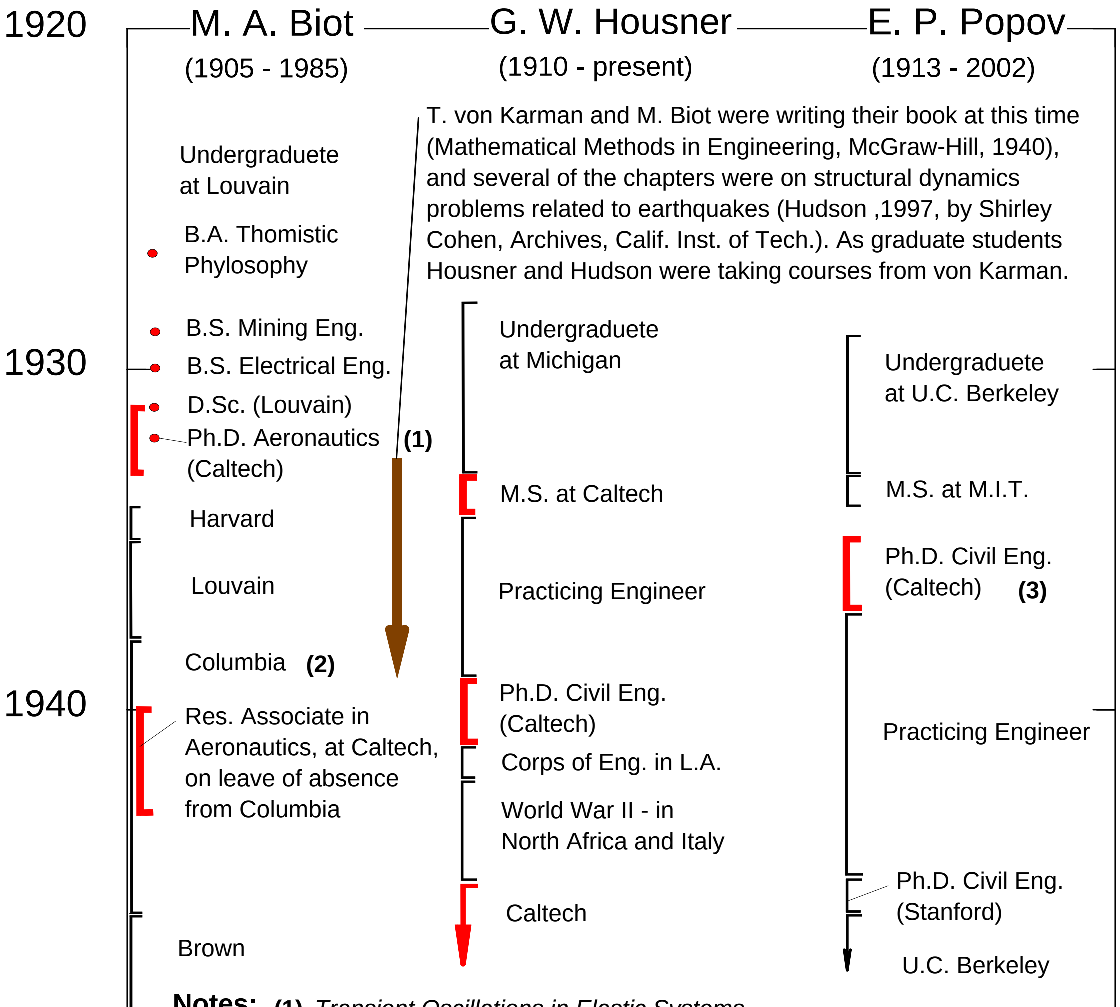
The terminology used in connection with the response spectrum concept has been evolving since 1932. Different scaling functionals were considered and used by those writing the papers on this subject during 1930s (energy spectra, spectral intensity curves, etc.), and others, who worked with the torsional pendulum (Neumann 1936; 1937; Savage 1939) or with the models of buildings on a shaking table (Jacobsen at Stanford), were also doing essentially the same thing—computing the relative displacement response spectra—even when they called it by different names.

Today, Biot is well known to almost everybody working in mechanics, primarily for his contributions to poromechanics (Tolstoy 2006), the theory of folding, and the second-order theory of elasticity (Biot 1965). The year 2005 marked the 100th anniversary of Biot's birth, and papers, special issues of journals, and conferences were organized to celebrate the occasion. An international conference was held in Norman, Oklahoma (May 2005) the *Biot Centennial* (Trifunac 2005)—and a special issue (Vol. 26, No. 6-7) of *Soil Dynamics and Earthquake Engineering* entitled "Biot Centennial—Earthquake Engineering" was published in 2006. While this monograph is being written, the special issue entitled "Response Spectra" of the *Indian Society of Earthquake Technology Journal* (Vol. 44, No. 1, 2007) is in the final stages of printing. This special issue was prepared for the occasion of the 75th anniversary of the response spectrum method and contains fourteen papers, which are all devoted to various aspects of response spectra.

Biot did not write journal papers on earthquake engineering after 1942, although he helped and advised committees working on the development of the design codes (EERI 1996), and he maintained interest and followed the developments in the field. His unpublished discussion, written after the publication of the study by Merchant and Hudson (1962) and recently discovered by his wife Nady Biot among his papers, is reproduced later in this monograph. It shows his keen interest in and exceptional understanding of the subject.

Biot and other students at Caltech in 1930s

Two other future earthquake engineers were among the graduate students at Caltech soon after Biot graduated in 1932 (Fig. 2). G.W. Housner arrived in 1933 and completed his M.S. studies during the 1933/34 academic year, and E.P. Popov started his Ph.D. studies in 1935. Both were students of R.R. Martel in civil engineering. After graduation, Housner became practicing engineer, from 1934 to 1939, and then he returned to Caltech and graduated with a Ph.D. degree in 1941. He briefly worked with Corps of Engineers in



Notes: **(1)** *Transient Oscillations in Elastic Systems* by M.Biot, Thesis No. 259, Aeronautics Dept. Caltech, 1932

(2) Applications of the concept of eathquake spectrum, using Biot's analytical method, have been made by R.R. Martel and M.P.White in 1939 (see p. 367 of Biot, 1942)

(3) Popov left Caltech in 1937 (see EERI (2002). Oral History Series - E.P.Popov, p 39-40.)

Fig. 2. Early careers of the three young earthquake engineers, M. Biot, G. Housner, and E. Popov, covering their undergraduate and graduate educations and the periods they were at Caltech.

Los Angeles before taking part in the World War II, in Northern Africa and in Italy. Housner returned to Caltech as a faculty member in 1945. E. Popov interrupted his Ph.D. studies in 1937 (EERI 2002, pages 39–40) to work as practicing engineer from 1937 to 1945, and then he returned to graduate school at Stanford in 1945. He completed his Ph.D. work in 1946, as the last student of S. Timoshenko (Fig. 2). After graduation, Popov joined the University of California at Berkeley, where he played the key role in creation of one of the leading earthquake engineering departments. He made numerous original contributions to the subject of structural design to resist earthquake forces.

From mid- to late 1930s, Von Kármán and Biot were writing their book *Mathematical Methods in Engineering* (McGraw-Hill 1940), which had several chapters directly applicable to the structural dynamics problems related to earthquakes (Hudson 1997). As graduate students Biot, Housner, Hudson, Popov, and many others all took courses from von Kármán, whose style of teaching, with emphasis on the essential physical nature of the problem, left a strong and enduring impression. In EERI (1997) Housner recalls, "when I started to work on my Ph.D. thesis on the dynamics of buildings, Prof. Martel asked von Kármán about the differential equation for a vibrating beam."

Housner introduces his dissertation as "a continuation of the work done by M.A. Biot and M.P. White." In the first part of his thesis, he reviews the response spectrum method, along the lines formulated by Biot (1932, 1933, 1934), but he places emphasis on a practical engineering viewpoint. Housner also presents the plots of the response spectra (Trifunac 2003) computed from accelerograms, which were available at that time. The last chapter of Housner's thesis deals with the energy flow from an earthquake source. It complements and extends the representation of the earthquake ground shaking problem in terms of wave energy flow, which was previously done only for structures by Sezawa and Kanai (1935, 1936).

Computation of response spectra

Computation of response spectra can begin with the solution of Duhamel's integral (Trifunac and Lee, 1973) and then selection of the maximum response. Prior to the age of digital computers, execution of these tasks was difficult and very time consuming. For example, before the 1940s, direct numerical integration (Martel and White 1939) and semi-graphical procedures using Intergraph instruments (Hudson 1956) were used.

"The first use of a mechanical analyzer for finding oscillator response to an earthquake motion was by Frank Neumann (1936; 1937) of the U.S. Coast and Geodetic Survey in 1936. In this work, the earthquake displacement curve, obtained by double integration of an accelerogram, was used to govern the motion of a torsional pendulum" (see discussion by M.P. White, 1942).

Response spectra were evaluated mechanically at Stanford University. "The acceleration record was integrated twice to give ground displacements. A cam cut in the pattern of these displacements actuated a shaking table upon which a simple oscillator was placed." The maximum relative displacement of such an oscillator multiplied by its natural frequency, ω_n, then gave the required value of pseudo-spectral velocity (Housner 1941a; Hoff 1942).

White and Byrne (1939) suggested a method by which an accelerogram can be used directly to actuate a mechanical analyzer. This principle is the same as the one later employed by Biot (1941, 1942) and Housner (1941a,b).

The first practical method for computation of spectral amplitudes was based on the torsional pendulum analog (Savage 1939; Biot 1941). In this method, an oscillator is represented by an eccentric mass supported by stretched wire, one end of which is forced to twist through angles proportional to the acceleration amplitude, versus time (Biot 1941; Housner 1941a; Alford et al. 1964). The most time-consuming difficulty associated with the use of such a torsional pendulum was the inconvenience of changing the natural period of torsional response. Gross changes in period were made by using torsional wire of different diameters. Fine adjustments were made by selecting the eccentricity of the mass on the inertia bar. Damping was also difficult to control. At first, it was thought to be zero, but later it was discovered to be in the range of a few percent of critical. The damping in the torsional pendulum came from the internal friction of the torsional spring and from air damping of the inertia bar (Alford et al. 1964). With Biot's torsional

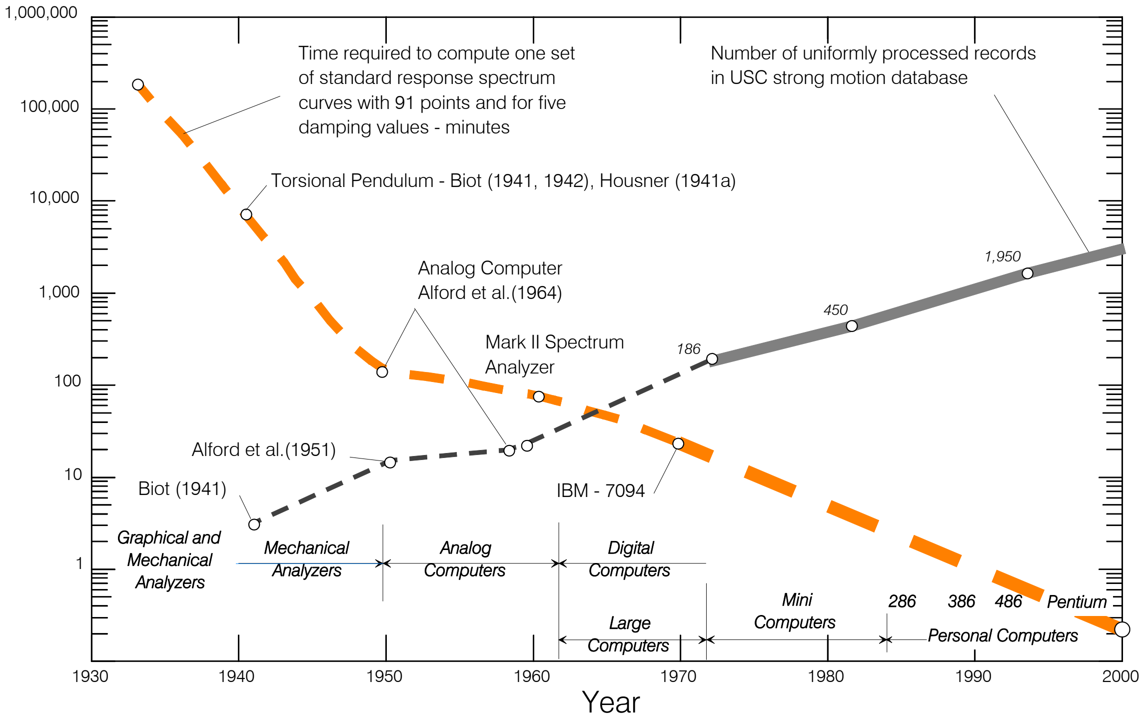


Fig. 3 Time required to compute one set of standard response spectrum curves (in minutes), and the cumulative number of accelerograms in strong-motion databases (light dashed line for the period prior to 1970) and in the uniformly processed strong-motion databases (wide gray line for the period after 1970).

pendulum at Columbia University, it took about 8 hours to construct one spectrum curve consisting of about 30 points (Biot 1942). At Caltech, it took about 15 minutes to construct one spectrum point (Alford et al. 1964). Prorating these durations to computation of spectra at 91 period points for five damping values (Trifunac and Lee 1973) results in a duration of work of about 7,000 minutes (167 hours; Fig. 3).

At the Earthquake Research Institute of Tokyo University, a moving coil galvanometer element was used as the mechanical torisonal system (Takahasi 1953). It had a torsional element with fixed frequency, and the period changes were effected by changing the speed of the film drive mechanism in the ground motion generator. By energy input into the torsional system, through an electrical feedback loop, effective zero damping of the system was possible.

The idea of using analog computers for computation of response spectra can be traced back to 1934: "The direct computation of…spectra might be tedious, but automatic electrical methods can be easily imagined, such as a photographic record passing in front of a photoelectric cell acting upon a tuned circuit" (Biot 1934). This idea was finally implemented, 20 years later, during the 1950s (Alford et al. 1964; Caughey et al. 1960).

In the late 1940s, an analog computer technique was introduced for solving the response of a single-degree-of-freedom system to arbitrary excitation. The significance of the analog computer was that it enabled, for the first time, systematic calculation of response spectra with assigned damping values. It was about 30 times faster than the torsional pendulum analog (Fig. 3). Crede et al. (1954) showed how a commercial electronic differential analyzer could be used for determination of response spectra. Then, a special-purpose spectrum analyzer using electronic operation techniques was described by Morrow and Riesen (1956). Using these ideas, a small special-purpose analog computer system, Mark II, designed for computation of response spectra, was developed in 1954 and tested through the mid-1950s (Caughey et al. 1960). Using this electric analog, response spectra were calculated for a series of strong-motion earthquakes in the western United States (Hudson 1956).

In the early 1960s the methods for computation of response spectra started to change, following the general availability of digital computers. Digitized accelerograms could be used in Duhamel integral, and integration could be performed numerically. Assuming that acceleration data can be approximated by piece-wise, straight-line segments between equally spaced points in time, the Duhamel integral can be integrated exactly over each time interval, thus reducing numerical integration to a sequential application of 2×2 matrices and two 2-component vectors. This required eight multiplications and six additions for each time step, or 14 N operations for an accelerogram defined by N points (Nigam and Jennings 1968).

As shown above, before the digital computer age computation of response spectra of strong-motion accelerograms was difficult and labor intensive, and the results had very uncertain accuracy. This, in combination with a very small number of available recorded accelerograms, made it impossible to carry out empirical studies on the scaling of earthquake spectral amplitudes. Also, it was difficult to explore the governing laws and to link the physical nature of the earthquake source mechanism with the amplitudes and shape of the response spectrum. It was primarily for these reasons that the response spectrum method was confined largely to the realm of academic research for almost 40 years (1932 to ~ 1972).

Response spectrum in design

In his 1934 paper, Biot stated that if a large enough number of seismogram spectra were available it would be possible to use their envelope as a standard spectral curve for evaluating the probable maximum effect on structures. In Biot (1941), he continued: “These standard curves…could be made to depend on the nature and magnitude of the damping and on the location. Although the previously analyzed data do not lead to final results, we…conclude that the spectrum will generally be a function decreasing with the period for values of the latter greater than about 0.2 s. A standard curve for earthquakes of the Helena and Ferndale…for values T > 0.2 s, could very well be the simple hyperbola $A = \frac{0.2g}{T}$, and for T < 0.2 s, A = g (4T + 0.2), where T is the period in seconds and g the acceleration of gravity. This standard spectrum is plotted in Figures [4] and [5]. Whether this function would fit other earthquakes can only be decided by further investigations.”

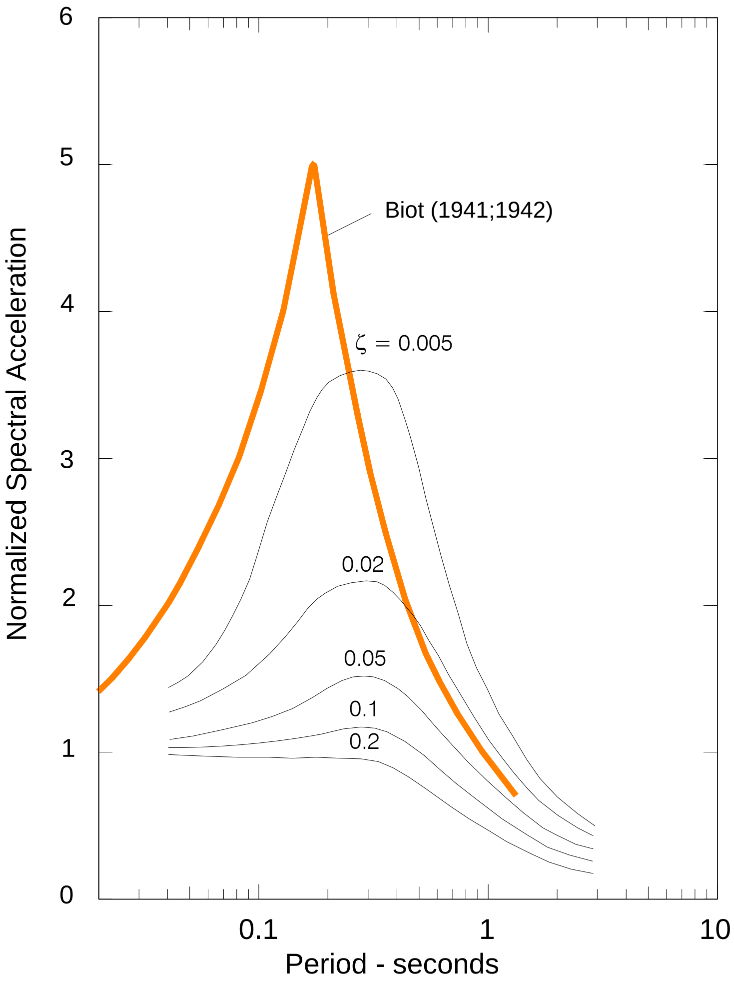


Fig.4 Comparison of Biot (1941; 1942) “standard spectrum” (heavy line) with average spectrum of Housner (1959, 1970), for five damping values (light lines).

Eighteen years later, Housner averaged and smoothed the response spectra of three strong-motion records from California (El Centro, 1934, *M* = 6.5; El Centro, 1940, *M* =

6.7; and Tehachapi, 1952, $M = 7.7$) and one from Washington (Olympia, 1949, $M = 7.1$). He advocated the use of this average spectrum shape in earthquake engineering design (five light lines in Fig. 4, for damping values $\varsigma =$ 0.005, 0.02, 0.05, 0.10, and 0.20; Housner 1959; 1970).

Newmark and co-workers (Newmark and Veletsos 1964; Veletsos et al. 1965) found that the shape of response spectra can be determined approximately by specifying peak acceleration, peak velocity, and peak displacement of strong ground motion. Spectrum shape was further studied by Mohraz et al. (1972) using 14 strong-motion records and by Blume et al. (1972), who analyzed 33 records. The joint recommendations of the Newmark and Blume studies of the shape of the response spectra (Newmark et al. 1973) were later adopted by the U.S. Atomic Energy Commission (now the U.S. Nuclear Regulatory Commission) (USAEC 1973) for use in the design of nuclear power plants (Fig. 5).

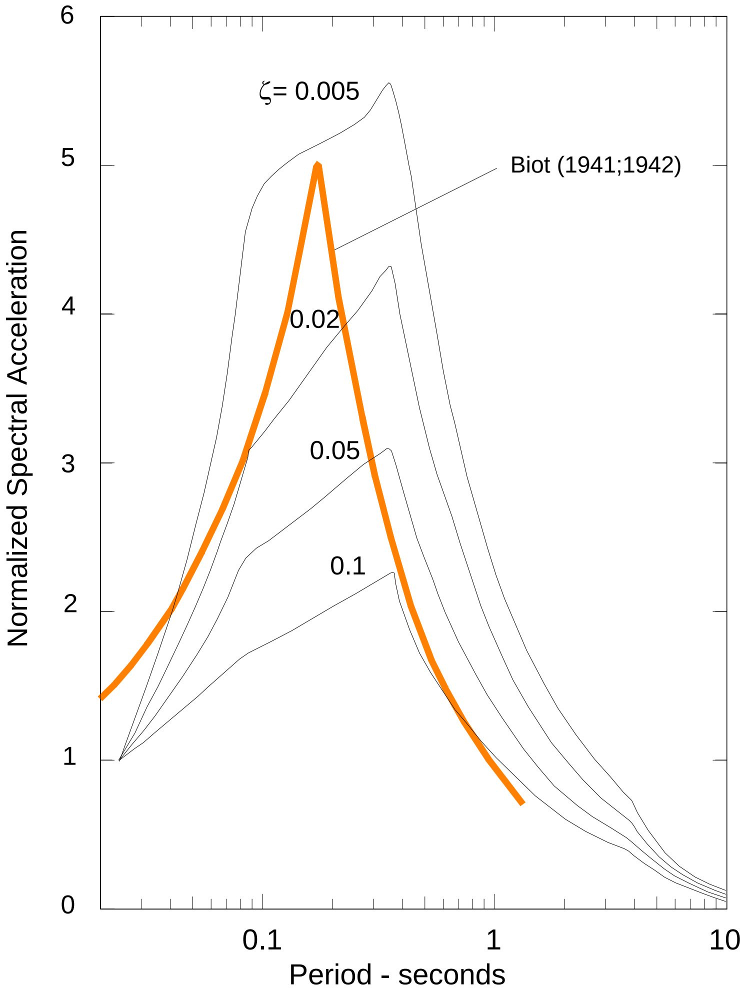


Fig. 5 Comparison of Biot (1941; 1942) “standard spectrum” (heavy line) with the regulatory guide 1.60 spectrum (USAEC 1973) (four light lines for damping values $\varsigma =$ 0.005, 0.02, 0.05, and 0.10).

In engineering design work, the fixed shapes of Housner and Newmark spectra, normalized to unit peak acceleration, were scaled by selecting the “design” peak acceleration. This procedure, which was first systematically used in the design of nuclear power plants, emerged as the “standard” scaling procedure for determination of design spectra in the late 1960s and early 1970s.

The rapid increase in the number of recorded strong motion accelerograms, which started with San Fernando earthquake, in California, in 1971, made possible advanced and more complete empirical scaling of the response spectral amplitudes. Detailed review of this subject is beyond the scope of this presentation, but the reader can find a detailed review of this subject in the work of Lee (2007).

The development of current seismic building code provisions started in the 1950s. A Joint Committee of the San Francisco Section of the ASCE and the Structural Engineers Association of Northern California prepared a "model lateral force provision" based on a dynamic analysis approach and response spectra (Anderson et al. 1952). The Proposed Design Curve, C = K/T, was based on a compromise between a standard acceleration spectrum by Biot (Biot 1941; 1942) and an El Centro analysis by E.C. Robison. The Biot curve for peak ground acceleration (PGA) of 0.2 g has a peak spectral acceleration of 1.0 g at a period of 0.2 seconds. The curve then descends in proportion to 1/T. If the peak spectral acceleration is limited to 2.5 times the PGA, the Biot spectrum is very close to the 1997 UBC design spectrum for a PGA of 0.2 g. The proposed design lateral force coefficient was C = 0.015/T, with a maximum of 0.06 and a minimum of 0.02. These values were considered consistent with the current practice, and the weight of the building included a percentage of live load (Freeman 2007).

Contemporary role of the Response Spectrum Method

As the purpose of this writing is to focus on the early history in the development of the response spectrum method, a review of its modern developments and uses is well beyond the present scope. Nevertheless, for completeness of this presentation, we will mention a few central topics and cite several sources that can help by providing some linkage of this work with the contemporary uses of the method.

An important development that preceded the widespread use of RSM in engineering design was carried out by N. Newmark and his co-workers and students. It introduced a simple, practical procedure, based on the comparative analysis of linear and nonlinear SDOF systems excited by the same strong ground motion record, which enabled simple approximate estimation of the nonlinear response spectral amplitudes for use in design. Implementation of this approach starts with the linear response spectrum amplitudes, which are then multiplied by the reduction factors to yield the nonlinear design spectra. A recent review of this work and of its validity near earthquake faults can be found in the papers by Jalali et al. (2007), and Jalali and Trifunac (2007a,b).

Since mid-1970s, numerous studies of the empirical scaling methods of spectral amplitudes have been developed. This work has typically occurred in cycles, which followed significant increases in the strong motion database after major earthquakes. The first successful scaling equations were developed in mid-1970s with less than 200 strong motion records, but by the mid-1990s there were about 2,000 records (Lee 2007).

Biot viewed the formulation of the standard design spectra as an enveloping process that depended upon the availability of many accelerograms recorded under different earthquake and site conditions (see Biot 1934). This approach was used extensively in numerous projects requiring site-specific design criteria, and it has also been responsible for influencing the spectral shapes used in design codes (Freeman 2007). After the mid-

1970s, with the accumulation of the recorded and processed strong-motion accelerograms (Fig. 3), and following the development of the concept of uniform hazard spectrum in 1977 (Gupta 2007), Biot's concept of searching for envelopes evolved into a process of finding the distribution functions of site-specific spectral amplitudes. After the detailed distribution functions of spectral amplitudes were developed (Lee 2007), it became possible to use the uniform hazard spectrum in both site-specific work and in seismic microzonation (Gupta 2007).

Other developments contributed to (1) better understanding of and refinement in the selection of the peaks of the relative response of structures (studies using order statistics for estimation of the largest peaks—e.g., Gupta and Trifunac 1988), (2) proposals on how to use the response spectrum method in the presence of soil-structure interaction (Gupta and Trifunac 1990), and (3) the generalization of the response spectrum method for long structures, which are sensitive to differential amplitudes of strong ground motion. (e.g., Gicev and Trifunac 2006; Jalali et al. 2007).

As already noted, a special issue (Vol. 44, No. 1, 2007) of the *Indian Society of Earthquake Technology (ISET) Journal* was devoted to the response spectrum method. It contains fourteen papers on various aspects of the use and development of the RSM and can serve as a valuable source for many contemporary references.

Acknowledgements

I am indebted to Mrs. Nady Biot for contributing the two newspaper clippings from June 1932, which describe von Kármán's informal presentation of Biot's research at Caltech, to the American Society of Mechanical Engineers, whose convention was held at Yale, New Haven, in June 1932.

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MAURICE ANTHONY BIOT – A BIOGRAPHICAL SKETCH

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Intuitive ability closely resembles artistic talent.
Biot (1963)

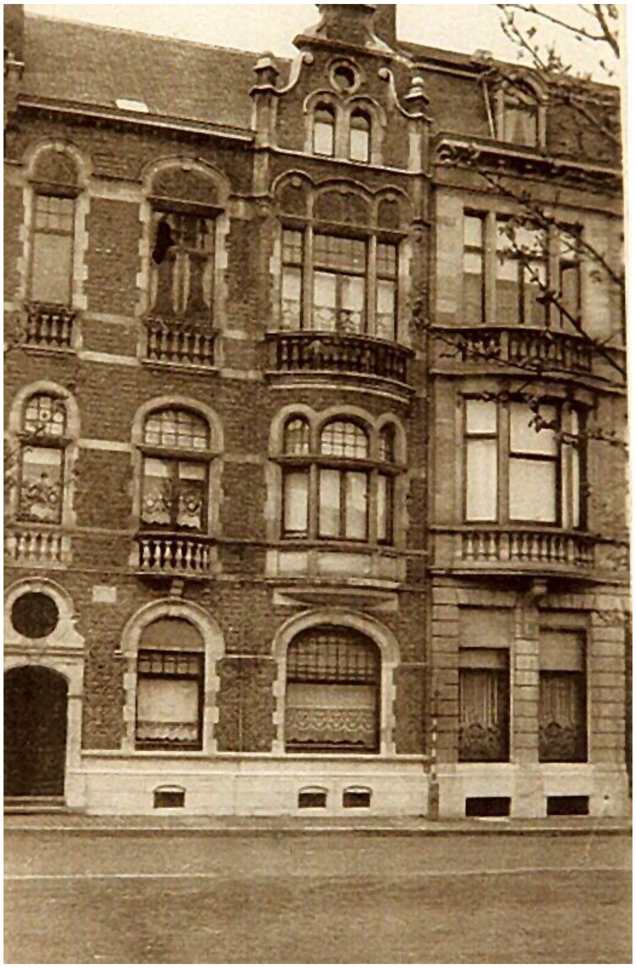
Introduction

The year 2007 marks the 75th anniversary of M.A. Biot's Ph.D. thesis, in which he formulated the concept of response spectrum. This presents us with an occasion to look back at some of the personal aspects of his life. His professional contributions are well known, and have been documented on more than one occasion, but little has been written about him as a person. M. Biot had unusual ability to produce much first-rate research, without students and essentially alone. He also had integrity, courage and spiritual qualities. In these modern times we may consider him as a Renaissance-type scientist, a real humanist, looking for truth. Our hope is that his influence will continue to grow through future generations of scientists and engineers, who recognize his many contributions. This would be the most fitting homage to pay to his memory.

House of birth, Antwerp 1905

Maurice Anthony Biot (or Tony, as he was known to his friends) was born in Antwerp on May 25, 1905. His parents were both from the French-speaking part of Belgium (Wallonia) and were married in Namur in 1900. His father, Arthur Biot, graduated in commercial and consular studies, and his mother, Elise Dethier, was the second child in a family of eight, in which the father, sons, and even the sons-in-law were well-liked physicians in Namur. Maurice had wonderful times during his holidays in the Ardennes whenever he, as the only child, was reunited with his numerous cousins. He stayed in close contact with all members of this warm-hearted family and saw much of them, especially following his return to Belgium from the U.S. in 1970. His parents, who had been married in the south of Belgium, moved to Antwerp in 1900 to set up a wholesale trade. At the beginning of the 20th century, Antwerp was a major trading center, particularly in the field of commodities, and was open to all foreign influences.

The life of the little boy, who was very imaginative and curious about many things, was upset on August 4, 1914 with the outbreak of World War I, as Belgium's neutrality was unscrupulously violated. The family managed to leave Antwerp and find refuge in Richmond, a suburb of London, where their son was immediately enrolled in school. Adaptation comes easily at that age, and Maurice very quickly learned to speak English like his school mates.



Biot House in Louvain

In 1915, the family left London for Paris. France was still at war, but Paris was not occupied by the enemy. This meant another school and fresh faces for Maurice, but it was also the time when he first displayed an interest in geometry. By the second year of the war, life for refugees in Paris had become difficult, and the family was in dire straits financially. An opportunity for his father to move to Chambéry and find a job there was a welcome one. Chambéry was a small town 40 miles from Grenoble, where the Red Cross offered Maurice's father a small job with which he could sustain the family.

The French high school did not suit young Tony well. The best professors had left for the front, and their substitutes taught without really caring about their charges. Tony, who had loved geometry so much the preceding year, lost heart after seeing the poor grades he earned in mathematics. Depression in a child can be severe when no one understands the need for encouragement. Maurice sought refuge in drawing or crafting little objects (for instance, he designed and produced a necklace for his mother, a surprising achievement for a child at that age). Fortunately, his good grades in English and in drawing provided some solace and proved very helpful later as the following anecdote shows.

After America joined the Allied powers in 1917 in their fight against Germany, the wounded soldiers were sent to convalesce in Aix-les-Bains near Chambéry. During their walks, some were astounded and very pleased to see a *little boy drawing landscapes—and speaking English to boot. He soon became their "little Belgian friend." Half in jest, one of the soldiers asked him if he could draw his portrait, and Tony answered yes, without hesitation. As a result, the soldier and his companions offered him a box of colors and were very happy with the results. Today, the portrait of this veteran may well hang on a wall somewhere in America. Although the "little Belgian friend" was only 12 years old in 1917, he would return 28 years later as an American citizen and participate in the liberation of Paris as a U.S. Naval officer.*

The Library, opposite of Biot House in Louvain

After four years, the war finally ended, and the family returned to Antwerp, which had suffered severe artillery bombardment, and they resettled in their house, which, despite looting, was still habitable. Maurice enrolled in the Jesuit high school, where the good pedagogues soon spotted the high potential of their new pupil. Fear, after the painful Chambéry experience, was forgotten—people had trust in him again. As Maurice pursued the Greek and Latin high school curriculum, Greek thought and poetry filled his need for the beautiful, and this became a lifelong influence. He also earned special awards, one for mathematics and another for physics. Music likewise held special attraction for him, especially through the organ concerts at the Antwerp cathedral, and he resumed his piano studies, which had been interrupted by the war. The pace of life then gave him ample time to look deeper into what was taught him.

Belgian Military Service (Air Corps), 1931

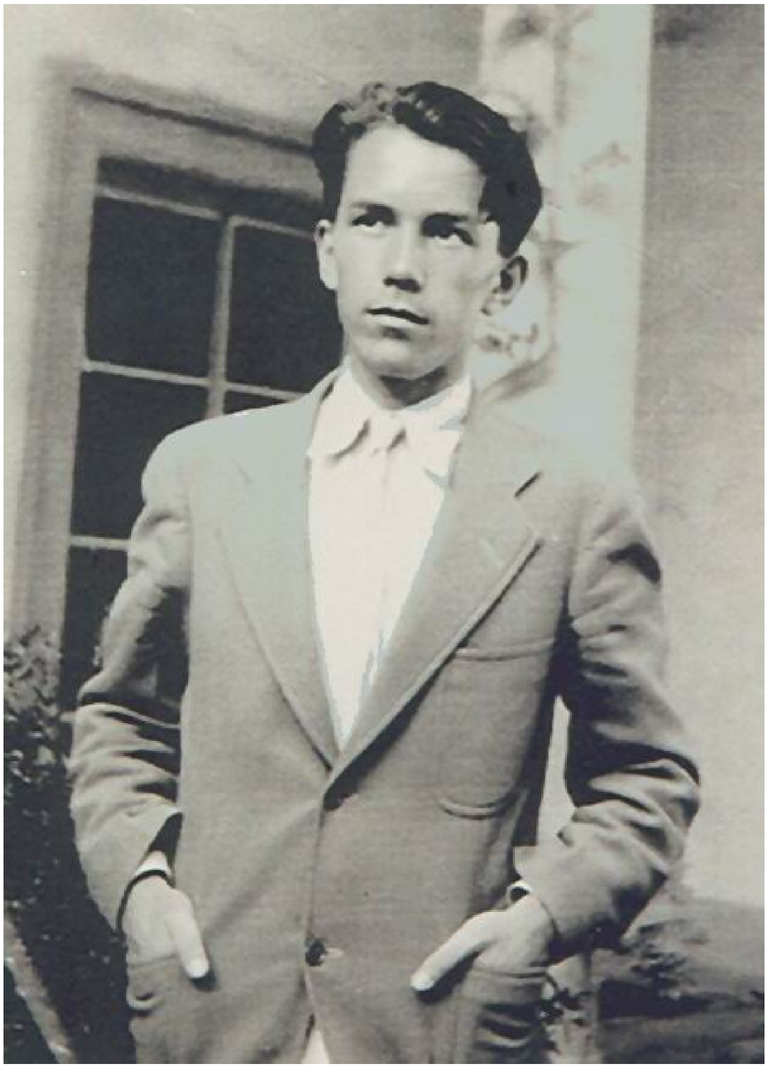
Later, he took a Sciences Preparatory year in Brussels to prepare for the entrance examination at the University of Louvain. This examination, which frightened him at first, was a success and, as he later explained, the thought came to him that "now, at last, at the University, I will have all the answers to so many questions I have been asking myself." And many answers he did receive, thanks to the excellent professors to whom he has always felt truly grateful.

He worked hard and with eagerness, and he earned degrees in mining engineering and electrical engineering, while concurrently completing a full curriculum in economics. These studies brought him technical mastery in mathematics and taught him the beauty of geometry and physics—very good tools indeed. Still, these did not completely satisfy him, for he was always concerned with the philosophical issues raised by science. Accordingly, he also attended classes in Thomistic Philosophy.

His parents retired from the business and moved to Louvain in 1924, a major university town at the time. They bought a house, which provided their son with a view from his study of the remarkable university Library, which had been burned down during the war but had later been rebuilt thanks to funds collected in the American schools. During the Second World

War, the library was once more destroyed and later rebuilt thanks to the gifts of another generation. Today, memorial plaques honoring numerous American states bear witness to this generosity.

Maurice had many student friends. He was an amusing companion who could show genuine friendliness. Often the students would gather at the "Biot house" to study and to review their work. Later in life, they often commented that "Biot made things clearer than the teachers themselves could." Throughout his life, he was neither vain nor pretentious, but he was a born teacher with an exceptional pedagogical talent.



At Caltech in 1931

During his studies at Louvain, Maurice published his first article titled "Modern Hydrodynamics and Aerodynamics." He was happy, as he felt he had found his own way. In 1930, he patented his article "Radio-guidance Systems for Ships and Aircraft."

Military service was mandatory in Belgium in those years, and Maurice enlisted in the Belgian Air Corps on August 18, 1930. At that time, Belgium only had a few biplanes. During his leisure time, he prepared his doctoral thesis, and on July 22, 1931 he was given a one-day special furlough to defend it — wearing his uniform. Undoubtedly this was on the strange side for the distinguished dons, who wore their gowns. Nevertheless, the result was that Tony graduated magna cum laude—he was now a Doctor of Sciences, Physics, and Mathematics. His military service ended on August 18, 1931, when he was 26 years old. He was on the eve of the turning point in his life.

The Way to America

Maurice was awarded a two-year fellowship from the Belgian American Educational Foundation[1], and September 1931 saw the great departure from Antwerp on the SS Belgienland. There were a dozen other graduates in the group, from various disciplines, and by all accounts they had a merry crossing.

At Professor von Kármán’s House.

Tony often recalled later their arrival at dawn, as they peered into the fog to catch sight of the Statue of Liberty and the tip of Manhattan. Their hearts were beating a little faster as the steamer slowly sailed into the harbor. How different are travels nowadays...

Life in the New World seemed wonderful. After a few days in New York, he was ready

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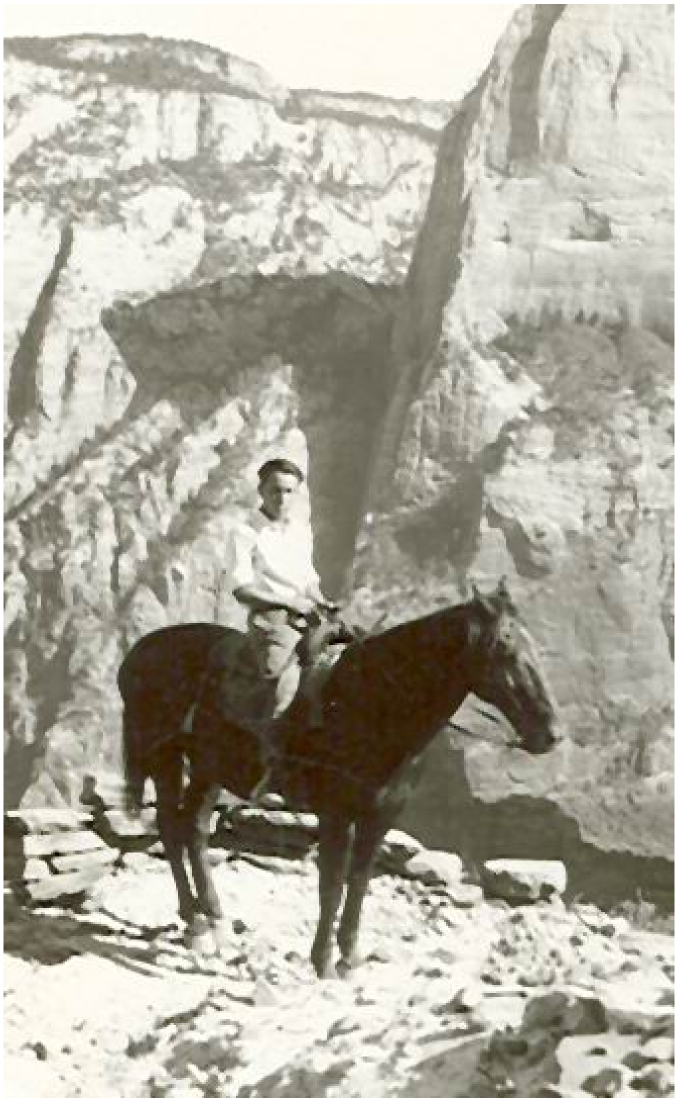
[1] Herbert Hoover, the late U.S. President, set up for Belgium, which had been ravaged during World War I, the Commission for Relief of Belgium. Four Belgian universities received funds, and in 1920 the first ship took 24 Belgian students to the United States. So far, 2,500 students have benefited from fellowships at the prestigious American universities.

for the long train trip to Southern California. In Pasadena, he was welcomed at the California Institute of Technology and stayed in the Athenaeum. The next day, he met Theodore von Kármán, who headed the Guggenheim Aeronautical Laboratory. Von Kármán was a great and beloved teacher. The clarity of his lectures and the stimulating discussions between the Professor and his students, often at his own house, were unforgettable to Maurice. He liked to recall a number of anecdotes from this period. One of them took place in 1932.



In Zion National Park

Einstein and von Kármán were involved in an animated discussion about earthquakes (not infrequent in Pasadena), when suddenly a shock was felt. It was a small one, to be sure, but the students were quite surprised when they realized, from the answers the two professors gave to their worried questions, that the shock had gone past them completely unnoticed: "A shock ... when?"

It was a perfect environment, in which Blot worked with enthusiasm. Theodore von Kármán was at all times a benevolent and stimulating advisor. In June 1932, Biot defended his American Ph.D. dissertation, "Theory of Elastic Systems Vibrating Under Transient Impulse With Application to Earthquake-Proof Buildings," in which he formulated the "response spectrum" concept. During the latter part of that summer, Maurice took part in a session at the University of Michigan, like so many other students from across the U.S., to learn the new applications of mechanics from great professors like Timoshenko, Southwell, and others.

Thanks to his fellowship grant, Maurice was able to explore the Los Angeles area and the national parks of the Southwest. He was also able to buy a small Chevrolet automobile, of which he was very proud. Together with two friends, he embarked on a journey to the North, during which they visited over ten states. In those years, traveling was an adventure, where you had to rough it, but it was also an experience that you would not forget. There were more than 300 pictures to tell about this tour and Tony loved seeing them again and showing them to his friends.

Catalina Island (summit)

In March 1933, Maurice was a Graduate Fellow at Caltech (his term finishing in August). The occurrence of the Long Beach earthquake was of major interest to him, and he visited the epicentral area and photographed the damaged buildings.

Damage caused by March 10, 1933, Long Beach Earthquake (photo by M. Biot)

Damage caused by March 10, 1933, Long Beach Earthquake (photo by M. Biot)

In 1934, having returned to Europe, Tony took a study trip subsidized by the National Belgian Foundation. Over several months, he visited the universities of Delft, Zurich, Cambridge, and Gottingen. His stay in Gottingen was especially productive, but trouble was brewing, even at this university, because of the rise of Nazism in 1933.

Washington 1943

Between 1934 and 1940, Maurice initiated his work on the "Non-Linear Theory of Elasticity," with the basic theory being developed in seven papers. Twenty-five years later, his monograph "Mechanics of Incremental Deformation" would embody this theory with new developments and applications, many of them appearing for the first time.

Toward the end of 1934, he crossed the Atlantic again because he was expected at Harvard, where he became an instructor in the Graduate School of Engineering, in the same department where his friend J. den Hartog taught.

Then, in 1936, the University of Louvain offered him a professorship. For a year, he taught Analytical Mechanics in French and the Theory of Electricity in Dutch. The workload was enormous and left him little opportunity for research.

In September 1937, Maurice was appointed Assistant Professor of Mechanics at Columbia University, where he taught Analytical Dynamics, Elasticity, Hydrodynamics, Vibration Theory, and Aerodynamics. Because Theodore von Kármán regularly shuttled between New York and California, they had the opportunity to work on the textbook, *Mathematical Methods in Engineering* (von Kármán and Biot, 1940). This book was later translated into nine languages. It was also at Columbia that he published the paper "A Mechanical Analyzer for the Prediction of Earthquake Stresses." It presented an application of the general method Tony had developed at Caltech in 1932.

World War II

Europe was being confronted with the opening tragedies of World War II, and in the spring of 1940 hostilities took a turn for worse. Belgium was invaded, and its armies collapsed in 18 days under the overwhelming power of the German aggressor. Twenty days later, France surrendered. Maurice was terribly distressed by the tragedy that had befallen Europe, and he faced difficult choices. Going back to Belgium seemed both impossible and useless.

A little earlier, in 1938, he had applied for the U.S. citizenship. America was at peace, and life there continued as normal. For many, the rallying cry was "keep the U.S. out of the war." For his part, Maurice was already well aware of the enormous threat to world peace and was very worried. In September of 1940, Robert Millikan, the president of Caltech, requested that Columbia grant a leave of absence to Maurice Biot so that his knowledge of vibration problems and high-speed aircraft might be utilized. Caltech had undertaken these researches under a contract with the Office of Scientific Research and Development (OSRD), and Biot was put in charge of teaching and research. During this time, he developed a three-dimensional theory of aircraft flutter that was later patented as the "flutter predicting apparatus."

With the Members of the Combined Field Team in Europe

In August 1941, Tony became a U.S. citizen. Then, on December 7, 1941 Japan attacked the United States at Pearl Harbor, which meant that America was at war. There was a torrent of activity, as the entire country joined in the war effort in a remarkable display of patriotism. Tony immediately decided to enlist, while continuing to pursue his research. He was assigned to the U.S. Navy and was sent to Key West, Florida, where his hydro-

The Central Park West (New York)

mechanical knowledge was needed for work on anti-submarine warfare. In 1943, as a Lieutenant Commander, he headed the Structural Section of the Bureau of Aeronautics in Washington.

As a member of the Office of Strategic Services (OSS), he assisted in London in preparing the target lists against German research activities. In 1944, he operated as a member of the Combined Advanced Field Team (CAFT) as the U.S. army advanced through Southern Germany. His mission was to gather scientific intelligence on aircraft and missile technology. He was mainly responsible for the capture of a number of Research Institutes in working order, complete with files and personnel. At that time, many scientists, including Biot, were on the front line and risked their lives to bring back information and equipment for the Allies.

During the war years, the flow of mail between the U.S. and Belgium had been interrupted or delayed. Because his last letter to his parents was dated August 26, 1940, they were led to believe that their son had been safely teaching at Columbia, far from the war zone.

Biot's father died in Brussels in 1941, and Maurice could not even write to his mother. In fact, it was only through the condolences sent to him via the Red Cross by one of his cousins, who happened to be in India, that he was apprised of his father's death.

On D-Day, June 6, 1944, Maurice was in London. He crossed the English Channel and joined the U.S. 6th army for the Allied push to the outskirts of Paris. On August 25, the initial entry into Paris was made by French troops, as the U.S. troops stood for a moment by the side of the road. The boundless elation of the liberated population left Tony with memories that he would often recall with undiminished emotion. The ALSOS mission (code name for military and scientific intelligence team) was headquartered in Paris, and Tony frequently shuttled between London and France to prepare future missions.

On September 5, Brussels was liberated by British troops. Tony obtained a short special furlough from Eisenhower's Headquarters, and alone he barreled down the road to Brussels with his jeep filled with supplies and medicines. Words cannot describe his mother's emotion and amazement to suddenly see her son in a US uniform. Together, they went to the cemetery to put flowers on his father's grave. Tony’s mother must have been quite a sight, sitting straight next to her ever astonishing son in a jeep crossing the city still cheering all liberators.

In America (1945-1970)

The war was not over in Europe and Tony returned to the task force and their mission of intelligence gathering about enemy’s scientific endeavors. On May 8, 1945, the war finally ended. Back in America, Maurice went through a period of personal grief and loneliness, but eventually his spirits rose again, and he got over it to continue his creative work.

He became a faculty member at Brown University in 1946. Two years later, Tony met Ivan Tolstoy, and their friendship extended over a period of forty years. He also made regular trips to Europe to visit his mother, which gave him the opportunity to meet Theodore von Kármán, who often stayed in Paris.

During this time, Maurice worked under a contract with the Cornell Aeronautical Laboratory on non-stationary aerodynamics and aero-elasticity in thin supersonic wings. In 1950, he devoted himself to basic research under the sponsorship of Shell Development. Then, in 1952, he was at his mother's deathbed in Brussels.

Avenue Paul Hymans, Brussels

Tony very much loved New York and settled there at Central Park West. He often went for walks in Central Park or spent time in the Metropolitan Museum, which he particularly liked. He said that most of his novel ideas came to him as he walked. We can only surmise how many of his theories came to light around the Central Park Reservoir. As he would say, his ideas were "in stand-by," and the irrational process of discovery was blossoming. In time, he wrote them down very quickly.

His Steinway piano held a large place in his life. He loved and played Bach, Beethoven, and Brahms, but he did not shy away from contemporary music, which he willingly played at parties for family and friends. Improvisations, played by ear, were his favorite way of expressing his joys and his sorrows. Painting was also a true source of joy for him. He regularly attended the Academy of Painting in New York, where he tirelessly practiced to improve his technique, and he continued to draw enormous pleasure from his artistic endeavors.

Tony loved the Italy of the Renaissance and often traveled to Tuscany and to other European countries, and he also spent some time in Latin America. He was an avid reader and was very interested in the Orient, where he unfortunately never had a chance to travel. However, he attended the conferences at the Asia House, in New York, with great interest. The many books in his personal library on oriental wisdom and Buddhism were a great source of inspiration for him. In addition, one of his pleasures was reading history books and poetry, as well as biographies. His choice of reading matter bore witness of his eclecticism and to his enthusiasm for new ideas.

Avenue Paul Hymans, Brussels

At the same time, Maurice was concerned about how slowly his ideas were gaining acceptance. As often in his life, he made up his mind in a determined way and decided that the best course would be to summarize many of his papers into a monograph form. In 1965, he produced the remarkable book *Mechanics of Incremental Deformation* (Biot, 1965), which included a part of his theory of the folding of stratified geological formations. He also traveled often to Europe, where he delivered numerous lectures on his ideas.

Return to Belgium

In 1970, Tony left New York to settle in Brussels, Belgium. The main reason for leaving America was to be close to his family in his final years. However, these years were in no sense a period of retirement. He maintained a high level of activity and creative work, devoting time to research and writing. As a consultant for Mobil research, he took trips to the U.S. twice a year, and he was always pleased to go back to America, where he had lived for over 40 years. He would stay for a few days in New York or in California, where he was able to meet again with his many friends.

In Belgium, he was a foreign member of the Royal Academy of Arts and Sciences; the monthly meetings were always interesting to him whether the debates addressed scientific, literary, or artistic matters. His beloved Steinway piano had followed him across the Atlantic, and he played often, even during his final years.

Young people always enjoyed meeting him. He never weighed in heavily with his views, but he always managed through his questions to lead them to a better understanding of their own questions. He understood the problems of youth, and in return they greatly admired and loved him. Tony was brilliant in conversation and was an amusing and articulate polemicist interested in all events, great and small, whether they were social, political, or cultural.

He liked his family and his friends, who appreciated his sense of humor and his kindness. He was a warm and sensitive human being.

His mind remained clear to the very end. On September 12, 1985, after writing a formula and talking to his wife with a smile, he died as he had lived, with dignity. He survived in the heart of those who loved him and through his numerous achievements that are accessible now throughout the world.

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ON THE CONCEPT OF RESPONSE SPECTRUM IN ENGINEERING SEISMOLOGY*

by

Maurice A. Biot

Central Park East, New York, N.Y.

Problems of engineering seismology have received considerable attention in recent years in the technical and scientific literature. In this development, the concept of response spectrum has played a central role. In the light of some recent work (Hudson 1962, Merchant and Hudson, 1962) it is of interest to re-emphasize some of the fundamental ideas, which have led to the development and use of the concept of response spectrum in earthquake-proof design. This may be useful and timely since the earlier work points to both limitations and unexplored possibilities, which more recently have had a tendency to be overlooked.

The concept of response spectrum based on the earthquake acceleration record was originally proposed and developed in my doctorial dissertation (Biot, 1932). This work contained a detailed physical and theoretical discussion of the nature of the spectrum along with numerical tables and procedures applicable to specific design problems. It was shown how the concept of response spectrum leads to drastic simplification in the analysis of very complex structures by representing the system in terms of a small number of fundamental modes of oscillation and treating each mode as a single degree of freedom system. A further simplification in this approach was provided by a separation of the response calculation into two simple parts, one embodying the essential features of the earthquake exciting force, the other representing the relevant dynamic properties of the structure. Suggestions were included for the practical evaluation of the response spectrum by means of tuned devices and, in particular, I mentioned as an example for this purpose, the use of a tuned electric circuit which has now become a standard procedure as part of electric analog methods.

This development was presented in a lecture, which I gave at the June 1932 meeting of the Seismological Society of America at the California Institute of Technology and published in two subsequent papers (Biot, 1933; 1934). As explicitly stated in these papers, the interest of the designer lies in evaluating maximum structural amplitudes and this leads to various approaches based on the spectrum concept, which for all practical purposes are equivalent although they may differ slightly in their formal definitions (Hudson, 1962). As mentioned in the paper (Biot, 1934), one may use, for example, a truncated Fourier spectrum or measure the response of a tuned electric circuit.

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[*] Discussion communicated posthumously by M.D. Trifunac

These concepts were essentially different from the stochastic approach, which was already in use at the time (in 1930) in electronics in connection with noise problems. It should be remembered that the response of an undamped oscillator to a random steady state excitation does not reach a steady state, but increases indefinitely as time goes on. It is only when damping is introduced that a steady state statistical response is reached. In engineering seismology, the problem is quite different. The transient character of the excitation is an essential factor in limiting the amplitude response. Also it should be kept in mind that the earthquake excitation is a mixture of randomness and coherence and that we are interested, not in averages, but in peak values of the response. The concept of response spectrum, which I introduced, was especially conceived for this purpose. In a later year, Benioff (1934) also discussed what are essentially some straightforward applications of the concept of response spectrum as already suggested earlier.

The results obtained by Merchant and Hudson are of considerable interest and confirm the significance of a design procedure, which I had originally proposed in the aforementioned papers, as are applications of the concept of response spectrum, and is based on adding the absolute values of the mode responses. Use of this procedure leading to the slightly modified one applying weight factors to each mode seems to provide a simple and flexible method of deriving safe criteria and at the same time avoids over-design.

My own approach to the problem was motivated by the thought that peak responses are extremely sensitive to phase differences. On the other hand, an evaluation of the phase differences will generally be very unreliable since they are also very sensitive to slight changes of earthquake excitation and structural rigidities. It must be kept in mind that during the earthquake itself, a weakening or incipient failure of the structure may induce a change in relative frequency with drastic results in the peak response. Such incipient failures in high intensity earthquakes are by no means excluded in safe design, since they may affect non-structural components, which contribute mainly to the rigidity and not the safety. To overlook this point may result in uneconomical over-design. This amounts to stating that a certain amount of uncertainty and randomness arises, not only from the earthquake excitation but also from the particular properties of the structure itself. For this reason, I suggested the use of spectral envelopes (Biot, 1933;1934) obtained by analyzing a family of typical earthquakes.

The technological context of earthquake-proof design is fundamentally different from that encountered in the electronics of communication where noise and information are of primary interest. It also differs from the problem of fatigue failure of structures under prolonged random excitation. We have here an excellent example of the caution to be exercised when transposing concepts and techniques from one area of technology to another.

The paper of Merchant and Hudson (1962) is an important contribution toward the formulation of reliable and practical design criteria in engineering seismology.

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POST SCRIPTUM

by

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In August of 2006 I received from Mrs. N. Biot a set of notes, which she felt might help to describe M. Biot’s continued interest in Earthquake Engineering. Among these notes was this unpublished discussion, which he probably wrote in early 1960s, after the paper by Merchant and Hudson appeared in 1962. Mrs. Biot found this note among her husband’s papers, after his death in 1985. It is not clear if this was intended as a draft of a formal discussion of Merchant and Husdon paper, which Biot may have considered publishing in the Bulletin of the Seismological Society of America, or as a more general discussion on the subject of the response spectrum superposition method. Because this discussion describes his views on the contemporary developments in the response spectrum method, I felt that it would be of interest to publish it, especially in the context of this monograph, which is written on the occasion of the 75th anniversary of the response spectrum method. The discussion is also of interest because it shows how Biot viewed the emerging stochastic approach to the response estimation.

PUBLICATIONS OF M. A. BIOT

During his remarkably productive career, spanning the fifty-seven years from 1928, when he was an engineering student at Louvain, until his death in 1985, M.A. Biot was the author of 179 published papers (Appendix A) and three books (Appendix B). Biot's publications cover an unusually broad range of subjects, and continue to make an extraordinary impact in science and engineering, as evidenced by the number of citations that they continue to receive (Trifunac, 2005). The principal subject areas to which Biot contributed can be summarized briefly as follows.

Earthquake Engineering (papers 10, 20, 41, and 45). Biot started to develop the concept of the response spectrum, based on recorded earthquake accelerations, in his doctoral dissertation of 1932. He proposed the use of spectral envelopes, based on a family of recorded accelerograms, and developed engineering design procedures using a 'standard' design spectrum and an associated spectrum superposition method (Trifunac, 2003).

Porous Media (e.g. papers 40, 42, 43, 44, 55, 57, 60, 61, 63, 68, 97, 99, 109, 118, 139, 142, 150, 155, 157, 160 and 161). From 1941, when he wrote his first fundamental paper on consolidation, Biot worked on a general and systematic theory to describe a porous solid containing a viscous fluid. Twenty-one of his papers on this subject have been collected in a book, *Twenty-one Papers by M. A. Biot,* edited by Ivan Tolstoy and published by the Acoustical Society of America (1992).

Wave Propagation (e.g. papers 33, 38, 69, 70, 74, 75, 77, 78, 106, 107, 108, 136 and 141). While working on radio-guidance in the mid-1950s, Biot laid the foundation for a theory of rough surface scattering in applied acoustics. In his work with Tolstoy he developed a new approach based on pulse-generated transient waves. Biot also investigated surface waves along a solid-fluid interface and the effect of initial stress on their dispersion.

Aerodynamics, Aeronautics and Fluid Mechanics (e.g. papers 35, 37, 39, 46, 47, 48, 49, 50, 51, 59, 71 and 176). Biot worked on aeronautical problems and fluid mechanics during the 1940s. He developed a three-dimensional aerodynamic theory of oscillating airfoils, patented an electrical analog flutter predictor, and applied his previous ideas on mechanical transients to the design of earthquake-resistant structures, to aircraft landing gear, and to the sound emitted by musical instruments.

Non-linear Elasticity (e.g. papers 17, 31, 32, 33, 34, 36 and 38). Between 1934 and 1940 Biot studied the second-order theory of elasticity, accounting for the effects of initial stress and large rotation. His book *Mechanics of Incremental Deformation* (Wiley, 1965) presented this theory and included many new developments and applications.

Theory of Folding (e.g. papers 72, 79, 81, 83, 84, 85, 90, 91, 92, 102, 110, 111, 113, 115, 116, 117, 118, 119, 120, 124, 125,126, 127, 128, 130, 132, 135 and 144). Biot's interest in the non-linear effects of initial stress and the inelastic behavior of solids led to his mathematical theory of folding of stratified rock. He verified his approach in the laboratory and successfully applied it to explain the dominant features of geological structures.

Thermodynamics (e.g. papers 54, 56, 62, 66, 76, 88, 93, 99, 109, 118, 127, 128, 130, 132, 139, 142, 143, 144, 145, 147, 148,149, 151, 153, 154, 155, 156, 157, 158, 160, 161, 163, 164, 167, 168, 169, 173 and 174). Biot's early work on thermal stresses *(e.g. papers 54, 62, 73, 76, 80, 82, 88, 95, 96, 129, 133 and 134)* developed into major advances in irreversible thermodynamics. These made it possible to establish variational principles which could be used to develop new methods for the solution of problems in heat conduction, diffusion, thermoelasticity, thermoviscoelasticity and chemical reactions. A review of this work was published in *Advances in Applied Mechanics* (Vol. 24, pp. 1-91; Academic Press, New York, 1984). Later Biot presented these developments more fully in a monograph entitled *Variational Principles in Heat Transfer* (Oxford University Press, 1970).

A bibliography of Biot's publications, including the three books, 179 scientific articles, seven Caltech Aeronautical Laboratory Reports, seven Cornell Aeronautical Laboratory Reports, two reports for the U.S. Air Force Office of Scientific Research and Development, four reports for the United States Air Force, and a description of seven patents (in Belgium, France and the United States) has been published as "Notice sur Maurice Anthony Biot," by A. Delmer and A. Janmolte, in the *Extrait de l'Annuaire de l'Académie Royale de Belgique* (Palais des Académies, Brussels, 1990).

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Appendix A

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Appendix B

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